CONTENTS

PREFACE

This new edition of Electronic In instrumentation and Measurement Techniques is a modernization of an old and effective text. The characteristics that has made this book suxessfu1 over the years have been retained while every effort was, taken to ensure a modern text that covers all aspects of instrumentation. To enforce this concept, the title has been changed to Modern Electronic Instrumentation and Measurement Techniques.

Basic measurement techniques such as accuracy, precision, standards, and so on, are retained, with some clarification and modernization to include new standards. Understanding these basics is an absolute prerequisite for the discussion of more sophisticated systems.

Some information concerning moving-coil meters was removed and modified, as these instruments find fewer applications in modern electronics. Some of the material is retained as an introduction to the general problems of measurement without bogging the reader down with excessively complex measuring systems.

The digital storage oscilloscope is a new subject, as its use has become more commonplace in recent years. The Fourier transform or digital spectrum analyzer is also included in this edition. These two digital instruments are gaining wide acceptance in electronic instrumentation.

Chapters 11 and 12 on transducers and data acquisition have received considerable overhaul to include more modern transducers and to include such important subjects as instrumentation and isolation amplifiers, and data transmission. An important inclusion in Chapter 12 is fiber optics data transmission, which is gaining rapid acceptance in the industrial environment.

Chapter 14 is totally new and covers fiber optics measurements. There is very little material available to the student on the subject of optical measurements relative to fiber optics, and this chapter makes this edition unique.

Above all, those items that make a book a textbook, such as worked-out examples, references, and review problems at the end of the chapters have been retained and expanded.

ACKNOWLEDGMENTS

Grateful appreciation is offered to the following individuals for reviewing our work: David G. Delker, Kansas State University; Val Feldkircher, Electronic Technology Institute; Dewey J. Gray, Nashville State College; Richard A. Hultin, Rochester Institute of Technology; Earl C. Iselin, Jr., University of Dayton; O. M. Kuritza; William Middendorf, University of Cincinnati; and Donald J. Poulin, Northeastern University.

MEASUREMENT AND ERROR

1.1 DEFINITIONS

Measurement generally involves using an instrument as a physical means of determining a quantity or variable. The instrument serves as an extension of human faculties and in many cases enables a person to determine the value of an unknown quantity which his unaided human faculties could not measure. An instrument, then, may be defined as *a device for determining the value or magnitude of or variable*. The electronic instrument, as its name implies, is based on electrical or electronic principles for its measurement function. An electronic instrument may be a relatively uncomplicated device of simple construction such as a basic dc current meter (see Chapter 4). As technology expands, however, the demand for more elaborate and more accurate instruments increases and produces new developments in instrument design and application. To use these instruments intelligently, one needs to understand their operating principles and to appraise their suitability for the intended application.

Measurement work employs a number of terms which should be defined here.

Instrument: a device for determining the value or magnitude of a quantity or variable.

Accuracy: closeness with which an instrument reading approaches the true value of the variable being measured.

Precision: a measure of the reproducibility of the measurements; i.e., given a fixed value of a variable, precision is a measure of the degree to which successive measurements differ from another.

Sensitivity: the ratio of output signal or response of the instrument to a change of input or measured variable.

Resolution: the smallest change in measured value to which the instrument will respond.

Error: deviation from the true value of the measured variable.

Several techniques may be used to minimize the effects of errors. For example, in making precision measurements, it is advisable to record a series of observations rather than rely on one observation. Alternate methods of measurement, as well as the use of different instruments to perform the same experiment, provide a good technique for increasing accuracy. Although these techniques tend to increase the *precision* of measurement by reducing error, they cannot account for instrumental error.*

This chapter provides an introduction to different types of error in measurement and to the methods generally used to express errors, in terms of the most reliable value of the measured variable.

1.2 ACCURACY AND PRECISION

Accuracy refers to the degree of closeness or conformity to the true value of the quantity under measurement. Precision refers to the degree of agreement within a group of measurements or instruments.

To illustrate the distinction between accuracy and precision, two voltmeters of the same make and model may be compared. Both meters have knife-edged pointers and mirror-backed scales to avoid parallax and they have carefully calibrated scales. They may therefore be read to the same *precision*. If the value of the series resistance in one meter changes considerably, its readings may be in error by a fairly large amount. Therefore the *accuracy* of the two meters may be quite different. (To determine which meter is in error, a comparison measurement with a standard meter should be made.)

