PROBLEMS

1. Express the following quantities by using prefixes.

Solution

(a) 4.0×10^{-4} m $= 0.40 \times 10^{-3} = 0.40$ mm

(b) 15.0×10^{-8} s $= 150 \times 10^{-9}$ s $= 150$ ns

(c) 7.5×10^{-7} g $= 0.75 \times 10^{-6}$ g $= 0.75$ µg

2. The length and width of a rectangular plate are (15.6 ± 0.1) cm and (10.80 ± 0.01) cm respectively. Calculate area of the plate and uncertainty in it.

Solution

$\ell = (15.6 \pm 0.1)$cm $w = (10.80 \pm 0.01)$ cm

$A = ?$ $\Delta A = ?$

Area $A = \ell \times w = 15.6 \text{ cm} \times 10.80 \text{ cm} = 168.5 \text{ cm}^2 = 168 \text{ cm}^2$

Percentage $\Delta \ell = \dfrac{0.1}{15.6} \times 100\,\% = 0.64\,\% = 0.6\,\%$

Percentage $\Delta w = \dfrac{0.01}{10.80} \times 100\,\% = 0.092\,\% = 0.09\,\%$

Total uncertainty $\Delta x = \%\,\Delta \ell + \%\,\Delta w = 0.6\,\% + 0.09\,\% = 0.69\,\% = 0.7\,\%$

$$\Delta x = \frac{0.7 \times 168}{100} = 1.18 = 1$$

Hence Area, $A = (168 \pm 1) \text{ cm}^2$

2. The length of a pendulum is (100.0 ± 0.1) cm. If acceleration of free fall is (9.8 ± 0.1) m s^{-2}, calculate the percentage uncertainty in time period of the pendulum.

Solution

$\ell = 100.0 \pm 0.1$ cm $g = 9.8 \pm 0.1 \text{ m/s}^2$

$$\%\Delta \ell = \frac{0.1}{100.0} \times 100\% = 0.1\,\%$$

$$\%\Delta g = \frac{0.1}{9.8} \times 100\% = 1.02\,\% = 1\,\%$$

The time period of the simple pendulum is;

$$T = 2\pi \sqrt{l/g} = 2\pi\,(L/g)^{1/2} = 2\pi\,(L)^{1/2} / (g)^{1/2}$$

Now the total uncertainty in time period T is;

$$\Delta T = 0.1\,\% \times \frac{1}{2} + 1\,\% \times \frac{1}{2}$$

$$\Delta T = 0.05\,\% + 0.5\,\% = 0.55\,\% = 0.6\,\%$$

3. Theory suggests that drag force depends upon the viscosity of the medium, average radius of the object and velocity of the object moving through the fluid. Derive a formula for dragging force of fluid by using dimensional analysis. (hint [viscosity] = [M L^{-1}T^{-1}])

Solution

The viscous force F acting on the spherical body depends on:

(i) Coefficient of viscosity "η" of the liquid such that $F_D \propto \eta^a$ (1)

(ii) Radius "r" of the sphere such that $F_D \propto r^b$ (2)

(iii) Velocity "V" of the spherical body and $F_D \propto V^c$ (3)

Combining Eq. (1), Eq. (2) and Eq. (3), we get:

$$F_D \propto \eta^a r^b V^c$$

$$\Rightarrow \quad F_D = k\,\eta^a r^b V^c \quad (4)$$

Where k = proportionality constant.

Putting dimensions in Eq. 4, we get;

$$[MLT^{-2}] = k\,[M L^{-1} T^{-1}]^a [L]^b [L T^{-1}]^c$$

$$\Rightarrow \quad [M][L][T^{-2}] = k\,[M]^a [L]^{-a} [T]^{-a} [L]^b [L]^c [T]^{-c}$$

$$\Rightarrow \quad [M][L][T^{-2}] = k\,[M]^a [L]^{-a+b+c} [T]^{-a-c} \quad (5)$$

Comparing coefficients on both sides of the above equation, we get;

$$a = 1,$$

$$-a - c = -2$$

$$\Rightarrow \quad -1 - c = -2 \quad \Rightarrow c = 1$$

And $\quad -a + b + c = 1 \quad \Rightarrow -1 + b + 1 = 1 \quad \Rightarrow b = 1$

Putting values of "a", "b", and "c" in Eq.4, we get;

$$F_D = k\,\eta^1 r^1 V^1 = k\,\eta\, r\, V$$

4. (a) Suppose that the displacement of an object is related to time according to the expression $x = B\,t^2$. (a) What are the dimensions of B? (b) The displacement is related to time as $x = A \sin(2\pi f t)$, where "A" and "f" are constants. Find the dimension of A?

