PROBLEMS

1. **A ship leaves a port P and travels 30 km due north. Then it changes course and travels 20 km in a direction 30° east of north to reach port R. calculate the distance from P to R.**

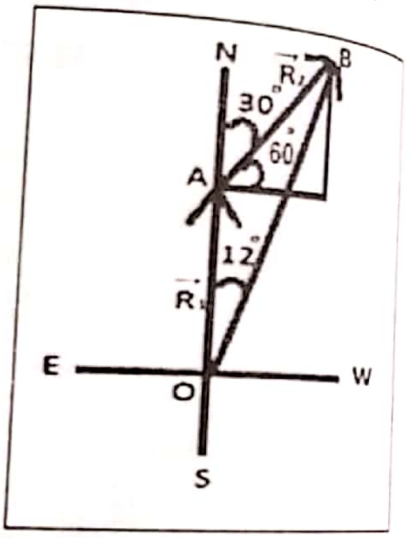
SOLUTION

From figure we can write:

$\vec{R}_1 = 30 \text{ km } \hat{j}$

$\vec{R}_2 = 20 \text{ km Cos } 60° \hat{i} + 20 \text{ km Sin } 60° \hat{j}$ $\vec{R}_2 = 20\,(1/2)\,\hat{i} + 20\,(0.866)\,\hat{j}$

$\vec{R}_2 = 10 \text{ km } \hat{i} + 17.32 \text{ km } \hat{j}$



Now the X-component of resultant R is;

$R_x = R_{1x} + R_{2x} = 0 \text{ km} + 10 \text{ km} = 10 \text{ km}$

And the Y-component of resultant R is;

$R_y = R_{1y} + R_{2y} = 30 \text{ km} + 17.32 \text{ km}$

$R_y = 47.32 \text{ km}$

The resultant is given by;

$R = \sqrt{R_x^2 + R_y^2}$

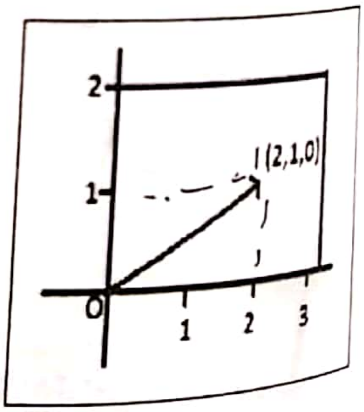
$R = \sqrt{10^2 + 47.32^2} = 48.37 \text{ km} = 48.4$ km

The angle of resultant with positive direction of X-axis is;

$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{47.32}{10}\right) = 78°$

The angle of resultant with $\vec{R}_1 = 90° - 78° = 12°$ east of north.

2. **A certain corner of the room is selected as origin of the rectangular coordinate system. If an insect is sitting on an adjacent wall at a point whose coordinates are (2,1,0) in units of meter, what is the distance of the insect from this corner of the room?**



SOLUTION

Let the coordinate of room corner O = (0,0,0)

The coordinates of the insect are I = (2,1,0) m

The distance of the insect from the room corner is given by;

$d = I - O = (2,1,0) - (0,0,0)$

$d = (2-0, 1-0, 0-0) = (2,1,0)$

$d = 2\hat{i} + \hat{j}$

$d = \sqrt{2^2 + 1^2} = \sqrt{5} \text{ m} = 2.2 \text{ m}$

3. The magnitude of dot and cross product of two vectors are $6\sqrt{3}$ and 6 respectively. Find the angle between the vectors.