Precision is composed of two characteristics: *Conformity* and the number of *significant figures* to which a measurement maybe made. Consider, for example, that a resistor, whose true resistance is 1,384,572 Ω, is measured by an ohmmeter which consistently and repeatedly indicates 1.4 MΩ. But can the observer "read" the true value from the scale? His estimates from the scale reading consistently yield a value of 1.4 MΩ. This is as close to the true value as he can read the scale by estimation. Although there are no deviations from the observed value, the error created by the limitation of the scale reading is a *precision* error. The example illustrates that conformity is a necessary, but not sufficient, condition for precision because of the lack of significant figures obtained. Similarly, precision is a necessary, but not sufficient, condition for accuracy.

Too often the beginning student is inclined to accept instrument readings at face value. He is not aware that the accuracy of a reading is not necessarily guaranteed by its precision. In fact, good measurement technique demands continuous skepticism as to the accuracy of the results.

In critical work, good practice dictates that the observer make an independent Set ‘of measurements, using different instruments or different measurement techniques not subject to the same systematic errors: He must also make sure that the instruments function properly and are calibrated against a known standard, and that no outside influence affects the accuracy of his measurements.

1.3 SIGNIFICANT FIGURES

An indication of the precision of the measurement is obtained from the number of significant figures in which the result is expressed. Significant figures convey actual information regarding the magnitude and the measurement precision of a quantity the more significant figures, the greater the precision of measurement.

For example, if a resistor is specified as having a resistance of 68Ω, its resistance should be closer to 68Ω than to 67Ω or 69Ω. If the value of the resistor is described as 68.0Ω, it means that its resistance is closer to 68.0Ω than it is to 67.9Ω or 68.1Ω. In 68Ω there are two significant figures; in 68.0Ω there are three. The latter, with more significant figures, expresses a measurement of greater precision than the former.

Often, however, the total number of digits may not represent measurement precision. Frequently, large numbers with zeros before a decimal are used for approximate populations or amounts of my. For example, the population of a city is reported in six figures as 380,000. This may imply that the true value of the population lies between 379,999 and 380,001, which is six significant figures. What is meant, however, is that the population is closer to 380,000 than to 370,000 or 390,000. Since in this case the population can be reported only to two significant figures, how can large numbers be expressed?

A more technically correct notation uses powers of ten, 38×10^4 or $3.8 \times l0^5$. This indicates that the population figure is only accurate to two significant figures. Uncertainty caused by zeros to the left of the decimal point is therefore usually resolved by scientific notation using powers of ten. Reference. to the velocity of light as 186,000 mi/s, for example, would cause no misunderstanding to anyone with a technical background. But 1.86 x 10 mi/s leaves no confusion.

It is customary to record a measurement with all the digits of which we are sure nearest to the true value. For example, in reading a voltmeter, the voltage may be read as 117.1 V. This simply indicates that the voltage, read by the observer to best estimation, is closer to 117.1 V than to 117.0 V or 117.2 V.

Another way of expressing result indicates the range of possible error. The voltage may be expressed 117.1 ± 0.05 V, indicating that the value of the voltage lies between 117.15 V and 117.15 V.

When a flu measurements are taken in an effort to obtain the best possible answer (closest to the true value), the result is usually expressed as the arithmetic mean of all the readings, with the range of possible error as the largest deviation from that mean. This is illustrated in Example 1.1.

EXAMPLE: 1-1

A set of independent voltage measurements taken by four observers was recorded as 117.02 V, 117.11 V, 117.08 V, and 117.03 V. Calculate (a) the average voltage; (b) the range of error

SOLUTION

a. $E_{av} = \frac{E_1 + E_2 + E_3 + E_4}{N}$

$$= \frac{117.02 + 117.11 + 117.08 + 117.03}{4} = 117.06V$$

b. Rang = $E_{max} - E_{av}$ = 117.11 – 117.06 = 0.05V

But also

$$E_{av} - E_{min} = 117.06 - 117.02 = 0.04 \text{ V}$$

The average range of error therefore equals

$$\frac{0.05 + 0.04}{2} = \pm 0.045 = \pm 0.05V$$

When two or more measurements with different degrees of accuracy are added, the result is only as accurate as the least accurate measurement. Suppose that two resistances are added in series as in Example 1-2.

