NUMERICAL PROBLEMS

Problem 14.1 Two identical coils A and B of 500 turns each has parallel planes such that 70% of flux produced by one coil links with the other. A current of 6 A flowing in coil A produces a flux of 0.06 mWb in it. If the current in coil A changes from 10 A to -10 A in 0.03 s, calculate (a) the mutual inductance and (b) the e.m.f. induced in coil B.

Solution

\[ N_A = N_B = 500 \quad i_A = 6 \, A \quad \Phi_A = 0.06 \, \text{mWb} \]
\[ \Phi_{\text{eff}} = 70\% \, \Phi_A = 0.70 \times 0.06 \, \text{mWb} = 0.042 \times 10^{-3} \, \text{Wb} \]
\[ i_1 = 10 \, A \quad i_2 = -10 \, A \quad \Delta t = 0.03 \, \text{sec} \]

(a) Mutual inductance is given by
\[ M = \frac{N_B \, \Phi_{\text{eff}}}{i_A} = \frac{500 \times (0.042 \times 10^{-3})}{6} \]
\[ M = 3.5 \, \text{mH} \]

(b) Induced emf produced in coil B due to change in flux in coil A is;
\[ \varepsilon = -M \left( \frac{\Delta i}{\Delta t} \right) \]
\[ \varepsilon = \frac{3.5 \times 10^{-3}}{0.03} \]
\[ \varepsilon = 2.33 \, \text{V} \]

\[ \Delta i = i_2 - i_1 \]
\[ \Delta i = -10 - 10 \]
\[ \Delta i = -20 \, A \]

Problem 14.2 A wheel with 12 metal spokes each 0.6 m long is rotated with a speed of 180 r.p.m in a plane normal to earth’s magnetic field at a place. If the magnitude of the field is 0.6 G, what is the magnitude of induced e.m.f. between the axle and rim of the wheel?

Solution

\[ \ell = 0.6 \, \text{m} \]
\[ r = \frac{\ell}{2} = 0.3 \, \text{m} \]
\[ \omega = 180 \, \text{r.p.m} = 180 \left( \frac{2 \pi}{60} \right) = 6\pi \, \text{rad/sec} \]
\[ B = 0.6 \, \text{G} = 0.6 \times 10^{-4} \, \text{T} \]

Emf produced due to motion of conductor inside a magnetic field is
\[ \varepsilon = B \, \ell \, v \]  
\[ \text{--- (1) ---} \]

but \[ v = r \omega \]
\[ \Rightarrow \]
\[ \varepsilon = B \, \ell \, (r \omega) \]
\[ \Rightarrow \]
\[ \varepsilon = (0.6 \times 10^{-4}) \, (0.6) \, (0.3 \times 6 \times 3.14) \]
\[ \Rightarrow \]
\[ \varepsilon = 2.035 \times 10^{-4} \, \text{V} \]
Problem 14.3 A circuit has 1000 turns enclosing a magnetic circuit 20 cm$^2$ in section with a 4 A current, the flux density is 1 Wb/m$^2$ and with a 9 A current, it is 1.4 Wb/m$^2$. Find the mean value of the inductance between these current limits and the induced e.m.f. if the current falls from 9 A to 4 A in 0.05 s.

Solution

\[ N = 1000 \quad A = 20 \text{ cm}^2 \]

Flux density \[ \frac{\Phi_1}{A} = 1 \text{ Wb/m}^2 \text{ when } i_1 = 4 \text{ A} \]

Flux density \[ \frac{\Phi_2}{A} = 1.4 \text{ Wb/m}^2 \text{ when } i_2 = 9 \text{ A} \]

\[ \Delta t = 0.05 \text{ sec} \]

\[ L = ? \quad \varepsilon = ? \]

Since \[ L = N \frac{\Delta \Phi}{\Delta i} \]  

As \[ \frac{\Phi_1}{A} = 1 \]

\[ \Rightarrow \quad \Phi_1 = 1 \times (20 \times 10^{-4} \text{ Wb}) = 20 \times 10^{-4} \text{ Wb} \]

Also \[ \frac{\Phi_2}{A} = 1.4 \text{ Wb/m}^2 \]

\[ \Rightarrow \quad \Phi_2 = 1.4 \times (20 \times 10^{-4} \text{ Wb}) = 28 \times 10^{-4} \text{ Wb} \]

Change in flux \[ \Delta \Phi = \Phi_2 - \Phi_1 = (28 - 20) \times 10^{-4} \]

\[ \Delta \Phi = 8 \times 10^{-4} \text{ Wb} \]  

(2)

As \[ \Delta i = i_2 - i_1 = 9 \text{ A} - 4 \text{ A} \]

\[ \Delta i = 5 \]  

(3)

Putting eq (2) and (3) in eq (1) we get

\[ L = \frac{1000 \times (8 \times 10^{-4})}{5} = 0.16 \text{ H} \]

Now induced emf is given by the relation

\[ \varepsilon = L \frac{\Delta i}{\Delta t} = 0.16 \times 5 \]  

\[ \frac{0.05}{0.05} = 16 \text{ V} \]

Problem 14.4 A coil of resistance 100$\Omega$ is placed in a magnetic field of 1 mWb. The coil has 100 turns and a galvanometer of 400 $\Omega$ resistance is connected in series with it: find the average emf and the current if the coil is moved in 1/10th s from the given field to a field of 0.2 mWb.

