

NUMERICAL PROBLEMS

15.1. The peak voltage of an AC supply is 300 V. What is the rms voltage?

Solution

$$V_o = 300 \text{ V}$$

$$V_{\text{rms}} = ?$$

$$V_{\text{rms}} = V_o / \sqrt{2} = 300 / \sqrt{2} = 212 \text{ V}$$

15.2. Rms value of current in an AC circuit is 10A. What is the peak current?

Solution

$$i_{\text{rms}} = 10 \text{ A}$$

$$i_o = ?$$

$$i_o = i_{\text{rms}} \times \sqrt{2} = 14.1 \text{ A}$$

15.3. The A.C voltage across a $0.5 \mu\text{F}$ capacitor is $16 \sin (2 \times 10^3 t) \text{ V}$. Find (a) the capacitive reactance (b) the peak value of current through capacitor.

Solution

(a) $X_c = ?$

(b) $i_o = ?$

$$C = 0.5 \times 10^{-6} \text{ F}$$

$$V = 16 \sin (2000t) \quad (1)$$

(a) Equation for voltage is;

$$V = V_o \sin (\omega t) \quad (2)$$

Comparing (1) and (2), we get;

Now as capacitive reactance

$$\omega = 2000 \text{ Hz}$$

$$X_c = \frac{1}{\omega C} = \frac{1}{2000 \times 0.5 \times 10^{-6}} = 1000 \Omega$$

(b) Again comparison of eq (1) and (2) shows

$$V_o = 16 \text{ V}$$

$$i_o = \frac{V_o}{X_c} = \frac{16}{1000} = 16 \text{ mA}$$

15.4. Voltage across a $0.01 \mu\text{F}$ capacitor is $240 \sin (1.25 \times 10^4 t - 30^\circ) \text{ V}$. Write the mathematical expression for the current through it.

Solution

$$C = 0.01 \mu\text{F}$$

$$V = 240 \sin (1.25 \times 10^4 t - 30^\circ) \quad (1)$$

Expression for $i = ?$ just like eq (1)

We need i_o , ω and Φ to represent current "i" as an equation;
We have equation for sinusoidally varying voltage as

$$V = V_0 \sin(\omega t + \Phi) \quad (2)$$

Compare (1) and (2) shows that

$$\omega = 2\pi f = 1.25 \times 10^4 \text{ rad/sec}$$

$$f = \frac{\omega}{2\pi} = \frac{1.25 \times 10^4}{2(3.14)} = 1990.4 \text{ Hz}$$

$$\Rightarrow X_c = \frac{1}{2\pi f C} = \frac{1}{2(3.14)(1990.4)(0.01 \times 10^{-6})} = 8000 \Omega$$

$$i_0 = \frac{V_0}{X_c} = \frac{240}{8000} = 0.03 \text{ A}$$

For Φ : In AC capacitive circuit, current leads voltage by 90° . So the phase constant for current will be equal to

$$\Phi = -30^\circ + 90^\circ = 60^\circ$$

Now our required parameters are $i_0 = 0.03 \text{ A}$, $\Phi = 60^\circ$ whereas " ω " will remain the same for current and voltage.

Now our required expression for instantaneously varying current is;

$$i = i_0 \sin(\omega t + \Phi)$$

Putting all the values

$$i = 0.03 \sin(1.25 \times 10^4 t + 60^\circ)$$

15.5. An inductor with an inductance of $100 \mu\text{H}$ passes a current of 10 mA when its terminal voltage is 6.3 V . Calculate the frequency of A.C supply.