SOLUTION

Let the two vectors be $\vec{A}$ and $\vec{B}$

Magnitude of dot product = $|\vec{A}.\vec{B}| = 6\sqrt{3}$ (1)

Magnitude of cross product = $|\vec{A} \times \vec{B}| = 6$ (2)

We also know that

$$|\vec{A}.\vec{B}| = AB\ Cos\ \theta \qquad \& \qquad |\vec{A} \times \vec{B}| = AB\ Sin\ \theta$$

Putting the values above in Eq.1 and Eq.2, we get;

$$AB\ Cos\ \theta = 6\sqrt{3}\ \& \qquad (3)$$

$$AB\ Sin\ \theta = 6 \qquad (4)$$

Diving Eq.4 by Eq.3, we get;

$$\frac{AB\ Sin\ \theta}{AB\ Cos\ \theta} = \frac{6}{6\sqrt{3}} \Rightarrow \frac{Sin\ \theta}{Cos\ \theta} = \frac{1}{\sqrt{3}} \Rightarrow tan\ \theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = tan^{-1}\frac{1}{\sqrt{3}} = 30^\circ$$

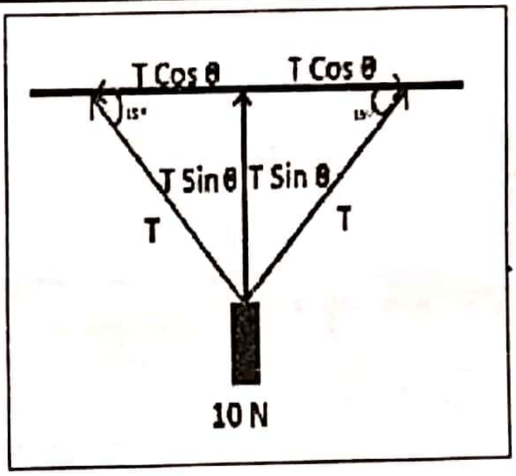
4. A load of 10.0 N is suspended from a clothes line. This distorts the line so that it makes an angle of 15° with the horizontal at each end. Find the tension in the clothes line.

SOLUTION

Load W = 10 N

Angle θ = 15°

Tension T =?



Let we resolve the forces along X-axis and Y-axis.

Applying 1st condition of equilibrium,

Sum of Forces acting along X-axis $\sum F_x = 0$

$$T\ Cos\ 15^\circ - T\ Cos\ 15^\circ = 0$$

Sum of Forces acting along Y-axis $\sum F_y = 0$

$$T\ sin\ 15^\circ + T\ Sin\ 15^\circ - 10\ N = 0$$

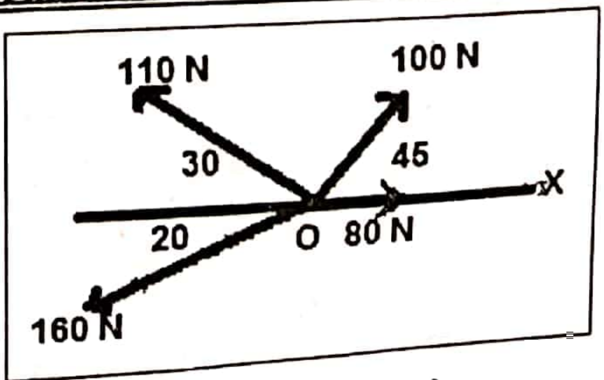
$$T\ (sin\ 15^\circ + Sin\ 15^\circ) = 10\ N$$

$$T\ (0.26 + 0.26) = 10\ N$$

$$T = (10/0.52)\ N = 19.3\ N$$

5. Four coplanar forces act on a body at a point O as shown in figure. Find their resultant.

SOLUTION



From the given figure, we obtain the following data:

F_1 = 80 N making an angle $\theta_1 = 0^\circ$ with +X-axis

F_2=100N making an angle $\theta_2 = 45^\circ$ with +X-axis

F_3 = 110 N making an angle $\theta_3 = 30^\circ$ with −X-axis or 150° with + X-axis