EXAMPLE: 1-2

Two resistors, R_1 and R_2, are connected in series. Individual resistance measurements, using a digital multimeter, give R_1 = 18.7Ω and R_2 3.624Ω. Calculate the total resistance to the appropriate number of significant figures.

SOLUTION

R_1 = 18.7Ω (three significant figures)

R_2 = 3.624Ω (four significant figures)

$R_T = R_1 + R_2$ = 22.324 Ω (five significant figures) = 22.3Ω

The doubtful figures are written in italics to indicate that in the addition of R_1 and R_2 the last three digits of the sum are doubtful figures. There is no value whatsoever in retaining the last two digits (the 2 and the 4) because one of the resistance is accurate only to three significant figures or tenths of an ohm. The result should therefore also be reduced to three significant figures or the nearest tenth, i.e., 22.3 Ω.

The number of significant figures in multiplication may increase rapidly, but again only the appropriate figures are retained in the answer, as shown in Example 1-3.

EXAMPLE: 1 – 3

In calculating voltage drop, a current of 3.18 A is recorded in a resistance of 35.68 Ω. Calculate the voltage drop across the resistor to the appropriate number of significant figures.

SOLUTION

$$E = IR = (35.68) \times (3.18) = 113.4624 = 113V$$

Since there are three significant figures involved in the multiplication, the answer can be written only to a maximum of three significant figures.

In Example 1-3, the current, I, has three significant figures and R has four; and the result of the multiplication has only three significant figures. This illustrates that the answer cannot be known to any accuracy greater than the least poorly defined of the factors. Note also that if extra digits accumulate in the answer, they should be discarded or rounded off. In the usual practice, if the (least significant) digit in the first place to be discarded is less than five, it and the following digits are dropped from the answer. This was done in Example 1-3. If the digit in the first place to be discarded is five or greater, the previous digit is increased by one. For three-digit precision, therefore, 113.46 should be rounded off to 113; and 113.74 to 114.

Addition of figures with a range of doubt is illustrated in Example 1-4.

EXAMPLE: 1 – 4

Add 826 ± 5 to 628 ± 3

SOLUTION

N_1 = 826 ± 5 (= ±0.605%)

N_2 = 628 ± 3 (= ±0.477%)

Sum = 1,454 ± 8 (= ±0.55%)

Note in Example 1-4 that the doubtful parts are added, since the ± sign means that one number may be high and the other low. The worst possible combination of range of doubt should be taken in the answer. The percentage doubt in the original figure N_1 and N_2 does not differ greatly from the percentage doubt in the final result.

If the same two numbers are subtracted, as in Example 1-5, there is an interesting comparison between addition and subtraction with respect to the range of doubt.

EXAMPLE: 1-5

Subtract 628 ± 3 from 826 ± 5 and express the range of doubt in the answer as a percentage.

SOLUTION

N_1 = 826 ± 5 (= ± 0.605%)

N_2 = 437 ± 4 (= ± 0.92%)

Difference = 198 ± 8 (= ± 4.04%)

Again, in Example 1-5, the doubtful parts are added for the same reason as in Example 1-4. Comparing the results of addition and subtraction of the same numbers in Examples 1-4 and 1-5, note that the precision of the results, when expressed in percentages, differs greatly. The final result after subtraction shows a large increase in percentage doubt compared the percentage doubt after addiction. The percentage doubt increases even more when the difference between the number is relatively small. Consider the case illustrated in Example 1-6.

EXAMPLE: 1-6

Subtract 437 $\pm$ 4 and express the range of doubt in the answer as a percentage.

SOLUTION

N_1 = 462 $\pm$ 4 (= $\pm$ 0.87%)

N_2 = 437 $\pm$ 4 (= $\pm$ 0.92%)

Difference = 25 $\pm$ 8 (= $\pm$ 32%)

Example 1-6 illustrates clearly that one should avoid measurement techniques depending on subtraction of experimental results because the range of doubt in the final result may be greatly increased.

1.4 TYPES OF ERROR

No measurement can be made with perfect accuracy, but it is important to find out what the accuracy actually is and how different errors have entered into the measurement. A study of errors is a first step in finding ways to reduce them. Such a study also allows us to determine the accuracy of the final test result.

Errors come from different sources and are usually classified under three main headings:

Gross errors: largely human errors, among them misreading of instruments incorrect adjustment and improper application of instruments, and computational mistakes.

Systematic errors: shortcomings of the instruments, such as defective or worn parts, and effects of the environment on the equipment or the user.

