PROBLEMS

1. **An object travels 60m in the first 6sec. Then keeps uniform motion for the next 9sec.In the last 10sec of the journey it decelerates uniformly to rest. (a) Find the maximum speed attained in the journey. (b)Find the total distance traveled in the whole journey.**

SOLUTION $S_1 = 60$ m $t_1 = 6$ s $V_i = 0$

$V_f = ?$

Using $S_1 = V_i t_1 + \frac{1}{2} a\, t_1^2$

$60\text{ m} = 0 \times 6 + \frac{1}{2} a(6)^2$ or $60\text{ m} = \frac{1}{2} a(36) = 18\,a$

$a = 60/18 = 3.33\ \text{m/s}^2$

Since $V_f = V_i + a\,t = 0 + 3.333 \times 6 = 20\ \text{m/s}$ (1)

Now after attaining a velocity of 20 m/s, it continue to move for another 9 s with the same velocity therefore, we proceed as;

$V_i = V_2 = 20\ \text{m/s}$

$t_2 = 9\text{sec}$

$S_2 = V_2 \times t_2 = 20 \times 9 = 180\ \text{m}$

Now in the last 10sec of the journey it decelerates uniformly to rest from V_i = 20 m/sec to $V_f = 0$. Therefore the maximum speed attained during the journey is 20 m/s.

$V_i = 20\ \text{m/s}$ & $V_f = 0$

$t_3 = 10$ s

$a = \Delta V/\Delta t = V_f - V_i / 10 = 0 - 10/10 = -2\ \text{m/s}^2$

The distance travelled during $t_3 = 10$ s is;

$S_3 = V_i t_3 + \frac{1}{2} a\, t_3^2 = 20 \times 10 - \frac{1}{2} \times 2 \times 10^2 = 200 - 100 = 100\ \text{m}$

The total distance covered is calculated as follow;

$S = S_1 + S_2 + S_3 = 60 + 180 + 100 = 340\ \text{m}$

2. **An object is traveling with a constant acceleration of 10m/sec². How much distance will it travel in 3rd second of its journey?**

SOLUTION

1st method

Given that $a = 10\ \text{m/s}^2$

$S = V_{avg} \times \Delta t$

Since the object is moving with constant acceleration so the velocity in first second is 10 m/s, 2nd second is 20 m/s and 3rd second is 30 m/s.

Hence the average velocity for 3rd second of it journey is;

$$V_{avg} = (20 + 30)/2 = 25 \text{ m/s}$$

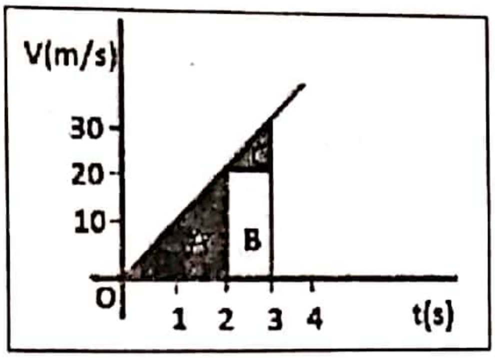
Therefore $S = 25 \text{ m/s} \times (3\text{-}2) \text{ s} = 25 \text{ m}$

2nd method

Given that $a = 10 \text{ m/s}^2$

This means that the velocity of the body changes 10 m/s each second.

The V-t graph of motion of the body is shown. The distance travelled by the body during the 3rd second of its journey is the sum of the area under the V-t graph. Thus:



$$S = \text{area of rectangle B} + \text{area of triangle C}$$

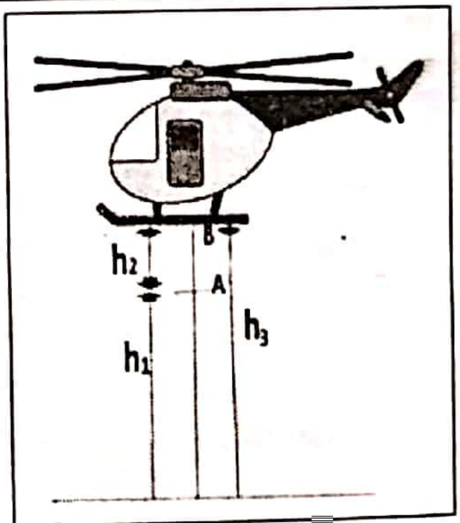
$$S = \text{height} \times \text{width} + \frac{1}{2}\Delta V_B \times \Delta t_B$$

$$S = 20 \times (3 - 2) + \frac{1}{2}(30 - 20) \times (3 - 2)$$

$$S = 20 \times (1) + \frac{1}{2}(10) \times (1) = 20 + 5 = 25 \text{ m}$$

3. **A helicopter is ascending vertically at a speed of 19.6 m/sec. When it is at a height 156.8 m above the grounds, a stone is dropped. How long does the stone take to reach the ground?**

SOLUTION



Initial of helicopter and stone = $V_i = 19.6$ m/sec

Initial height = $h_1 = 156.8$m

Time taken by stone to hit the ground =?