F_4 = 160 N making an angle $\theta_4 = 20°$ with –X-axis or 220° with +X-axis

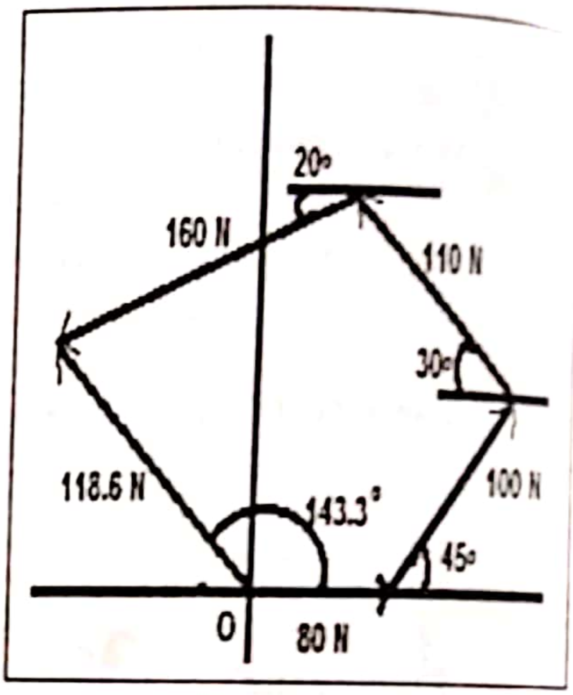
Now the X- component of the resultant is; $F_x = F_{1x} + F_{2x} + F_{3x} + F_{4x}$

$F_x = F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3 + F_4 \cos\theta_4$

$F_x = 80 \cos 0° + 100 \cos 45° + F_3 \cos 150° + F_4 \cos 200°$

$F_x = 80 (1) + 100 (0.707) + 110 (-0.866) + 160 (-0.939)$

$F_x = -94.8\ N = -95\ N$



Now the Y- component of the resultant is;

$F_y = F_{1y} + F_{2y} + F_{3y} + F_{4y}$

$F_y = F_1 \sin\theta_1 + F_2 \sin\theta_2 + F_3 \sin\theta_3 + F_4 \sin\theta_4$

$F_y = 80 \sin 0° + 100 \sin 45° + F_3 \sin 150° + F_4 \sin 200°$

$F_y = 80 (0) + 100 (0.707) + 110 (0.5) + 160 (-0.34)$

$F_y = 0 + 70.7 + 55\ N - 54.7\ N = 71\ N$

The magnitude of the resultant is given as;

$F = \sqrt{F_x^2 + F_y^2} = \sqrt{-95^2 + 71^2} = 118.6\ N$

The direction of the resultant is calculated as follow;

$$\varphi = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{71}{95}\right) = 36.77°$$

Since $F_x = -95\ N$ & $F_y = 71\ N$;

Therefore $\theta = 180° - \varphi = 180° - 36.77° = 143.3°$

6. A force of 5 N is applied perpendicular to the plane of a uniform door 2 m high and 0.6 m wide. Find the torque about the line joining the hinges.

SOLUTION

$F = 5\ N$ $r = 0.6\ m$ $\theta = 90°$ $\tau = ?$

Since $\tau = r F \sin\theta = 0.6\ m \times 5\ N \times \sin 90° = 3\ N\ m\ (1) = 3\ N\ m$

7. Find the magnitudes of the forces provided by the supports A and B, if shown a balanced condition. The weight of the plank is 500 N and is uniform in shape. The weight of block = 100N and weight of student is 500 N.

SOLUTION

Weight of Plank = W = 500N Weight of Block = W_b = 100N

Weight of student = W_s = 500 N F_A = ? & F_B = ?