Random errors: those due to causes that cannot be directly established because of random variations in the parameter or the system of measurement.

Each of these classes of errors will be discussed briefly and some methods will be suggested for their reduction or elimination.

1-4.1 Gross Errors

This class of errors mainly covers human mistakes in reading or using instruments and in recording and calculating measurement results. As long as human beings are involved, some gross errors will inevitably be committed. Although complete elimination of gross errors is probably impossible, one should try to anticipate and correct them. Some gross errors are easily detected; others may be very elusive. One common gross error, frequently committed by beginners in measurement work, involves the improper use of an instrument., In general, indicating instruments change conditions to some extent when connected into a complete circuit, so that the measured quantity is altered by the method employed For example, a well-calibrated voltmeter may give a misleading reading when connected across two points in a high-resistance circuit (Example 1-7). The same voltmeter, when connected in a low-resistance circuit, may give a more dependable reading Example 1-8). These examples illustrate that the voltmeter has a "loading effect" on the circuit, altering the original situation by the measurement process.

EXAMPLE: 1-7

A voltmeter, having a sensitivity of 1,000 Ω/V, reads 100 V on its 150-V scale when connected across an unknown resistor in series with a milliammeter. When the milliammeter reads 5 mA, calculate (a) the apparent resistance of the unknown resistor; (b) the actual resistance of the unknown resistor; (c) the error due to the loading effect of the voltmeter.

SOLUTION

a. The total circuit resistance equals

$$R_T = \frac{V_T}{I_T} = \frac{100V}{5mA} = 20k\Omega$$

Neglecting the resistance of the milliammeter, the value of the unknown resistor is R_x 20 kΩ,

b. The voltmeter equals

$$RV = 1{,}000\frac{\Omega}{V} \times 150V = 150k\Omega$$

Since the voltmeter is in parallel with the unknown resistance, we can write

$$RX = \frac{R_T R_V}{R_V - R_T} = \frac{20 \times 150}{130} = 23.05k\Omega$$

c. % Error = $\frac{\text{actual} - \text{apparent}}{\text{actual}} \times 100\% = \frac{23.05}{23.05} \times 100\%$

= 13.23%

EXAMPLE: 14 – 8

Repeat Example 1-7 if the milliammeter reads 800 mA and the voltmeter reads 40 V on its 150V scale.

a. $R_T = \frac{V_T}{I_T} = \frac{40V}{0.8A} = 50\Omega$

b. $RV = 1{,}000\frac{\Omega}{V} \times 150V = 150k\Omega$

$$RX = \frac{RTRV}{RV - RT} = \frac{50 \times 150}{149.95} = 50.1\Omega$$

c. $\%\,\text{Error} = \frac{50.1 - 50}{50.1} \times 100\% = 0.2\%$

Errors caused by the loading effect of the voltmeter can be avoided by using it intelligently. For example, a low-resistance voltmeter should not be used to measure voltages in a vacuum tube amplifier. In this particular measurement, a high-input impedance voltmeter (such as a VTVM or TVM) is required.

A large number of gross errors can be attributed to carelessness or bad habits, such as improper reading of an instrument, recording the result differently from actual reading taken, or adjusting the instrument, incorrectly. Consider the case in which a multirange voltmeter uses a single set of scale markings with different number designations for the various voltage ranges. It is easy to use a scale which does not correspond to the setting of the range selector of the voltmeter 4 gross error may also occur when the instrument is not set to zero before the measurement is taken; then all the readings are off.

Errors like these cannot be treated mathematically. They can be avoided only by taking care in reading and recording the measurement data. Good practice requires making more than one reading of the same quantity, preferably by a different observer. Never place complete dependence on one reading but take at least three separate readings, preferably under conditions in which instruments are switched off-on.

1- 4.2 Systematic Errors

This type of error, is usually divided into two different categories: 1. instrumental error, defined as shortcomings of the instruments; 2. environmental errors, due to extern conditions affecting the measurement.

Instrumental errors are errors inherent in measuring instruments because of their mechanical structure. For example the d'Arsonval movement friction in bearings of various moving components may cause incorrect readings. Irregular spring tension, stretching of the spring, or reduction in tension due to improper handling or overloading of the instrument will result in errors. Other instrumental errors are calibration errors, causing the instrument to read high or low along its entire scale. (Failure to set the instrument to zero before making a measurement has a similar effect.)

