PROBLEMS

1. Water flows through a 1 cm diameter pipe with a speed of 1 ms^{-1}. What should be the diameter of the nozzle if the water is to emerge at 2.1 ms^{-1}?

SOLUTION Diameter of pipe, $d_1 = 1$ cm $= 0.01$ m

Speed of water in pipe, $V_1 = 1\ ms^{-1}$

Diameter of the nozzle, $d_2 = ?$

Speed of water at nozzle, $V_2 = 2.1\ ms^{-1}$

According to equation of continuity

$$A_1V_1 = A_2V_2$$

$$(\pi r_1^2)\ V_1 = (\pi r_2^2)\ V_2$$

$$(\pi d_1^2/4)\ V_1 = (\pi d_2^2/4)\ V_2$$

$$d_1^2\ V_1 = d_2^2\ V_2 0.01^2 \times 1 = d_2^2 \times 2.1$$

$$\frac{0.0001}{2.1} = d_2^2 = 0.000048$$

$$d_2 = \sqrt{0.000048} = 0.0069 = 0.69 \times 10^{-2}\ m$$

$$d_2 = 0.7\ cm$$

2. Water is flowing smoothly through a closed pipe system. At one point speed of water is 3 m/s, while at another point 3 m higher, the speed is 4.0 m/s. At the lower point, the pressure is 80 k pa. Find the pressure at the upper point.

SOLUTION Speed at lower end, $V_1 = 3.0ms^{-1}$

Height of lower end, $h_1 = 0$ m

Height of upper end, $h_2 = 3$ m

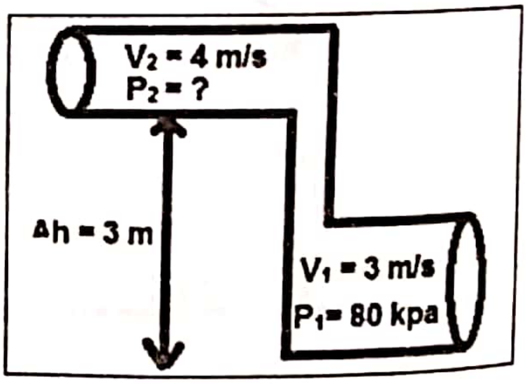
Speed at upper end, $V_2 = 4$ m/s

Pressure at lower end, $P_1 = 80$ kpa

$= 80 \times 10^3$ pa

Pressure at upper end, $P_2 = ?$

Density of water, $\rho = 1000\ kgm^{-3}$



Since we know that,

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

$$P_2 = P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 - \frac{1}{2}\rho V_2^2 - \rho g h_2$$

$$P_2 = P_1 + \frac{1}{2}\rho(V_1^2 - V_2^2) + \rho g(h_1 - h_2)$$

$$P_2 = 80\times10^3 + \frac{1}{2} \times 1000(3^2 - 4^2) + 1000 \times 9.8(0 - 3)$$

$$P_2 = 80\times10^3 - 3500 - 29400 = 47\times10^3 = 47\ kPa$$

3. An airplane wing is designed so that when the speed of the air across the top of the wing is 450 m s^{-1}, the speed of air below the win is 410 ms^{-1}. What is the pressure difference between the top and bottom of the wings?

SOLUTION

Speed of air above the wing, $V_1 = 450ms^{-1}$

Speed of air below the wing, $V_2 = 410ms^{-1}$

Pressure of air above the wing $= P_1$

Pressure of air below the wing $= P_2$

Difference of pressure, $\Delta P = P_2 - P_1 = ?$

Density of air, $\rho = 1.29\ kgm^{-3}$

Now, the aeroplane wings behave like horizontal region (pipe) So, P.E remains constant;

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_2 - P_1 = \frac{1}{2}\rho V_1^2 - \frac{1}{2}\rho V_2^2$$

$$\Delta P = \frac{1}{2}\rho(V_1^2 - (V_2^2) = \frac{1}{2} \times 1.29(450^2 - 410^2)$$

$$\Delta P = 0.645(202500 - 168100) = 22\times10^3\ kPa = 22\ kPa$$

4. **Water flows through a pipe whose internal diameter is 2 cm at a speed of $1 m s^{-1}$. What should be the diameter of the nozzle if the water is to emerge at a speed of $4\ m\ s^{-1}$.**