Applying 1st condition of equilibrium

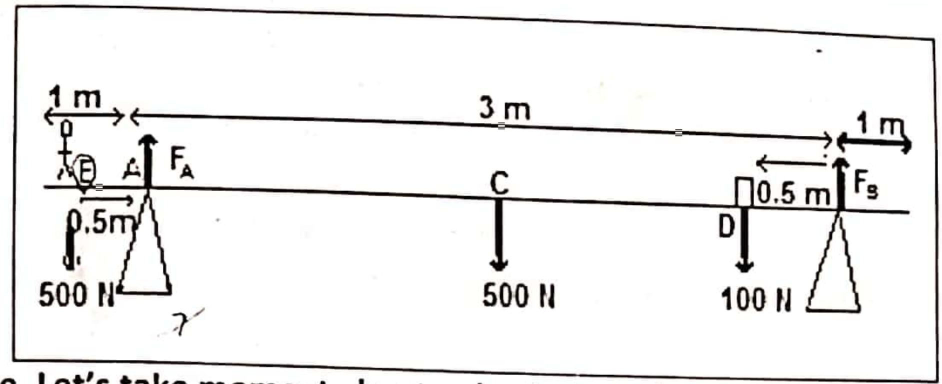
$\sum F_x = 0$ & $\sum F_y = 0$

$F_A + F_B - W - W_b - W_s = 0$ ⇒ $F_A + F_B = W + W_b + W_s$

$F_A + F_B = 500 + 100 + 500$ ⇒ $F_A + F_B = 1100\ N$

Now to find the magnitude of forces we apply 2nd condition of equilibrium.

Anticlockwise moments are taken as positive and clockwise moments are taken as.



negative. Let's take moment about point A, then the moment due to force F_A is zero. Thus

$\tau_E - \tau_C - \tau_D + \tau_B = 0 \quad \Rightarrow \quad \tau_E + \tau_B = \tau_C + \tau_D$

$500\ N \times 0.5\ m + F_B \times 3\ m = 500\ N \times 1.5\ m + 100\ N \times 2.5\ m$

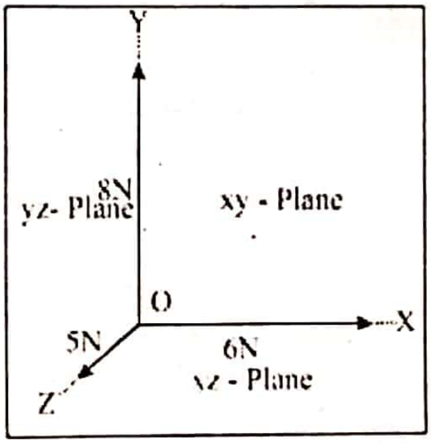
$250\ Nm + 3\ F_B\ (m) = 750\ N\ m + 250\ N\ m$

$3\ F_B\ (m) = \cancel{750}\ N\ m \quad \Rightarrow F_B = 250\ N$

By putting values in equation 1, we get

$F_A + 250\ N = 1100\ N \quad \Rightarrow F_A = 850\ N$

8. **Three forces are acting on a body as shown. Find the magnitude of their resultant and also state in which plane the final resultant lies.**



SOLUTION

From the given figure,

$F_x = 6N \quad F_y = 8\ N\ F_z = 5\ N$

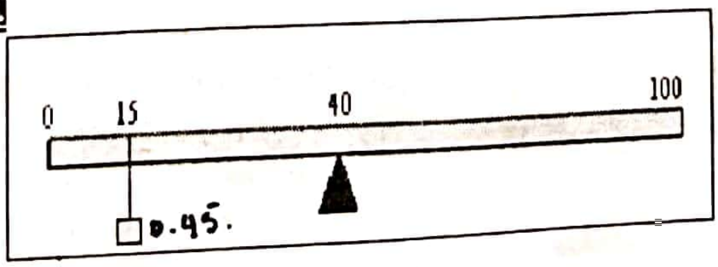
Resultant force = F =?.

$F = \sqrt{F_x^2 + F_y^2 + F_z^2} \Rightarrow F = \sqrt{(6)^2 + (8)^2 + (5)^2} = \sqrt{36 + 64 + 25} = 11.2\ N$

The resultant lies in space.