There are many kinds of instrumental errors, depending on the type of instrument used. The experimenter should always take precautions to insure that the instrument he is using is operating properly and does not contribute excessive errors for the purpose at hand. Faults in instruments may be detected by checking for erratic behavior, and stability and reproducibility of results. A quick and easy way to check an instrument is to compare it to another with the same characteristics or to one that is known to be more accurate.

Instrumental errors may be avoided by (1) selecting a suitable instrument for the particular measurement application; (2) applying correction factors after determining the amount of instrumental error; (3) calibrating the instrument against a standard.

Environmental errors are due to conditions external to the measuring device, including conditions in the area surrounding the instrument, such as the effects of changes in temperature, humidity, barometric pressure, or of magnetic or electrostatic fields. Thus a change in ambient temperature at which the instrument causes a change in the elastic properties of the spring in a moving-coil mechanism and so affects the reading of the instrument. Corrective measures to reduce these effects include air conditioning, hermetically sealing certain components in the instrument, use of magnetic shields, and the like.

Systematic errors can also be subdivide into static or dynamic errors. Static errors caused by limitations of the measuring device or the physical laws governing its behavior. A static error is introduced in a micrometer when excessive pressure is applied in torquing the shaft. Dynamic errors are caused by fast enough to follow the changes in a measured variable.

1-4.3 Random Errors

These errors are due to unknown causes and occur even when a1l systematic errors have been accounted for. In we-design experiments, few random errors usually occur, but they become important in high-accuracy work. Suppose a voltage is being monitored by a voltmeter which is read at half-hour intervals. Although the instrument is operated under ideal environmental conditions and has been accurately calibrated - measurement, it will be found that the readings vary slightly over the observation. This variation cannot be corrected by any method ‘or other known method of control and it cannot be explained till investigation. The only way to offset these errors is by increasing the number of readings and using statistical means to obtain the best approximation of the true value of the quantity under measurement.

1-5 STATISTICAL ANALYSIS

A statistical analysis of measurement data is common practice because it allows an analytical determination of the uncertainty of the final test result. The outcome of a certain measurement method may be predicted on the basis of sample data without having detailed information on all the disturbing factors. To make statistical methods and interpretations meaningful, a large number of measurements is usually required. Also, systematic errors should be small compared with residual or random errors, because statistical treatment of data cannot remove a fixed bias contained in all the measurements.

1-5.1 Arithmetic Mean

The most probable value of a measured variable is the arithmetic mean of the number of readings taken. The best approximation will be made when the number of readings of the same quantity is very large. Theoretically, an infinite number of readings would give the best result, although in practice, only a finite number of measurements can be made. The arithmetic mean is given by the following expression:

$$\bar{x}\frac{x_1 + x_2 + x_3 + x_4 \ldots\ldots + x_n}{n} = \frac{\sum x}{n} \qquad (1\text{-}1)$$

Where $\bar{x}$ = arithmetic mean

x_1, x_2, x_n = readings taken

n = number of readings

Example 1-1 showed how the arithmetic mean is used.

1-5.2 Deviation from the Mean

Deviation is the departure of a given reading from the arithmetic mean of the group of readings. If the deviation of the first reading, x_1, is called d_1, and that of the second reading, x_2, is called d_2, and so on, then the deviations from the mean can be expressed as

$$d_1 = x_1 - \bar{x} \qquad d_2 = x_2 - \bar{x} \qquad d_n = x_n - \bar{x} \qquad\qquad (1\text{-}2)$$

Note that the deviation from the mean may have a positive or a negative value and that the algebraic sum of all the deviations must be zero.

Example 1 – 9 illustrates current measurements was taken by six observers and recorded as 12.8 mA, 12.2 mA, 12.5 mA, 13.1 mA, 12.9 mA, and 12.4 mA.

Calculate (a) the arithmetic mean; (b) the deviations from the mean.

SOLUTION:

a. using Eq. (1-1), we see that the arithmetic mean equals

$$\bar{x} = \frac{12.8 + 12.2 + 12.5 + 13.1 + 12.9 + 12.4}{6} = 12.65\text{mA}$$

b. Using Eq. (1-2), we see that the deviations are

d_1 = 12.8 – 12.65 = 0.15 mA

d_2 = 12.2 – 12.65 = 0.45 mA

d_3 = 12.5 – 12.65 = 0.15 mA

d_4 = 13.1 – 12.65 = 0.45 mA

d_5 = 12.9 – 12.65 = 0.25 mA

d_6 = 12.4 – 12.65 = -0.25 mA

Note that the algebraic sum of all the deviations equals zero.