SOLUTION

Internal diameter of the pipe, $d_1 = 2\ cm = 0.02\ m$

Speed of water in the pipe, $V_1 = 1\ ms^{-1}$

Internal diameter of the nozzle, $d_2 = ?$

Speed of water at nozzle, $V_2 = 4\ ms^{-1}$

Now, from equation of continuity

$$A_1V_1 = A_2V_2$$

$$\Rightarrow (\pi r_1^2)\ V_1 = (\pi r_2^2)\ V_2$$

$$(\pi d_1^2/4)\ V_1 = (\pi d_2^2/4)\ V_2$$

$$\Rightarrow d_1^2\ V_1 = d_2^2\ V_2$$

$$0.02^2 \times 1 = d_2^2 \times 4$$

$$\frac{0.0004}{4} = d_2^2 = 0.0001$$

$$d_2 = \sqrt{0.0001} = 0.01\ m = 1\times10^{-2}\ m = 1\ cm$$

5. **Eight equal drops of oil are falling through air with a steady velocity of $0.1\ m\ s^{-1}$. If the drops recombine to from a single drop, what should be the new terminal velocity?**

SOLUTION

Terminal velocity of one drop, $V_t = 0.1\ ms^{-1}$

Radius of each drop $= r$

Volume of each drop, $V = 4/3\ \pi\ r^3$

Combine terminal velocity, $V_t' = ?$

When eight drops of equal size combine together the volume will increase 8-times, then the Combined volume is,

$$V' = 8\ V = 8 \times \frac{4}{3}\pi(r')^3$$

$$V' = 8\,V = \frac{4}{3}\pi(2r)^3$$

The new radius is; $r' = 2r$ (because $V \propto r^3$)

Therefore $V' = 8 \times \frac{4}{3}\pi(r')^3$

The Terminal velocity of a single drop is given by;

$$V_t = \frac{2\rho g r^2}{9\eta}$$

$$V_t' = \frac{2\rho g(r')^2}{9\eta} = \frac{2\rho g(2r)^2}{9\eta} = 4 \times \frac{2\rho g r^2}{9\eta} = 4\,V_t$$

Thus $V_t' = 4\,(0.1) = 0.4\ m/s$

6. Calculate the speed of efflux of kerosene oil from a narrow hole of a tank, in which pressure is 4 atm. Density of kerosene oil is 720 kg m^{-3}. (1 atm = 1.03×10^5 pa).

SOLUTION

Speed of efflux, V=?

Pressure in tank, $P = 4\ atm = 4 \times 1.3 \times 10^5\ pa = 412000\ pa$

Density of kerosene oil, $\rho = 720\ kgm^{-3}$

As $P = \rho g h \Rightarrow h = \frac{P}{\rho g} = \frac{412000}{720 \times 9.8} = 58.39$

The speed of efflux is; $V_{eff} = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 58.39} = 33.82\ m/s$

7. A small sphere of volume V falling in a viscous medium, acquires a terminal velocity V_t. What will be the terminal velocity of a sphere of same material and volume 8V, falling through the same medium?

SOLUTION

Terminal velocity of one drop = V_t

Radius of each drop = r

Volume of each drop $V = 4/3\ \pi\ r^3$

Combine terminal velocity, $V_t' = ?$

When eight drops of equal size combine together the volume will increase 8-times.

The combine volume is;

$$V' = 8\,V = \frac{4}{3}\pi\,(2r)^3$$

The new radius is; $r' = 2r$

Therefore (because $V \propto r^3$)

$$V' = 8 \times \frac{4}{3}\pi\,(r')^3$$

The Terminal velocity of a single drop is given by;

$$V_t = \frac{2\rho g r^2}{9\eta}$$

$$V_t' = \frac{2\rho g(r')^2}{9\eta} = \frac{2\rho g(2r)^2}{9\eta} = 4 \times \frac{2\rho g r^2}{9\eta}$$

$$V_t' = V_t$$

8. **Determine the radius of a water drop falling through air with a terminal velocity of 0.012 m s^{-1}. [Viscosity of air = 0.019 × 10^{-3} N s m^{-2}. Density of water = 1.0 × 10^{3} kg m^{-3}, Density of air = 1.29 kg m^{-3}]**

SOLUTION

Radius of the water drop, r =? $V_t = 0.012\ ms^{-1}$

Viscosity of air $\eta = 0.019 \times 10^{-3}$ N s m^{-2}

Density of water, $\rho = 1.0 \times 10^3$ kg m^{-3}

Density of air, $\sigma = 1.29$ kg m^{-3}

When water drop is falling through air then we also consider the buoyant force of air on the drop. In such case the terminal velocity is given by;

$$V_t = \frac{2(\rho-\sigma)gr^2}{9\eta}$$

$$r^2 = \frac{9\eta}{2\rho g} \times V_t$$

$$r^2 = \frac{9 \times 0.019 \times 10^{-3}}{2 \times (1.0 \times 10^3 - 1.29) \times 9.8} \times 0.012$$

$$r^2 = 1.0 \times 10^{-10}$$

$$r = \sqrt{1.0 \times 10^{-10}} = 1.0 \times 10^{-5}\ m = 0.01 \times 10^{-3} m$$

$$r = 0.01\ mm$$