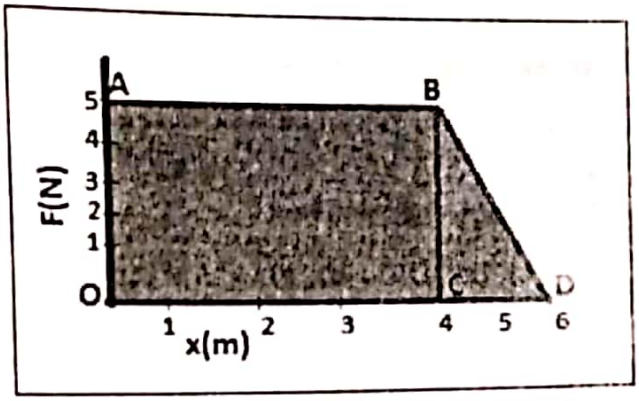
PROBLEMS

1. A force acting on an object varies with distance 'd' as shown. Calculate the work done by the force as the object is displacement from d = 0 to d = 6m.

Solution

The total work done can be calculated from the area under the curve of the given graph.

Consider rectangle OABC;



As Area = length × width

F = 5 N d = 4 m

W =?

$W = F\,d = 5 \times 4 = 20\ J$

The area of triangle BCD is;

$\text{Area} = \frac{1}{2}$ base × height

$W = \frac{1}{2} \times 2\ m \times 5\ N = 5\ J$

Total work done = Area of rectangle + area of triangle = 20 J + 5 J = 25 J

2. A 70 kg man runs up a long flight of stairs in 4.05. The vertical height of the stair is 4.5m. Calculate his power.

Solution Mass of man m = 70 kg Time = t = 4.0 s h = 4.5 m

$g = 9.8\ m/s^2$

Power P =?

Work W = mgh

$\text{Power } P = \frac{mgh}{t} = \frac{70 \times 9.8 \times 4.5}{4} = 7.7 \times 10^2 \text{ watts}$

3. A body of mass 2.0 kg is dropped from a rest position 5m above the ground. What is its velocity at height of 3.0 m above the ground.

Solution

Mass m = 2.0 Kg

Effective height through which body falls h = 5 - 3 = 2 m

Gravitational acceleration $g = 9.8\ m/s^2$

Initial Velocity $V_i = 0$ m/s Final Velocity V_f =?

Since $2gs = V_f^2 - V_i^2$

$2 \times 9.8 \times 2 = V_f^2 \Rightarrow V_f = \sqrt{39.2} = 6.3$ m/s

4. A man pulls a trolley through a distance of 50m by applying a force of 100N which makes an angle of 30° with –axis. Calculate the work done by the man.

Solution

Distance through which trolley is moved d = 50 m

Applied force F = 100 N Angle made by force θ = 30°

Work done W =?

$W = \vec{F}.\vec{d} = Fd \cos 0 = 100 \times 50 \times \cos 30° = 4330$ J

5. A man whose mass is 70kg walks up to the third floor of a building which is 12m above the ground in 20 s. Find his power in watts and hp.

Solution

Mass of the man m = 70 kg

Height h = 12 m

Time taken t = 20 s

Power P =?

Since $P = \frac{W}{\Delta t} = \frac{mgh}{\Delta t} = \frac{70 \times 9.8 \times 12}{20} = 411.6$ watts

And since 1 hp = 746 watts $\Rightarrow P = 411.6 \times \frac{1}{746}$ hp = 0.55 hp

6. A ball of mass 100 g is thrown vertically upward at a speed of 25 ms^{-1}. If no energy is lost, determine the height it would reach. If the ball only rises to 25 m, calculate the work done against air resistance. Also calculate the force of friction.

Solution

(a) Ideal case: when no energy is lost, the KE at point of projection is converted only in to the PE at highest point.

Given that Mass of ball m = 100 g = 0.1 kg Speed = V = 25 m/s

$g = 9.8$ m/s² height = h =?