Solution

$$L = 100 \mu\text{H}$$

$$i = 10 \text{ mA}$$

$$V = 6.3 \text{ V}$$

$$f = ?$$

We have relation for inductive reactance as

$$X_L = 2\pi f L$$

As

$$X_L = \frac{V}{i}$$

$$\Rightarrow \frac{V}{i} = 2\pi f L$$

$$f = \frac{V}{2\pi L i} = \frac{6.3}{2(3.14)(100 \times 10^{-6})(10 \times 10^{-3})} = 10^6 \text{ Hz}$$

15.6. Calculate the inductive reactance of a 3 mH inductor, when 60 Hz and 10 kHz AC voltages are applied. (b) What is rms current at each frequency if the applied rms voltage is 120 V ?

$$(a) X_L = ? \text{ for } f_1 \text{ \& } f_2 \quad L = 3 \times 10^{-3} \text{ H} \quad f_1 = 60 \text{ Hz} \quad f_2 = 10 \text{ kHz}$$

$$(b) i_{\text{rms}1} = ? \text{ \& } i_{\text{rms}2} = ? \text{ If } V_{\text{rms}} = 120 \text{ V}$$

Solution

$$X_{L1} = 2\pi f_1 L$$

$$X_{L1} = 2(3.14)(60)(3 \times 10^{-3}) = 1.13 \Omega$$

$$X_{L2} = 2\pi f_2 L$$

$$X_{L2} = 2(3.14)(10 \times 10^3)(3 \times 10^{-3}) = 188.4 \Omega$$

$$i_{\text{rms}1} = \frac{V}{X_{L1}} = \frac{120}{1.13} = 106 \text{ A}$$

(b)

Similarly

$$i_{rms 2} = \frac{V}{X_{L 2}} = \frac{120}{188.4} = 0.637 \text{ A}$$

15.7. For the same RLC series circuit having a 40 Ω resistor, a 3 mH inductor and a 5 μF capacitor: (a) Find resonant frequency. (b) Calculate i_{rms} at resonance if V_{rms} is 120 V.

$R = 40 \Omega$ $L = 3 \text{ mH}$

$C = 5 \mu\text{F}$

(a) $f_o = ?$ (b) $i_{rms} = ?$ at resonance if $V_{rms} = 120 \text{ V}$

Solution

(a)
$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2(3.14)\sqrt{(3 \times 10^{-3})(5 \times 10^{-6})}} = 1300 \text{ Hz}$$

(b) At resonance impedance $Z = R$

$$i_{rms} = \frac{V_{rms}}{R} = \frac{120}{40} = 3 \text{ A}$$

15.8. A coil of pure inductance 318 mH is connected in series with a pure resistance of 75Ω. The voltage across resistor is 150 V and the frequency of power supply is 50 Hz. Calculate the voltage of power supply and phase angle.

$L = 318 \text{ mH}$

$R = 75 \Omega$

$V_R = 150 \text{ V}$

$f = 50 \text{ Hz}$

(a) $V = ?$

(b) $\Phi = ?$

Solution

(a)

$$V_R = i R$$

⇒

$$i = \frac{150}{75} = 2 \text{ A}$$

In series current "i" remains same for each component

$$V_L = i X_L = i (2\pi f L)$$

$$V_L = 2 (2 \times 3.14 \times 50 \times 318 \times 10^{-3})$$

$$V_L = 199.7 \text{ V} \dots\dots\dots (1)$$

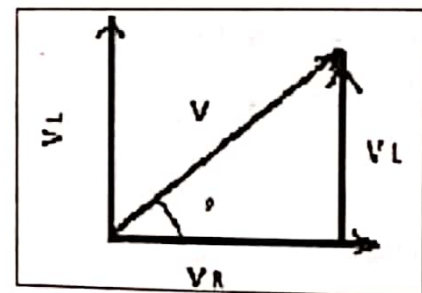
From figure, using Pythagoras theorem

$$V^2 = V_R^2 + V_L^2 = \sqrt{(150)^2 + (199.7)^2} = 250 \text{ V}$$

(b) From figure,

$$\cos \theta = \frac{V_R}{V} = \frac{150}{250} = 0.6$$

$$\theta = 53.1^\circ$$



15.9. A resistor of resistance 30 Ω is connected in series with a capacitor of capacitance 79.5 μF across a power supply of 50 Hz and 100 V. Find (a) impedance (b) current (c) phase angle and (d) equation for the instantaneous value of current.

