NUMERICAL PROBLEMS

Problem 13.1 At what distance from a long straight wire carrying a current of 10 A is the magnetic field equal to the earth’s magnetic field of $5 \times 10^{-5}$ T?

Solution

$$r = ? \quad i = 10 \, \text{A} \quad B = 5 \times 10^{-5} \, \text{T}$$

Using Ampere’s circuital law

$$B = \frac{\mu_0 i}{2\pi r}$$

$$r = \frac{\mu_0 i}{2\pi B} = \frac{\left(4\pi \times 10^{-7}\right)(10)}{2\pi \left(5 \times 10^{-5}\right)} = 0.04 \, \text{m}$$

Problem 13.2 A long solenoid has 1000 turns uniformly distributed over a length of 0.5 m produces a magnetic field of $2.5 \times 10^{-3}$ T at the center. Find the current in the solenoid?

Solution

$N = 1000 \quad \ell = 0.5 \, \text{m} \quad B = 2.5 \times 10^{-3} \, \text{T} \quad i = ?$

For a solenoid $B = \mu_0 n i$

$$i = \frac{B}{\mu_0 n} = \frac{B\ell}{\mu_0 N} = \frac{(2.5 \times 10^{-3})(0.5)}{4\pi \times 10^{-7}(1000)} = 1 \, \text{A}$$

Problem 13.3 A proton moving at right angle to magnetic field of $0.1 \, \text{T}$ experience a force of $2 \times 10^{-12} \, \text{N}$. What is the speed of the proton?

Solution

$\theta = 90^\circ \quad B = 0.1 \, \text{T} \quad F = 2 \times 10^{-12} \, \text{N} \quad V = ? \quad q = e \, \text{(proton)}$

Force on a charge entering in a magnetic field is

$$F = q \, v \, B \, \sin \theta$$

$$v = \frac{F}{q \, B \, \sin \theta} = \frac{2 \times 10^{-12}}{(1.6 \times 10^{-19})(0.1)(\sin 90^\circ)}$$

$$v = 1.25 \times 10^8 \, \text{m/s}$$

Problem 13.4 An 8 MeV proton enters perpendicularly into a uniform magnetic field of $2.5 \, \text{T}$. Find (a) the force on the proton (b) what will be the radius of the path of proton?

Solution

K.E. = 8 MeV = $8 \times 10^6 \, \text{eV}$

$\theta = 90^\circ \quad B = 2.5 \, \text{T}$

$m \, (mass \, of \, proton) = 1.67 \times 10^{-27} \, \text{kg}$

(a) $F = ? \quad (b) r = ?$

(a) $F = q \, v \, B \, \sin \theta$  \hspace{1cm} (1)

Velocity “$v$” is unknown in above equation. We find it by following way;

Given that kinetic energy $K.E. = \frac{1}{2} \, m \, v^2$

$$v = \sqrt{\frac{2 \, (K.E.)}{m}} = \sqrt{\frac{2 \, (8 \times 10^6)(1.6 \times 10^{-19})}{1.67 \times 10^{-27}}} = 3.9 \times 10^7 \, \text{m/s}$$

Put value of “$v$” in equation (1) =>
\[ F = q v B \sin \theta = (1.6 \times 10^{-19}) \times (3.9 \times 10^7) \times (2.5) \sin 90^0 \]
\[ F = 1.6 \times 10^{-11} \text{ N} \]

(b) The force \( F \) that the magnetic field exerts on proton provides necessary centripetal force given by
\[ F = \frac{m v^2}{r} \]
\[ r = \frac{m v^2}{F} = \frac{1.67 \times 10^{-27}}{(1.6 \times 10^{-11})} = 0.16 \text{ m} \]

**Problem 13.5** A wire carrying current 10 mA experiences a force of 2 N in a uniform field. What is the force on it when current rises to 30 mA?

**Solution**
\[ i_1 = 10 \text{ mA} \quad F_1 = 2 \text{ N} \quad i_2 = 30 \text{ mA} \quad F_2 = ? \]
\[ F = i \varepsilon B \]
For first conductor
\[ F_1 = i_1 \varepsilon B \]
\[ \Rightarrow \varepsilon B = \frac{2}{10 \times 10^{-3}} = 200 \text{ N A}^{-1} \]
Now for second conductor
\[ F_2 = i_2 \varepsilon B \]
Putting value of \( \varepsilon B \) in above equation we get
\[ F_2 = (30 \times 10^{-3})(200) = 6 \text{ N} \]

**Problem 13.6** What is the time period of an electron projected into a uniform magnetic field of 10 mT and moves in a circle of radius 6 cm?

