PROBLEMS

1. A string 1 meter long is used to whirl a 100 grams stone in a horizontal circle at a speed of 2 m s^{-1}. Find the tension in the string.

SOLUTION

$V = 2$ m/s $m = 100$ g $= 0.1$ kg length = radius of circle $r = 1$ m

Tension T=?

The required centripetal force must be provided to the mass through a tension in the string.

$$T = F_c = m\frac{V^2}{r} = 0.1 \times \frac{2^2}{1} = 0.4 \text{ N}$$

2. The moon revolves around the earth in almost a circle of radius 382400km in 27.3 days. What is the centripetal acceleration?

SOLUTION

Radius, $r = 382400$ km $= 382400 \times 10^3$ m $= 3.8 \times 10^8$ m

Time period, $T = 27.3$days $= 2.36 \times 24 \times 60 \times 60$ s $= 2358720$ s $= 2.36 \times 10^6$s

Centripetal acceleration a_c=? $V = r\omega$ $a = \frac{r^2\omega^2}{r}$

$$a_c = \frac{V^2}{r} = r\,\omega^2 = r \times \frac{4\pi^2}{T^2} = 3.8 \times 10^8 \times \frac{4\pi^2}{(2.36 \times 10^6)(2.36 \times 10^6)} = 0.0027 \text{ m/s}^2$$

$r\ \frac{2\pi}{T}$

3. A ball of mass 'm' tied to one end of string and is whirled from the other end in a vertical circle of radius 'r' with constant speed. Find out the tension in the string, when the ball is at;

a. The highest point and

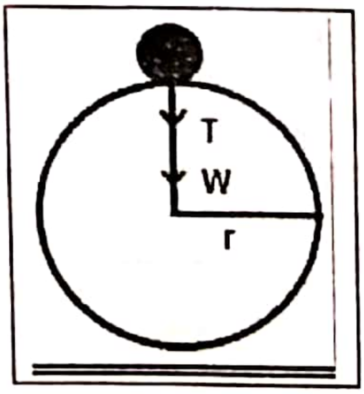
b. The lowest point in its path of motion.

SOLUTION

Mass of the body = m Radius = r

Constant speed = V

Tension in the string =T=?



a) WHEN STRING IS AT HIGHEST POINT

At the top two radially inward forces act on the body,

i. Weight, $W = mg$

ii. Tension = T

The resultant (net) of these two forces will be required centripetal force to keep the body in its circular path.

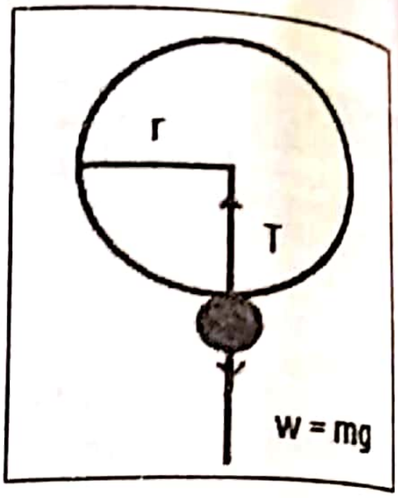
$$F_c = T + W$$

$$m\frac{V^2}{r} = T + W$$

$$T = m\frac{v^2}{r} - W$$

$$T = m\frac{v^2}{r} - mg$$

$$T = m\left(\frac{v^2}{r} - g\right) \qquad (1)$$



b) At the lowest point (bottom)

i. Tension is radially inward

ii. W = mg is acting outward

$$F_c = T - W$$

$$m\frac{v^2}{r} = T - W$$

$$T = m\frac{v^2}{r} + W = m\frac{v^2}{r} + mg$$

$$T = m\left(\frac{v^2}{r} + g\right) \qquad (2)$$

3. A motor car is traveling at a speed of 30 m s^{-1}. If its wheel has a diameter of 1.5m, find its angular speed in rad s^{-1} and rev s^{-1}.

SOLUTION Speed of the car V = 30 m/s Diameter of the wheel, d = 1.5 m

⟹ Radius r = d/2 = 1.5/2 = 0.75 m Angular speed ω =?

Since, $V = r\omega$

$$\omega = \frac{V}{r} = \frac{30}{0.75} = 40 \text{ rad/s}$$

As we know that 1 rad $= \frac{1}{2\pi}$ rev

Therefore $\omega = 40 \times \frac{1}{2\pi}$ rev/s = 6.36 rev/s

5. An electric motor is running at 1800 rev min^{-1}. It comes to rest in 20 s. If the angular acceleration is uniform find the number of revolutions it made before stopping.

SOLUTION Initial angular velocity ω_i = 1800 rev/min = (1800/60) rev/s = 30 rev/s

Final angular velocity ω_f = 0 rev/s

Time taken, Δt = 20 s

Number of revolutions =?

Now, Average angular velocity is;

$$\langle\omega\rangle = \frac{\omega_i + \omega_f}{2} = \frac{30 + 0}{2} = 15 \text{ rev/s}$$

Since $\omega = \frac{\Delta\theta}{\Delta t}$

⟹ $\Delta\theta = \omega\,\Delta t = 15 \times 20 = 300$ rev

6. What is the moment of inertia of a 100 kg sphere whose radius is 50 cm.

SOLUTION Moment of inertia of the sphere I =?

