PROBLEMS

1. A force of 0.04 N is required to displace a body attached to a spring through 0.1 m from its mean position. Calculate the spring constant of spring.

Solution

Force F = 0.04 N

Displacement x = 0.1 m

Spring constant K =?

As we know, from Hooke's law

$$F = k\,x$$

⇒ $$k = F / x = 0.04\ N / 0.1\ m = 0.4\ N/m$$

2. A body of mass 0.025 kg attached to a spring is displaced through 0.10 m to right of mean position. If spring constant of spring is 0.4 Nm^{-1} and its velocity at the end of this displacement be 0.4 ms^{-1}. Calculate (i) Time period 'T' (ii) Frequency 'f' (iii) Angular speed 'ω' (iv) The total energy (v) the amplitude of its motion (vi) the maximum velocity (vii) the maximum accelerations.

Solution

Mass m = 0.025 kg Displacement x = ~~10 cm = 0.01 m~~ 0.10 m

Spring Constant K = 0.4 N/m Velocity V = 0.4 m/s

1. T =? 2. f =? 3. ω =? 4. T.E =? 5. x_o =?

6. V_{max} =? 7. a_{max} =?

i. The time period of mass-spring system is calculated as follow:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.025}{0.4}} = 1.57\ s$$

ii. The frequency is given by:

$$f = 1/T = 1 / 1.57 = 0.637\ Hz$$

iii. To find angular speed, we proceed as;

$$\omega = 2\pi / T = 2 \times 3.14 / 1.57 = 4\ rad/s$$

iv. Now the total energy is;

$$E = K.E + P.E = ½\, mV^2 + ½\, Kx^2$$

$$E = ½\ 0.025 \times (0.4)^2 + ½\ 0.4\ (0.10)^2 = 4 \times 10^{-3}\ J$$

v. As we know that the total energy of mass attached to the spring is conserved.

$$P.E_{max} = E$$

$$½\, kx_o^2 = 4 \times 10^{-3}\ J$$

$$x_o^2 = 8 \times 10^{-3}\ J / 0.4 = 2 \times 10^{-2}\ J$$

$$x_o = 0.14\ m$$

vi. The maximum velocity

$$V_{max} = \omega x_o = 4 \times 0.14 = 0.56\ m/s$$

vii. The maximum acceleration, $a = -kx_o/m = -0.4 \times 0.14 / 0.025 = -2.24\ m/s^2$

3. A simple pendulum completes one vibration in one second calculate its length when $g = 9.81\ m\ s^{-2}$.

Solution

Frequency $f = 1\ Hz$ $\quad g = 9.81\ m/s^2$

Length of pendulum $L = ?$

Since $T = 1/f = 1\ s$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

⇒ $$T^2 = 4\pi^2\frac{L}{g}$$

⇒ $$L = gT^2/4\pi^2$$

⇒ $$L = 9.8 \times 1^2/4\pi^2 = 0.248\ m = 24.8\ cm$$

4. Calculate the length of a second pendulum having time period 2 seconds at a place where $g = 9.8\ ms^{-2}$.

Solution

Acceleration due to gravity $g = 9.8\ m/s^2$

Time period $T = 2\ sec$

Length $L = ?$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T^2 = 4\pi^2\frac{L}{g}$$

$$L = 9.8 \times 2^2/4\pi^2 = 0.99\ m = 99\ cm$$

5. A body of mass 'm', suspended from a spring with force constant k, vibrates with 'f_1'. When its length is cut into half and the same body is suspended from one of the halves, the frequency is 'f_2'. Find out $\frac{f_1}{f_2}$.

Solution

mass suspended = m $\quad$ Spring constant = k

Displacement before cutting $X_1 = X$

Displacement after cutting $= X_2 = X/2$

Frequency before cutting $= f_1$

Frequency after cutting $= f_2$

Frequency Ratio $\frac{f_1}{f_2} = ?$

a) Before cutting to half, the time period T_1 of the spring is

$$T_1 = 2\pi\sqrt{\frac{m}{k}} \qquad (1)$$

Since

$$K = \frac{F}{X_1} = \frac{mg}{X} \qquad (2)$$

Putting value from Eq. (2) in Eq. (1)

$$T_1 = 2\pi\sqrt{\frac{m}{\frac{mg}{X}}} = 2\pi\sqrt{\frac{mX}{mg}}$$

$$T_1 = 2\pi\sqrt{\frac{X}{g}} \quad (3)$$

Now the frequency f_1 is given by;

$$f_1 = 1/T_1 = \frac{1}{2\pi}\sqrt{\frac{g}{x}} \quad (4)$$

Similarly cutting the spring into half, the frequency f_2 of the mass spring system is;

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{g}{x_2}} = \frac{1}{2\pi}\sqrt{\frac{g}{X/2}}$$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{2g}{X}} \quad (5)$$

Dividing Eq. (4) by Eq. (5), we get;

$$\frac{f_1}{f_2} = \frac{\frac{1}{2\pi}\sqrt{\frac{g}{x}}}{\frac{1}{2\pi}\sqrt{\frac{2g}{X}}} = \sqrt{\frac{1}{2}}$$

$$\frac{f_1}{f_2} = \sqrt{\frac{1}{2}} = 0.707$$

6. A mass at the end of spring describes S.H.M with T = 0.40 s. Find out acceleration when the displacement is 0.04 m.

Solution

Time period T = 0.04 s

Displacement x = 4 cm = 0.04 m

Acceleration a =?