9. **A meter rule is supported on a knife edge placed at the 40 cm graduation. It is found that the meter rule balances horizontally when a mass which has a weight of 0.45 N is suspended at the 15 cm graduation, as shown in the diagram. Calculate the moment about the knife edge in this balanced condition of the force due to the mass of the rule. If the weight of the rule is 0.90N, calculate the position of its center of gravity.**

SOLUTION



Given that the balancing point is at 40 cm.

Weight suspended = W_s = 0.45 N

Weight of ruler = W_r = 0.9

Moment arm of W_s = r_s = 40-15 = 25 cm = 0.25 m

Moment arm of W_r = r =?

Applying 2[nd] condition of equilibrium

$$\sum \tau = 0$$

$$W_r \times r = W_s \times r_s \quad (1)$$

Moment about knife edge = $W_s \times r_s = 0.45\ N \times 0.25\ m = 0.1125\ Nm$

Now When $W_r = 0.9\ N$, then Eq.1 becomes;

$$0.9\ N \times r = 0.1125\ Nm$$

The center of gravity is;

$$r = \frac{0.1125}{0.9} = 0.125\ m = 12.5\ cm$$

Hence the position of center of gravity from zero point or left side is;

$$R = 40\ cm + 12.5\ cm = 52.5\ cm$$

10. A uniform plank AB of length 4.0 m and weight 500 N is suspended by a vertical rope at each end. A girl of weight 300 N stands in the position shown, 1.2 m from the end A. By taking moments about A, calculate the tension in the rope supporting the end B. Would you expect the tension in the rope at A to be larger or smaller than that in the rope at B? State a reason for your answer.

SOLUTION

Given that

Length of plank $L = 4\ m$

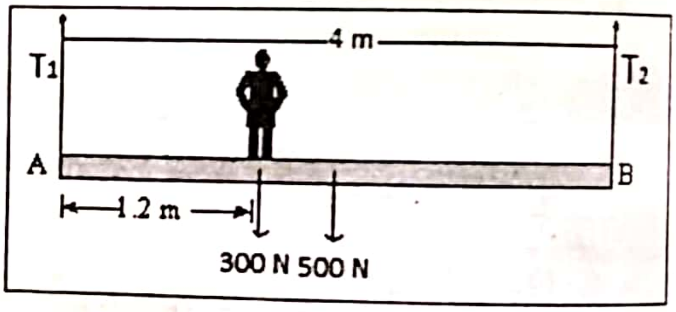
Weight of plank $W_1 = 500\ N$

Distance of W_1 from A, $r_1 = 2\ m$

Weight of girl $W_2 = 300\ N$

Distance from A $r_2 = 1.2\ m$

Tension at end B =?



Let T_1 be the tension at end A and T_2 be the tension attend B.

Since the rod is supported at both points A and B, the tension at each end can be found by taking moments about these points.

Let we take moment about A.

Moment due to T_1

Since the distance of T_1 from A is zero, hence moment due to T_1 is zero.

Moment due to W_1, W_2 and T_2

W_1 and W_2 produces clockwise moment about A and T_2 produces anticlockwise moment.

Since sum of anti-clockwise moments = sum of clockwise moments

$\Rightarrow$ $W_1 \times r_1 + W_2 \times r_2 = T_2 \times 4\ m$

$500\ N \times 2\ m + 300\ N \times 1.2\ m = T_2 \times 4\ m$

$1000\ Nm + 360\ Nm = T_2 \times 4\ m$

$1360\ Nm = T_2 \times 4\ m$

$T_2 = 340\ N$

Also total upward force = total downward force

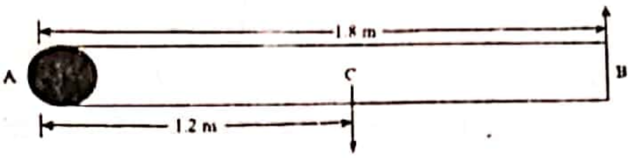
$$T_1 + T_2 = W_1 + W_2$$

Hence $T_1 = W_1 + W_2 - T_2 = 500\ N + 300\ N - 340\ N = 460\ N$

This shows that tension at end A is greater than tension at end B.