Mass of the sphere m = 100 kg

Radius r = 50 cm = 0.5 m

Now, we know that the moment of Inertia of a sphere is given by;

$$I = \frac{2}{5} mr^2 = \frac{2}{5} \times 100 \times 0.5^2 = 10 \text{ kg m}^2$$

7. What is the kinetic energy of a 5.0 kg solid ball whose diameter is 15cm if it rolls across a level surface with a speed of 2m s^{-1}?

SOLUTION

K.E of the solid ball (sphere) =?

Diameter of the ball, d = 15 m

Radius of the ball, r = d/2 = 15/2 = 7.5 m

Linear speed, V = 2 m/s

Mass of the ball, m = 5 kg

Since, we know that for rolling motion

$$K.E_{Total} = K.E_{linear} + K.E_{rotational} = \frac{1}{2} m V^2 + \frac{1}{2} I \omega^2$$

Moment of inertia of sphere $I = \frac{2}{5} mr^2$ and $\omega = \frac{V}{r}$

$$K.E_{Total} = \frac{1}{2} m V^2 + \frac{1}{2} (\frac{2}{5} mr^2) (\frac{V}{r})^2$$

$$K.E_{Total} = \frac{1}{2} m V^2 + \frac{1}{5} mV^2 = \frac{7}{10} mV^2$$

$$K.E_{Total} = \frac{7}{10} \times 5 \times 2^2 = 14 \text{ J}$$

8. A cylinder of 50cm diameter at the top of an incline 29.4cm high and 10m long is released and rolls down the incline. Find its linear and angular speeds at the bottom. Neglect friction.

SOLUTION

Diameter of the cylinder (solid), d = 50 cm = 0.5 m

Radius of the cylinder, r = d/2 = 0.25 m

Height of the inclined plane, h = 29.4 cm= 0.294 m

Length of the inclined plane, l= 10 m

Linear speed at the bottom, V =?

Angular speed at the bottom ω =?

Since the motion of the solid cylinder is rolling motion, therefore; P.E at the top will be converted into $K.E_{linear}$ and $K.E_{rotational}$

Loss of P.E total = gain of K.E total

$$m g h = \frac{1}{2} m V^2 + \frac{1}{2} I \omega^2$$

Moment of inertia of cylinder is $I = \frac{1}{2} mr^2$ and $\omega = \frac{V}{r}$

$$m g h = \frac{1}{2} m V^2 + \frac{1}{2} (\frac{1}{2} mr^2)(\frac{V}{r})^2 = \frac{1}{2} m V^2 + \frac{1}{4} mV^2 = \frac{3}{4} mV^2$$

$$V^2 = \frac{4}{3} gh = \frac{4}{3} \times 9.8 \times 0.294 = 3.84 m^2/s^2$$

$$V = 1.96 \text{ m/s}$$

Also

$$\omega = \frac{V}{r} = \frac{1.96}{0.25} = 7.84 \text{ rad/s}$$

9. Show that when a hoop rolls down on an inclined plane, half of kinetic energy is rotational and the other half is translational.

SOLUTION

At the top of the inclined plane the total energy of the hoop (ring) is only P.E. when it rolls down the inclined plane, some of the energy is translational and some is rotational so,

$$\text{Loss of P.E} = \text{Gain in K.E}_{lin} + \text{Gain in K.E}_{rot}$$

$$mgh = \frac{1}{2} m V^2 + \frac{1}{2} I \omega^2$$

Moment of inertia of hoop $I = mr^2$ and $\omega = \frac{V}{r}$

$$mgh = \frac{1}{2} m V^2 + \frac{1}{2} (mr^2)(\frac{V}{r})^2$$

$$mgh = \frac{1}{2} m V^2 + \frac{1}{2} mV^2 \qquad (1)$$

From Eq.1 it is clear that half of energy is translational energy and other half is rotational energy.

11. A disc without slipping rolls down a hill of vertical height 1000cm. if the disc starts from rest at the top of the hill, what is its magnitude of velocity at the bottom?

SOLUTION

Vertical height of hill is h = 1000 cm = 10 m

Since

$$\text{Loss in P.E} = \text{Gian in K.E}_{tran} + \text{Gain in K.E}_{rot}$$

$$m\,g\,h = \frac{1}{2} m V^2 + \frac{1}{2} I \omega^2$$

Moment of inertia of disc is $I = \frac{1}{2} mr^2$ and $\omega = \frac{V}{r}$

$$m\,g\,h = \frac{1}{2} m V^2 + \frac{1}{2} (\frac{1}{2} m r^2)(\frac{V}{r})^2 = \frac{1}{2} m V^2 + \frac{1}{4} mV^2 = \frac{3}{4} mV^2$$

$$V^2 = \frac{4}{3} gh = \frac{4}{3} \times 9.8 \times 10 = 129.96\ m^2/s^2$$

$$V = 11.4\ m/s$$