1-5.3 Average Deviation

The average deviation is an indication of the precision of the instruments used in making the measurements. Highly precise instruments will yield a low average deviation between readings. By definition, average deviation is the absolute values of the deviations divided by the number of readings. The absolute value of the deviation is the value without respect to sign. Average deviation may be expressed as

$$D = \frac{|d_1| + |d_2| + |d_3| + ...|d_n|}{n} = \frac{\sum |d|}{n} \qquad (1\text{-}3)$$

Example 1-10 shows how average deviation is calculated.

EXAMPLE: 1-10

Calculate the average deviation for the data given in Example 1-9.

SOLUTION

$$\mathrm{D} = \frac{0.15 + 0.45 + 0.15 + 0.45 + 0.25 + 0.25}{6} = 0.283\mathrm{mA}$$

1-5.4 Standard

In statistical analysis the root-mean-square deviation or standard deviation is a very valuable aid. By definition, the standard deviation of an infinite number of data is the square root of the sum of all the individual deviations squared, divided by the number of readings. Expressed mathematically:

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + ...d_n^2}{n}} = \sqrt{\frac{\sum d_t^2}{n}} \qquad (1\text{-}4)$$

In practice, of course, the possible number of observations is finite. The standard deviation of a finite number of data is given by

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \dots\dots d_n^2}{n-1}} = \sqrt{\frac{\sum d_t^2}{n-1}} \qquad (1\text{-}5)$$

Equation (1-5) will be used in Example 1-11.

Another expression for essentially the same quantity is the variance or mean square deviation, which is the same as the standard deviation except that the square root is not extracted. Therefore

variance (V) = mean square deviation = o.2

The variance is a convenient quantity to use in many computations because variances are additive. The standard deviation, however, has the advantage of being of the same units as the variable, making it easy to compare magnitudes. Most scientific results are now stated in terms of standard deviation.

1-6 PROBABILITY OF ERRORS

1-6.1 Normal Distribution of Errors

Table 1-1 shows a tabulation of 50 voltage readings that were taken at small time intervals and recorded to the nearest 0.! V. The nominal value of the measured voltage was 100.0 V. The result of this series of measurements can be presented

TABLE 1-1 Tabulation of Voltage Readings

Voltage reading	**Number of readings**

(volts)	
99.7	1
99.8	4
99.9	12
100.0	19
100.1	10
100.2	3
100.3	1
	50

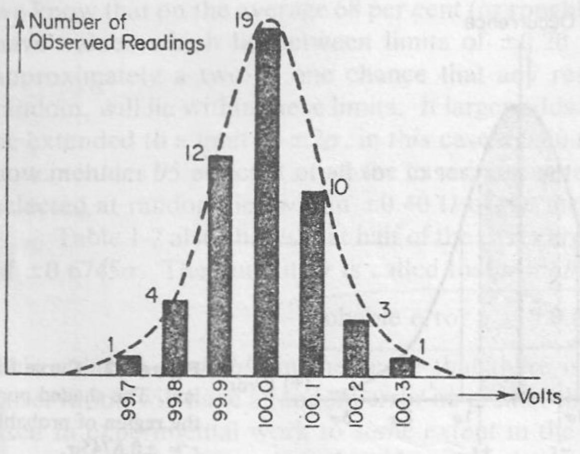

Figure 1-1 Histogram showing the frequency of occurrence of the 50 voltage readings of Table 1-1. The dashed curve represents the limiting case of the histogram when a large number of readings at small increments are taken.

graphically in the form of a block diagram or histogram in which the number of observations is plotted against each observed voltage reading. The histogram of Fig. 1-1 represents the data of Table 1-1.

Figure 1-1 shows that the largest number of readings (19) occurs at the central value of 100.0 V, while the other readings are placed more or less symmetrically on either side of the central value. If more readings were taken at smaller increments, say 200 readings at 0.05-V intervals, the distribution of observations would remain approximately symmetrical about the central value and the shape of the histogram would be about the same as before. With more and more data, taken at smaller and smaller increments, the contour of the histogram would finally become a smooth curve, as indicated by the dashed line in Fig. 1-1. This bell-shaped curve is known as a Gaussian curve. The sharper and narrower the curve, the more definitely an observer may state that the most probable value of the true reading is the central value or mean reading.