11. **The diagram below shows the plan view of a door hinged at A. if a man applies a force F of 40 N at the end marked B, calculate the moment of this force about A. What is the minimum force X that must be applied at C in order to stop the door form turning? Name the principle applied to solve this problem.**

SOLUTION



Force at end B is $F_B = 40$ N

Moment of Force at B $r_B = 1.8$ m

Torque about B τ =?

$$\tau = r_B \times F_B = 1.8$$

m×40 N = 72 Nm

Now minimum force X =?

$$r_c = 1.2 \text{ m}$$

$$\tau = 72 \text{ Nm}$$

Applying 2[nd] condition of equilibrium;

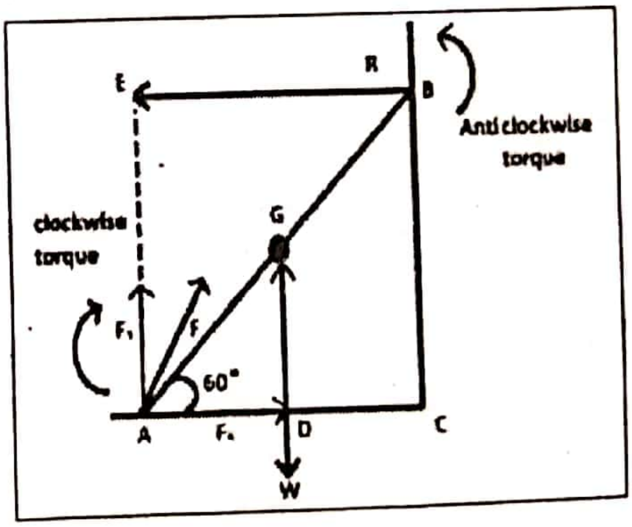
Sum of anti-clockwise moments = sum of clockwise moments

$$72 \text{ Nm} = 1.2 \text{ m} \times X \quad \Rightarrow \quad X = 60 \text{ N}$$

The principle applied to solve this problem is 2[nd] condition of equilibrium.

12. **Consider a ladder weighing 200N resting against a smooth wall such that it makes an angle of 60° with the horizontal. Find the reaction on the ladder due to the wall and ground.**

SOLUTION



Let the Length of ladder AB = L

Weight of ladder W = 200 N

Center of gravity from ground AG = L/2

Force of Reaction on the ladder due to wall R =?

Force of Reaction on the ladder due to ground F =?

Applying first condition of equilibrium,

$$\sum F_x = 0$$

$$F_x - R = 0$$

$$F_x = R \qquad (1)$$

$$\sum F_Y = 0$$

$$F_Y - W = 0$$

$$F_Y = W = 200 \text{ N} \qquad (2)$$

Now consider moments about point A. Applying the second condition of equilibrium;

Sum of anti-clockwise moments = Sum of clockwise moments

$\Rightarrow$ $AE \times R = AD \times W$ (3)

From figure $AE = BC = AB \sin 60°$ & $AD = AG \cos 60°$

Hence Eq.3 becomes;

$$AB \sin 60° \times R = AG \cos 60° \times W$$

But AB = L & AG = L/2;

Therefore $\cancel{L} \sin 60° \times R = (\cancel{L}/2) \cos 60° \times W$

$$0.866 \times R = (0.5)\, 0.5 \times 200 \text{ N}$$

$$R = 57.73 \text{ N}$$

This gives the reaction force due to wall. The reaction force due to ground is given by; $F = \sqrt{F_x^2 + F_y^2} = \sqrt{57.73^2 + 200^2} = 208 \text{ N} = 208 \text{ N}$