Solution Given that $x = B\,t^2 \Rightarrow B = x/t^2$ has units "m/s^2"

Hence the dimensions of $B = [LT^{-2}]$

Given that $x = A \sin(2\pi f t) \quad \Rightarrow A = x / \sin(2\pi f t)$

Since $\sin(2\pi ft)$ is purely a number, Therefore $A = x / \sin(2\pi f t)$ has unit "m". Hence the dimensions of A is [L].

5. Carry out the following conversions.

(a) Calculate the density $1.33 \times 10^{-7}\ g\,cm^{-3}$ into $kg\,m^{-3}$.

(b) Calculate a speed of $20\ m\,s^{-1}$ in $km\,h^{-1}$.

Solution

$$\rho = 1.33 \times 10^{-7}\ g\,cm^{-3} = \frac{1.33 \times 10^{-7} \times 10^{-3}}{10^{-2} \times 10^{-2} \times 10^{-2}}\ kg\,m^{-3}$$

$$\rho = 1.33 \times 10^{-4}\ kg\,m^{-3}$$

(a) $V = 20\ m\,s^{-1} = \dfrac{20 \times 3600}{1000} = 72\ km\,h^{-1}$

6. If there are $N_o = 6.02 \times 10^{23}$ atoms in 4.0 g of helium, what is the mass of helium atom?

Solution Number of atoms in 4.0 g of He = 6.02×10^{23} atoms

Mass of 1 atom = $\frac{4.0\ g}{6.02 \times 10^{23}} = 0.66\times 10^{-23}$ g

Mass of 1 atom = 6.6×10^{-24} g

7. Compute the following to correct significant digits

(a) 3.85 m × 3.9 m (b) 1023 kg + 8.5489 kg

(c) 22 / 7 (d) $\frac{m_p}{m_e} = \frac{1.67 \times 10^{-27}}{9.1096 \times 10^{-31}}$

Solution

(a) $3.85\ m \times 3.9\ m = 15.015\ m^2 = 15\ m^2$

(b) 1023 kg + 8.5489 kg = 1031. 5489 kg = 1032 kg

(c) 22 / 7 = 3.142857143 ………… infinite no. of Sig. figures

(d) $\frac{m_p}{m_e} = \frac{1.67 \times 10^{-27}}{9.1096 \times 10^{-31}} = 0.1833 \times 10^4 = 1.83 \times 10^3$

8. A rectangular metallic piece is 3.70 ± 0.01 cm wide, and (7.20 ± 0.01) cm Long.
(a) Find the area of the rectangular metallic piece and uncertainty in area.
(b) Verify that the sum of the percentage uncertainty in the length and in the width is equal to percentage uncertainty in area A.

Solution

(a) w = (3.70 ± 0.01) cm L = (7.20 ± 0.01) cm

Area A = L × w = (3.70) cm × (7.20) cm = 26.6 cm^2

Percentage Uncertainty in w = $\frac{0.01}{3.70} \times 100\ \% = 0.27\ \% = 0.3\ \%$

Percentage Uncertainty in L = $\frac{0.01}{7.20} \times 100\ \% = 0.14\ \% = 0.1\ \%$

Total uncertainty in A = % Δw + % ΔL

Total uncertainty, ΔA = 0.3 % + 0.1 % = 0.4 %

$\Delta A = \frac{0.4 \times 26.6}{100} = 0.906 = 0.1$

Hence Area, A = (26.6 ± 0.1) cm^2

(b) The maximum width & length are:

w_{max} = (3.70 + 0.01) cm = 3.71 cm & L_{max} = (7.2 + 0.01) cm = 7.21 cm

Hence the maximum area is:

$A_{max} = L_{max} \times w_{max} = 3.71\ cm \times 7.21\ cm = 26.7\ cm^2$

The minimum width & length are:

w_{min} = (3.70 – 0.01) cm = 3.69 cm & L_{min} = (7.2 – 0.01) cm = 7.19 cm

Hence the minimum area is:

$$A_{min} = L_{min} \times w_{min} = 3.69 \text{ cm} \times 7.19 \text{ cm} = 26.5 \text{ cm}^2$$

The area is now given by;

$$A = \frac{A_{max} + A_{min}}{2} \pm \frac{A_{max} - A_{min}}{2} = \frac{26.7 + 26.5}{2} \pm \frac{26.7 - 26.5}{2}$$

$$A = (26.7 \pm 0.1) \text{ cm}^2 = (26.7 \pm 0.1) \text{ cm}^2$$

Thus the percentage uncertainty $\Delta A = \frac{0.1 \times 100\,\%}{26.7} = 0.37\,\% = 0.4\,\%$

10. Calculate the answer up to appropriate numbers of significant.

(a) 168.99×9 (b) $23.5 + 234.09$ (c) $984.25 / 80.0$

Solution

a. $168.99 \times 9 = 1520.9 = 1.5209 \times 10^3 = 2 \times 10^3$

b. $23.5 + 234.09 = 257.59 = 257.6$

c. $984.25 / 80.0 = 12.3$