Since, the stone will move up with initial velocity of the helicopter due to inertia, hence it will gain some height say h_2, as shown in figure;

The vertical distance h_2 covered by stone from point A to point B at top where its final velocity $V_f = 0$ is;

$V_i = 19.6$ m/s $\qquad V_f = 0 \qquad g = -9.8 \text{ m/sec}^2$

$h_2 = ?$

$$2\,g\,h_2 = V_f^2 - V_i^2$$

$$2 \times -9.8 \times h_2 = -19.6^2$$

$$h_2 = 19.6 \text{ m}$$

Time taken from point A to point B is;

$$V_f = V_i + a\,t$$

$$0 = 19.6 + g\,t = 19.6 - 9.8\,t_1$$

$$t_1 = 19.6/9.8 = 2\ s$$

Now the total height from ground = $h_3 = h_1 + h_2 = 156.8 + 19.6\ m = 176.4\ m$

The time taken by stone to fall from point B to ground through height h_3 is;

$$h_3 = V_i t_2 + \frac{1}{2} a t_2^2$$

At point B, $V_i = 0$ $a = g = 9.8 m/s^2$ $h_3 = 176.4\ m$

Therefore $175.4 = 0 \times t_2 + \frac{1}{2} \times 9.8 \times t_2^2$

$$t_2^2 = 2 \times 175.4 / 9.8 = 36\ s$$

$$t_2 = 6\ s$$

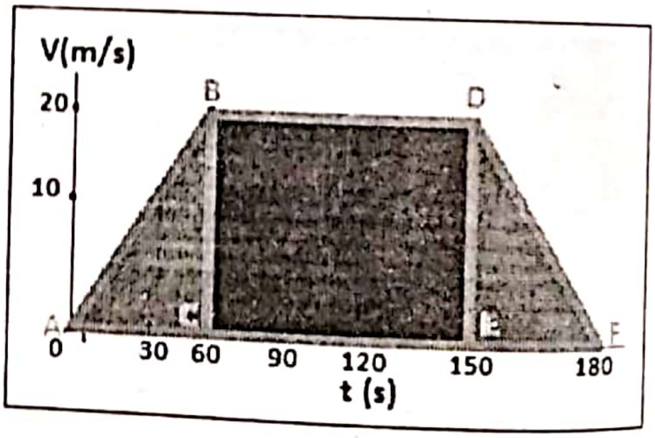
Hence the total time $t = t_1 + t_2 = 2 + 6 = 8\ s$

4. **Using the following data and draw a velocity - time graph for a short journey on a straight road of a motorbike. Using v-t graph to calculate (a) the initial acceleration. (b). the final acceleration (c) and total distance traveled.**

Velocity (m/sec)	0	10	20	20	20	20	0
Time(sec)	0	30	60	90	120	150	180

SOLUTION

The (V-t) graph of the given data is shown.



(a) The initial acceleration is the slope of the triangle ABC which is given by;

$$a = BC/AC = (20 - 0)/(60 - 0)$$

$$a = 0.33\ m/s^2$$

(b) The final acceleration is the slope of triangle DEF, which is given by;

$$a = DE/EF = (0 - 20)/180 - 150$$

$$a = -0.67\ m/s^2$$

(c) The total distance traveled is equal to the area under the V-t graph.

Distance = Area of ΔABC + Area of parallelogram BCDE + Area of ΔDEF

$$S = ½ (60 \times 20) + (90 \times 20) + ½ (30 \times 20) = 600 + 1800 + 300 = 2700\ m$$

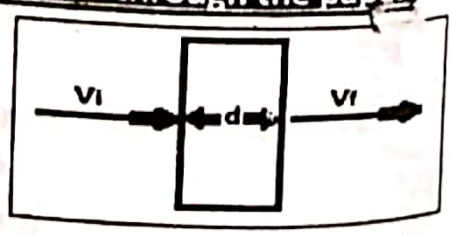
5. **A proton moving with a speed of 1.0×10^7 m/sec passes through a 0.020 cm thick sheet of paper and emerges with a speed of 2.0×10^6 m/sec. Assuming a uniform deceleration, find the retardation and time taken to pass through the paper**

SOLUTION

$V_i = 1.0 \times 10^7\ m/s$

$S = 0.020\ cm = 0.02 \times 10^{-2}\ m$

$V_f = 2.0 \times 10^6\ m/s$



Thickness of the sheet = d = 0.02 cm = 2×10^{-4} m

Retardation a =?

Time taken to pass through the sheet t =?