Solution

\[ R = 100 \text{ } \Omega \quad \Phi_1 = 1 \text{ mWb} \quad N = 100 \quad R_g = 400 \text{ } \Omega \text{ (in series)} \]

\[ \varepsilon = ? \text{ & } i = ? \text{ in } \Delta t = 0.1 \text{ sec} \quad \Phi_2 = 0.2 \text{ mWb} \]

According to Faraday's law of electromagnetic induction, induced emf “$\varepsilon$”

\[ \varepsilon = -N \frac{\Delta \Phi}{\Delta t} \]  

(1)

Where \[ \Delta \Phi = \Phi_2 - \Phi_1 = 0.2 - 1 = -0.8 \text{ mWb} \]
Eq (1) => \[ \epsilon = -100 \frac{-0.8 \times 10^{-3}}{0.1} = 0.8 \text{ V} \]

Total resistance in series combination will be \( R + R_g \)

So the current \( i = \frac{\epsilon}{R + R_g} = \frac{0.8}{100 + 400} = 1.6 \text{ mA} \)

**Problem 14.5** A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5 m/s at right angles to the horizontal component of earth’s magnetic field 0.30 \( \times \) \( 10^{-4} \) Wb/m²

(a) What is instantaneous value of emf induced in wire? (b) What is the direction of the emf? (c) Which end of the wire is at the higher electrical potential?

**Solution**
\[ \ell = 10 \text{ m} \quad \nu = 5 \text{ m/sec} \quad B = 0.30 \times 10^{-4} \text{ Wb/m}^2 \]
\[ \theta = 90^\circ \] (Induced emf in a conductor is maximum when it moves perpendicular both to its own length and to the B field)

(a) \( \epsilon = ? \)  
(b) Direction of \( \epsilon = ? \)  
(c) Which end is at high potential?

(a) Motional emf in a conductor moving in a magnetic field is
\[ \epsilon = B\nu \sin \theta \]
\[ \epsilon = (0.30 \times 10^{-4}) (10)(5) \sin 90^\circ \]
\[ \epsilon = 1.5 \times 10^{-2} \text{ V} \]

(b) Applying Fleming Right hand rule shows that direction of this induced emf is from east to west.

(c) The end towards east will be at higher electric potential.

**Problem 14.6** Current in a circuit falls from 5 A to 0 A in 0.1 s. If an average emf of 200 V induced, give an estimate of the self-inductance of the circuit.

\[ i_1 = 5 \text{ A} \quad i_2 = 0 \text{ A} \quad \Delta t = 0.1 \text{ s} \quad \epsilon = 200 \text{ V} \quad L = ? \]

**Solution**
\[ \epsilon = -L \frac{\Delta i}{\Delta t} \]
\[ \Rightarrow \]
\[ L = -\epsilon \frac{\Delta t}{\Delta i} = -\frac{200 \times 0.1}{-5} = 4 \text{ H} \]

**Problem 14.7** A long solenoid with 15 turns per cm has a small loop of area 2 cm² placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from 2 A to 4 A in 0.1 s, what is the induced emf in the loop while the current is changing?

**Solution**
\[ n = 15 \text{ per cm} = 1500 \text{ per meter} \]
\[ \Delta A = 2 \text{ cm}^2 = 2 \times 10^{-4} \text{ m}^2 \]
\[ \Delta i = 4 - 2 = 2 \text{ A} \]
\[ \Delta t = 0.1 \text{ sec} \]
\[ \epsilon = ? \]

Magnetic field inside a current carrying solenoid is
\[ \Delta B = \mu_0 n \Delta i = (4 \times 3.14 \times 10^{-7}) (1500) (2) = 3.77 \text{ mT} \]

So emf \( \varepsilon \) for this single loop (\( N = 1 \)) will be

\[ \varepsilon = \frac{\Delta B \cdot A}{\Delta t} = \frac{(3.77 \times 10^{-3})(2 \times 10^{-4})}{0.1} = 7.54 \times 10^{-6} \text{ V} \]

**Problem 14.8** A rectangular wire loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 T directed normal to loop. What is emf developed across the cut if velocity of loop is 1 cm/sec in a direction normal to (a) longer side (b) shorter side of loop? For how long does induced voltage last in each case?