$R = 30 \Omega$

$C = 79.5 \mu\text{F}$

$f = 50 \text{ Hz}$

$V = 100 \text{ V}$

(a) $Z = ?$

(b) $i = ?$

(c) $\phi = ?$

(d) Equation for $i = ?$

Solution

(a) In RC series circuit,

$$Z = \sqrt{R^2 + X_C^2}$$

Capacitive reactance

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2(3.14)(50)(79.5 \times 10^{-6})} = 40 \Omega \quad (1)$$

Now putting R and X_c in equation (1)

$$Z = \sqrt{(30)^2 + (40)^2} = 50 \Omega$$

(b)

$$i = \frac{V}{Z} = \frac{100}{50} = 2 \text{ A}$$

(c) From figure

$$\sin \Phi = \frac{V_c}{V} = \frac{i X_c}{i Z}$$

$$\sin \Phi = \frac{40}{50} = 0.8$$

⇒

$$\Phi = \sin^{-1}(0.8) = 53.1^\circ$$

(d)

$$i = i_o \sin(\omega t + \Phi)$$

(2)

So we need to find i_o , ω and Φ

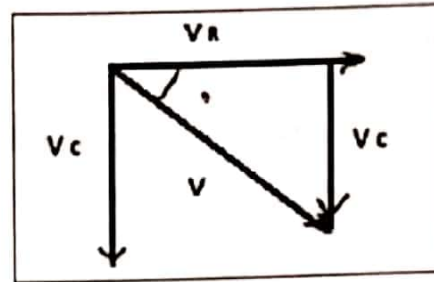
$$i_o = i \sqrt{2} = 2 \sqrt{2} = 2.828$$

Since

$$\omega = 2 \pi f = 2 (3.14) 50 = 314 \text{ rad/sec}$$

Now eq (2) ⇒

$$i = 2.828 \sin(314 t + 53^\circ)$$



15.10. A coil having a resistance of 7Ω and an inductance of 31.8 mH is connected to 230 V , 50 Hz supply. Calculate (a) the circuit current (b) phase angle (c) power factor and (d) power consumed.

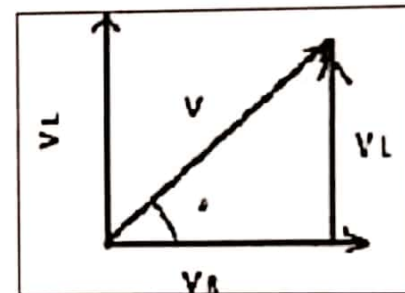
$$R = 7 \Omega \quad L = 31.8 \text{ mH}$$

$$V = 230 \text{ V}$$

$$f = 50 \text{ Hz} \quad \text{(a) } i = ?$$

$$\text{(b) } \Phi = ?$$

$$\text{(c) } \cos \Phi = ? \quad \text{(d) } P = ?$$



Solution

(a) Impedance "Z" for RL series circuit is given by

$$Z = \sqrt{R^2 + X_L^2} \quad (1)$$

$$X_L = 2 \pi f L = 2 (3.14) (50) (31.8 \times 10^{-3}) = 10 \Omega$$

Eq (1) ⇒

$$Z = \sqrt{(7)^2 + (10)^2} = 12.2 \Omega$$

As

$$i = \frac{V}{Z} = \frac{230}{12.2} = 18.8 \text{ A}$$

(b) From figure

$$\sin \Phi = \frac{V_L}{V} = \frac{i X_L}{i Z} = \frac{10}{12.2} = 0.820$$

$$\Phi = \sin^{-1}(0.820) = 55^\circ$$

(c) Power factor for RL series circuit is defined as

$$\cos \phi = \cos(55^\circ) = 0.574$$

(d) Power consumed in RL series circuit is

$$P = i V \cos \phi = (18.8) (230) (0.574)$$

$$P = 2481 \text{ Watt}$$

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