The average speed to pass through the sheet is;

$$V_{avg} = \frac{V_i+V_f}{2} = \frac{1\times10^7 + 2\times10^6}{2} = 6\times10^6 \text{ m/s}$$

Since $S = V_{avg}\,\Delta t$

$\Rightarrow$ $\Delta t = S/V_{avg} = 2\times10^{-4}/6\times10^6 = 3.33\times10^{-11}$ m/s

Now $$a = \Delta V/\Delta t = \frac{V_f - V_i}{3.33\times10^{-11}} = \frac{2\times10^6 - 1\times10^7}{3.33\times10^{-11}} = -2.4\times10^{17} \text{m/s}^2$$

6. **A constant force F changes the velocity of a 80 Kg sprinter from 3 ms^{-1} to 4 ms^{-1} in o.5 sec. calculate the acceleration of the sprinter.**

SOLUTION V_i = 3m/s V_f = 4m/s Δt = 0.5 s m = 80Kg

Acceleration = a =?

$$a = \Delta V/\Delta t = \frac{4-3}{0.5} = 2 \text{ m/s}^2$$

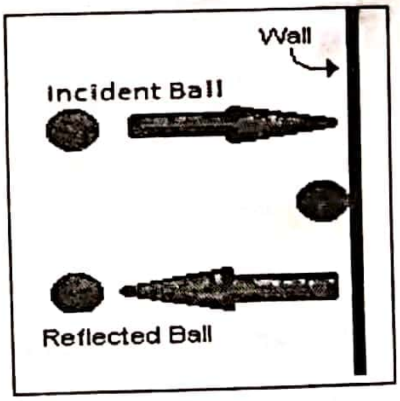
7. **A 4kg ball traveling with the speed of 2 ms^{-1} strikes a rigid wall and rebounds elastically if the ball is in contact with the wall for 0.050 sec, what is. (a) Momentum imparted to the wall? (b) Average force exerted on the wall?**

SOLUTION V_i = 2 m/s V_f = –2m/s m = 4 kg

Δt = 0.050 s

Momentum imparted to the wall =?

The average force exerted on the wall =?



The momentum imparted to the wall cannot be calculated directly, rather can be calculated indirectly from the momentum imparted to the ball. i.e.

$\Delta P_{ball} = P_f - P_i = mV_f - mV_i = m(V_f - V_i) = 4(-2(-2)) = 4(-4) = -16$ kg m/s

is the change in momentum of the ball.

Hence the momentum imparted to the wall $= -\Delta P_{ball} = -(-16$ kg m/s$) = 16$ kg m/s

The average force exerted on the wall is;

$$F = (\Delta P/\Delta t) = 16/0.05 = 320 \text{ N}$$

8. **A Projectile is thrown from the ground level with a speed of 100m/s in a direction 30° with the horizontal. Find the time of flight, range and height to which it rises. Take g = 9.8m/s².**

SOLUTION V_i = 100 m/s $\theta = 30^0$ g = 9.8 m/sec²

Time of flight T =? Height H =? Range R =?

Total time of flight of projectile,

$$T = 2V_i \frac{\sin\theta}{g} = 2 \times 100 \times \frac{\sin 30}{9.8} = 10.2\ s$$

The height reached by projectile is given by;

$$H = V_i^2 \frac{\sin^2\theta}{2g} = 100^2 \times \frac{\sin^2 30}{2 \times 9.8}$$

$$H = 510.2 \times (0.5)^2 = 127.6\ m$$

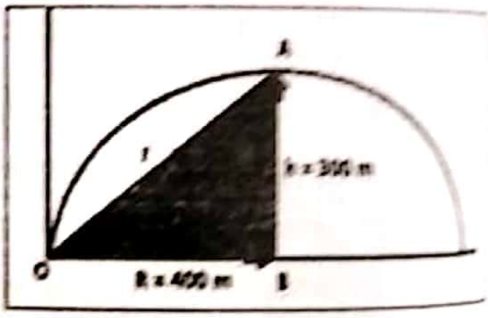
The range of projectile,

$$R = V_i^2 \frac{\sin(2\theta)}{g} = 100^2 \times \frac{\sin(2 \times 30)}{9.8}$$

$$R = 1020.4 \times \sin 60^0 = 883.6\ m$$

9. **The maximum height gained by a projectile is 300 m if is travels a range of 800m then find the displacement of the summit point form the point of projection**

SOLUTION



Maximum height = H = 300 m

Horizontal range = $R' = 800$ m

Horizontal distance from B is R = 400 m

Displacement r =?

Thus to find displacement r we use Pythagoras theorem;

$$r = \sqrt{R^2 + H^2} = \sqrt{400^2 + 300^2} = 500\ m$$

10. **Calculate the angle of projection for which K.E at the highest point of its trajectory equal to one fourth of its K.E at the point of projection.**

SOLUTION

Since we know that at summit point, the velocity has only x-component and $V_y = 0$. So

the KE at projection is; $KE_p = \frac{1}{2}mV^2$ (1)

The KE at summit is; $KE_s = \frac{1}{2}mV_x^2 = \frac{1}{2}mV^2\cos^2\theta$ (2)

We have to find the angle for which the following condition is satisfied:

$$\text{KE at summit} = \frac{1}{4}\text{ KE at point of projection}$$

Putting values from Eq.1 and Eq.2, we get;

$$\frac{1}{2}mV^2\cos^2\theta = \frac{1}{4}\left(\frac{1}{2}mV^2\right)$$

$$\cos^2\theta = \frac{1}{4}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$