**Solution**

Length \( l = 8 \text{ cm} = 0.08 \text{ m} \)

Width \( b = 2 \text{ cm} = 0.02 \text{ m} \)

\( B = 0.3 \text{ T} \)

\( \theta = 90^\circ \)

\( v = 1 \text{ cm/s} = 0.01 \text{ m/s} \)

(a) emf \( \varepsilon = ? \) for Longer side  

(b) emf \( \varepsilon = ? \) for Shorter side  

\( \Delta t_1 = \Delta t_2 = ? \)

(a) Emf developed due to the motion of wire loop inside magnetic field is

\[ \varepsilon = B \cdot l \cdot v \sin \theta \quad (l \text{ for longer side}) \]

\[ \varepsilon = (0.3 \times 0.08 \times 0.01) \sin 90^\circ \]

\[ \varepsilon = 2.4 \times 10^{-4} \text{ V} \]

As

\[ \varepsilon = N \frac{\Delta \Phi}{\Delta t} \]

re-arranging it, we get

\[ \Delta t_1 = N \frac{\Delta \Phi}{\varepsilon} = \frac{1 \times 4.8 \times 10^{-4}}{2.4 \times 10^{-4}} = 2 \text{ sec} \]

(b) Similarly emf developed due to motion of wire loop inside magnetic field

\[ \varepsilon = B \cdot b \cdot v \sin \theta \quad (b \text{ is used for shorter side}) \]

\[ \varepsilon = (0.3 \times 0.02 \times 0.01) \sin 90^\circ = 0.6 \times 10^{-4} \text{ V} \]

As

\[ \varepsilon = N \frac{\Delta \Phi}{\Delta t} \]

\[ \Rightarrow \Delta t_2 = N \frac{\Delta \Phi}{\varepsilon} = \frac{1 \times 4.8 \times 10^{-4}}{0.6 \times 10^{-4}} = 8 \text{ sec} \]

**Problem 14.9** A 90 mm length of wire moves with an upward velocity of 35 m/sec between the poles of a magnet. The magnetic field is 80 mT directed to the right. If the resistance in the wire is 5 m\( \Omega \) what are the magnitude and direction of the induced current?

**Solution**

\( l = 90 \text{ mm} = 0.09 \text{ m} \)

\( v = 35 \text{ m/sec} \)

\( B = 80 \text{ mT} = 80 \times 10^{-3} \text{ T} \)

\( \theta = 90^\circ \)

\( R = 5 \text{ m}\Omega = 0.005 \text{ \Omega} \)

\( i = ? \) (Magnitude and direction)
Using equation for motional emf is
\[ \varepsilon = B \ell v \sin \theta \]
\[ \varepsilon = (80 \times 10^{-3}) (0.09) (35) \sin 90^\circ = 0.252 \text{ V} \]
The current is
\[ i = \frac{\varepsilon}{R} = \frac{0.252}{0.005} = 50.4 \text{ A} \]

Direction of the current, according to Fleming Right hand rule, is into the plane of page (see figure).

**Problem 14.10** A pair of adjacent coils has mutual inductance of 1.5 H. If current in one coil changes from 0 to 20A in 0.5 s, what is change of flux linkage with other coil?

**Solution**
\[ M = 1.5 \text{ H} \]
\[ i_1 = 0 \quad i_2 = 20\text{A in } \Delta t = 0.5 \text{ sec} \]

Change in current \( \Delta i = i_2 - i_1 = 20\text{A} \)

From Faraday's law of electromagnetic induction, magnitude of induced emf
\[ \varepsilon = N \frac{\Delta \phi}{\Delta t} \]  \hspace{1cm} (1)

Also magnitude of emf due to mutual induction is given by
\[ \varepsilon = M \frac{\Delta i}{\Delta t} \]  \hspace{1cm} (2)

By comparing; we get;
\[ N \frac{\Delta \phi}{\Delta t} = M \frac{\Delta i}{\Delta t} \]

Multiplying both sides by \( \Delta t \)
\[ N(\Delta \phi) = M(\Delta i) \]

Hence
\[ N(\Delta \phi) = 1.5 \times 20 = 30 \text{ Wb} \]

**Problem 14.11** Back emf in a motor is 120V when motor is turning at 1680 rev/min. What is back emf when the motor turns at 3360 rev/min?

**Solution**

Back emf \( \varepsilon_1 = 120 \text{ V} \) when \( \omega_1 = 1680 \text{ r.p.m} = 1680 \times \frac{2\pi}{60} = 56\pi \text{ rad/sec} \)

Back emf \( \varepsilon_2 = ? \) when \( \omega_2 = 3360 \text{ r.p.m} = 3360 \times \frac{2\pi}{60} = 112\pi \text{ rad/sec} \)

In motor, back emf produced is given by equation \( \varepsilon = N A B \omega \sin \theta \)

First case:
\[ \varepsilon_1 = N A B \omega_1 \sin \theta \]

Second case:
\[ \varepsilon_2 = N A B \omega_2 \sin \theta \]  \hspace{1cm} (1)

Dividing eq. 1 by eq.2 and re-arranging, we get;
\[ \varepsilon_2 = \frac{\omega_2}{\omega_1} \varepsilon_1 = \frac{112\pi}{56\pi} \times 120 = 240 \text{ V} \]