**Solution**
\[ T = ? \]
\[ B = 10 \times 10^{-3} \text{ T} \quad r = 6 \text{ cm} = 6 \times 10^{-2} \text{ m} \]
As magnetic field provides centripetal force to a charged particle
\[ q v B = \frac{m v^2}{r} \] \[ \text{(1)} \]
also
\[ v = r w = r \left( \frac{2\pi}{T} \right) \] \[ \text{(2)} \]
Putting eq (2) in eq (1) and re-arranging them we get
\[ T = \frac{2\pi m}{q B} = \frac{2(3.14)(9.11 \times 10^{-31})}{(1.6 \times 10^{-19})(10 \times 10^{-3})} \]
\[ T = 3.6 \times 10^{-9} \text{ sec} = 3.6 \text{ n.sec} \]

**Problem 13.7** A 0.2 m wire is bent into a circular shape and is placed in uniform magnetic field of 2T, if the current in the wire is 20 mA then find the maximum torque acting on loop?

**Solution**
\[ \varepsilon = 0.2 \text{ m} \quad B = 2 \text{ T} \quad i = 20 \text{ mA} \quad \tau_{\text{max}} = ? \]
\[ \tau = B i N A \cos \theta \]
\[ N = 1 \text{ and for maximum torque } \cos \theta = 1 \]
\[ \tau_{\text{max}} = B i A \]
Area for a wire is \( A = \pi r^2 \)
\[ \tau_{\text{max}} = B i \pi r^2 \] \[ \text{(1)} \]
When wire is bent into circle then its length is equal to circumference
\[ \ell = 2 \pi r \]
\[ r = \frac{0.2}{2 \pi} = \frac{0.2}{2 (3.14)} = 0.0318 \text{ m} \]

Now putting this value of \( r \) in eq (1) above we get
\[ \tau_{\text{max}} = 2 (20 \times 10^{-3}) (3.14) (0.0318)^2 = 1.27 \times 10^{-4} \text{ N m} \]

**Problem 13.8** Full scale deflection for galvanometer is 10 mA. Its resistance is 100 \( \Omega \). How it be converted into ammeter of range 100 A?

**Solution**
\[ i_g = 10 \text{ mA} = 0.01 \text{ A} \quad R_g = 100 \Omega \quad R_s = ? \quad \text{for} \ i = 100 \text{ A} \]
In order to convert a galvanometer into an ammeter, we need a shunt resistor \( r_s \) in parallel with galvanometer given by
\[ R_s = \frac{i_g R_g}{(i - i_g)} = \frac{(10 \times 10^{-3}) 100}{(100 - 0.01)} \]
\[ R_s = 0.01 \Omega \]

**Problem 13.9** How 5 mA, 100\( \Omega \) galvanometer converted to 20 V voltmeter?
\[ i = 5 \text{ mA} \quad R_g = 100 \Omega \quad R_h = ? \quad \text{for} \ V = 20 \text{ Volts} \]

**Solution**
In order to convert galvanometer into a voltmeter of 20 V range we connect a high resistance \( r_h \) in series with galvanometer and is given by
\[ R_h = \frac{V}{i} - R_g = \frac{20}{0.005} - 100 = 3900 \Omega \]

**Problem 13.10** Two parallel wires 10 cm apart carry current in opposite direction of 8 A. What is the magnetic field halfway between them?

**Solution**
\[ d = 10 \text{ cm} \quad i = 8 \text{ A} \quad B = ? \text{ at mid point} \]
At mid-point \[ r = \frac{d}{2} = 5 \text{ cm} \]
Now \( B \) can be determined by Ampere's circuit law
\[ B = \frac{\mu_0 i}{2 \pi r} + \frac{\mu_0 i}{2 \pi r} = \frac{\mu_0 i}{\pi r} = \frac{(4 \pi \times 10^{-7}) (8)}{\pi (5 \times 10^{-2})} \]
\[ B = 6.4 \times 10^{-5} \text{ T} \]