Since

$$a = -x\,\omega^2 = -x\frac{4\pi^2}{T^2}$$

$$a = -0.04 \times \frac{4\pi^2}{(0.04)^2}$$

$$a = -0.04 \times 246.46$$

$$a = -9.8596\ m/s^2 = -9.8\ m/s^2$$

7. A block weighing 4.0 kg extends a spring by 0.16 m from its un-stretched position. The block is removed and a 0.50 kg body is hung from same spring. If the spring is now stretched and the released, what is its period of vibration?

Solution

Mass = m_1 = 4.0 kg

Extension x = 0.16 m

g = 9.8 m/s²

m_2 = 0.50 kg

[illegible]ing constant K =?

$$k = \frac{mg}{X} = \frac{4.0 \times 9.8}{0.16} = 245 \text{ N/m}$$

Now when mass $m_2 = 0.50$ kg $\quad K = 245$ N/m $\quad$ Time period T =?

$$T = 2\pi\sqrt{\frac{m_2}{k}} = 2\pi\sqrt{\frac{0.50}{245}} = 0.28 \text{ s}$$

8. What should be the length of simple pendulum whose time period is one second? What is its frequency?

Solution

Time period T = 1 s $\quad$ $g = 9.8 \text{ m/s}^2$

Length of pendulum L =? $\quad$ Frequency =?

Since

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T^2 = 4\pi^2\frac{L}{g}$$

$$T^2 = 4\pi^2\frac{L}{g}$$

$$L = 9.8 \times 1^2/4\pi^2 = 0.25 \text{ m}$$

$$f = 1/T = 1/1 = 1 \text{ Hz}$$

9. A spring, whose spring constant is 80.0 Nm^{-1} vertically supports a mass of 1.0 kg in the rest position. Find the distance by which the mass must be pulled down, So that on being released, it may pass the mean position with velocity of one meter per second.

Solution

Spring constant K = 80.0 N/m

Mass attached to spring m = 1.0 kg

Velocity V = 1 m/s

Since $\quad V^2 = \omega^2 X_o^2$

⇒ $\quad 1 = k/m\, X_o^2 \quad (\omega^2 = k/m)$

⇒ $\quad 1 = (80/1)\,(X_o)$

⇒ $\quad X_o = 0.11$ m

10. A 800 g body vibrates in S.H.M with amplitude 0.30 m. The restoring force is 60 N and the displacement is 0.30 m. Find out (i) T (ii) a (iii) V (iv) K.E and P.E when the displacement is 12 cm.

Solution

m = 800 g = 0.8 kg

1. T = ? $\quad$ 2. a = ? $\quad$ $x_o = 0.30$ m $\quad$ $F_r = 60$ N

3. V = ?

4. K.E and P.E =? When x = 12 cm = 0.12m

The spring constant K is;

$$K = F_r / x_o = 60 / 0.3 = 200 \text{ N/m}$$

Now

1. $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.8}{200}} = 0.4\ s$

2. $a = k\,x / m = 200 \times 0.12 / 0.8 = 30\ m/s^2$

3. Since $V = x_o\,\omega$ &

$\omega = \frac{2\pi}{T} = \frac{2\times 3.14}{0.4} = 15.7$ ~~4.18~~ rad/s

Therefore

$V = 0.30 \times$ ~~4.18~~ = ~~1.4 m/s~~ 4.71

4. $K.E = ½\,k\,(x_o^2 - x^2) = ½\ 20 \times (0.3^2 - 0.12^2) = 7.6\ J$

5. $P.E = ½\,k\,x^2 = ½ \times 20 \times 0.12^2 = 1.44\ J$

11. Find the amplitude, frequency and period of an object oscillating at the end of a spring if the equation for its position at any instant t is given by x = 0.25 Cos (π/8)t. Find the displacement of the object after 2.0 sec.

Solution

The equation for the SHM given is;

$$x = 0.25\ Cos\ (\pi/8)t \qquad (1)$$

But the general equation for the displacement of SHM is given by;

$$x = x_0\ Cos\ \omega t \qquad (2)$$

Comparing Eq. (1) and Eq. (2), we get;

The amplitude $x_0 = 0.25$ units &

The angular frequency $\omega = (\pi/8)$

To find the frequency f, we have;

$$\omega = 2\pi f$$

$$f = \omega / 2\pi = \pi/8 \times \pi = 1/16\ Hz$$

Thus the time period T is;

$$T = 1/f = 1/1/16 - 16\ s$$

The displacement after 2 s is;

$$x = 0.25\ Cos\ (\pi/8 \times t)$$

$$x = 0.25\ Cos\ (\pi/8 \times 2)$$

$$x = 0.25\ Cos\ (180°/4)$$

$$x = 0.25\ Cos\ 45^0$$

$$x = 0.25 \times 0.707$$

$$x = 0.18\ units$$