According to the law of conservation of energy, we have

Gain in P.E = loss in K.E

$mgh = (\frac{1}{2})\, mV^2$

$\Rightarrow h = (\frac{1}{2}) \times \frac{V^2}{g} = (\frac{1}{2}) \times \frac{25^2}{9.8} = 31.9$ m

(b) Real case: If the ball only rises to 25 m, then the work done against air resistance is calculated as follow;

Mass of ball m = 100 g = 0.1 kg

Height h_1 = 25 m $g = 9.8$ m/s²

Work against friction = W =?

Since Loss in KE = Gain in PE + Work done against force of friction

$\therefore = (\frac{1}{2})\, mV^2 + f\,h$

$0.1 \times 9.8 \times 25 = (\frac{1}{2})\, 0.1 \times 25^2 + W$

$31.25 = 24.5 + W$

$31.25 - 24.5 = W = 6.7\ J$

(c) Since, $W = fd = 6.7\ J$

Therefore, $f = \frac{6.75}{25} = 0.3\ N$

7. A force of 2000 N is exerted in lifting a 100 kg mass straight up to a height of 10 m.
(a) How much work is done?
(b) What are the K.E and P.E of the object when it gets to that height?

Solution

Force F = 2000 N Mass m = 100 kg

Height h = 10 m $g = 9.8\ m/s^2$

(a) Work done W =?

(b) K.E =? & P.E =?

(a) The work done is;

$$W = \vec{F}.\vec{h} = F\,h\,\text{Cos}\,\theta = F\,h\,\text{Cos}\,0^\circ = F\,h = 2000 \times 10 = 20000\ J$$

The P.E of the mass at that height h is;

$$PE = m\,g\,h = 100 \times 9.8 \times 10 = 9800\ J$$

(b) From law of conservation of energy

Work done = KE + PE

KE = Work Done – PE = 20000 – 9800 = 10200 J

8. An object of mass 1000 grams falls from a height of 30m on the sand below. If it penetrates 4 cm into the sand, what opposing force is exerted on it by the sand? Neglect air friction.

Solution

Mass of the object m = 1000 g = 1 kg $g = 9.8\ m/s^2$

Height h = 30 m Penetration depth d = 4 cm = 0.04 m

The opposing force f =?

Since Gain K.E = loss in P.E

K.E = m g h = 1 x 9.8 x 30 = 294 J

Now work done by opposing force is equal to loss in KE through a distance d in the sand;

Work = K.E

$fd = 294 \Rightarrow f \times 0.04 = 294$

$$f = \frac{294}{0.04} = -7350\ N$$

Where the negative signs shows the opposing force.

9. A body of mass 'm' drops from Nowshera Bridge into water of the river. The Bridge is 10 m high from the water surface. (a). Find the speed of the body 5 m above the water surface. (b). Find the speed of the body before it strikes the water.

Solution

Mass of body = m

Height from water surface = h_t = 10 m

Height 5m above water surface h = 10−5 = 5 m

a) Speed above water surface V=?

b) Speed before striking water surface = V =?

a) To find the value of Speed above water surface, we proceed as;

loss of PE = gain in KE

$\Rightarrow mgh = \frac{1}{2} mV^2$

$gh = \frac{1}{2} V^2$

$\Rightarrow$ $V = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 5} = 9.9$ m/s

b) The Speed before striking water surface

$V = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 10} = 14$ m/s

10. The engine of a JF-Thunder fighter (made by Pakistan and China) develops a thrust of 3000 N. What horsepower does it produces at velocity of 600 m s^{-1} .

Solution

Force of thrust F = 3000 N

Velocity of engine V = 600 m/Sec

P =?

$P = FV = 3000 \times 600 = 1800000$ watts

And since $1 \text{ hp} = 746 \text{ watts} \Rightarrow 1 \text{ watt} = \frac{1}{746} \text{ hp}$

$\Rightarrow$ $P = 1800000 \times \frac{1}{746} \text{ hp} = 2413 \text{ hp}$