The Gaussian or Normal law of error forms the basis of the analytical study of random effects. Although the mathematical treatment of this subject is beyond the scope of this text, the following qualitative statements are based on the Normal law:

a. All observations include small disturbing effects, called random errors.

b. Random errors can be positive or negative.

c. There is an equal probability of positive and negative random errors.

We can therefore expect that measurement observations include plus and minus errors in more or less equal amounts, so that the total error will be small and the mean value will be the true value of the measured variable.

The possibilities as to the form of the error distribution curve can be stated as follows

a. Small errors are more probable than large errors.

b. Large errors are very improbable.

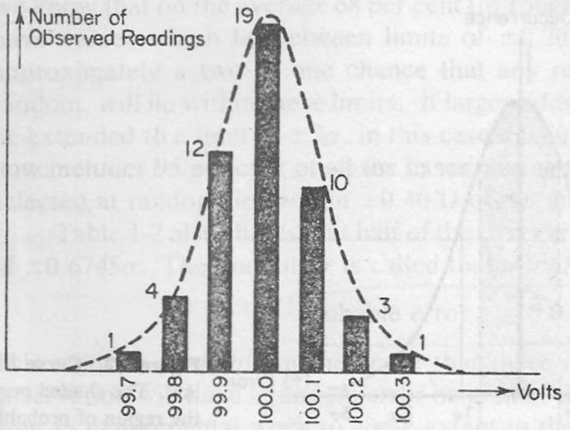

Figure 1-1 Histogram showing the frequency of occurrence of the 50 voltage readings of Table 1-1. The dashed curve represents the limiting case of the histogram when a large number of readings at small increments are taken.

c. There is an equal probability of plus and minus errors so that the probability of a given error will be symmetrical about the zero value.

The error distribution curve of Fig. 1-2 is based on the Normal law and shows a symmetrical distribution of errors. This normal curve may be regarded as the limiting form of the histogram of Fig. 1-1 in which the most probable value of the true voltage is the mean value of 100.0 V.

1-6.2 Probable Error

The area under the Gaussian probability curve of Fig. 1-2. between the limits $+\infty$ and $-\infty$, represents the entire number of observations. The area under the curve between the $+\sigma$ and $-\sigma$ limits represents the cases that differ from the mean by no more than the standard deviation. Integration of the area under the curve within the ± σ limits gives the total number of cases within these limits. For normally dispersed data, following the Gaussian distribution, approximately 68 percent of all the cases lie between the limits of + σ and - σ from the mean. Corresponding values of other deviations, expressed in terms of σ, are given in Table 1-2.

If, for example, a large number of nominally 100-Ω resistors is measured and the mean value is found to be 100.00σΩ, with a standard deviation (S.D.) of 0.20Ω.

TABLE 1-2 Area Under the Probability curve

Deviation (±), σ	Fraction of total area included
0.6745	0.5000
1.0	0.6828
2.0	0.9546
3.0	0.9972

We know that on the average 68 percent (or roughly two-thirds) of all the resistors have values which lie between limits of ± 0.20 Ω of the mean. There is then approximately a two to one chance that any resistor, selected from the lot at random, will lie within these limits. If larger odds are required, the deviation may be extended to a limit of ± 2σ, in this case ± 0.40Ω. According to Table 1-2, this now includes 95 percent of all the cases, giving ten to one odds that any resistor selected at random lies within ± 0.40Ω of the mean value of 100.00Ω.

Table 1-2 also shows that half of the cases are included in the deviation limits of ± 0.6745σ. The quantity *r* is called the *probable error* and is defined as

$$\text{Probable error } r = \pm\, 0.6745\sigma \qquad (1\text{-}6)$$

This value is probable in the sense that there is an even chance that any one observation will have a random error no greater than ± *r*. Probable error has been used in experimental work to some extent in the past, but standard deviation is more convenient in statistical work and is given preference.

EXAMPLE: 1-11

Ten measurements of the resistance of a resistor gave 101.2Ω, 101.7Ω, 101.3Ω, 101.0Ω, 101.5Ω, 101.3Ω, 101.2Ω, 101.4Ω, 101.3Ω, and 101.1Ω. Assume that only random errors are present. Calculate (a) the arithmetic mean; (b) the standard deviation of the readings; (c) the probable error.

SOLUTION With a large number of readings a simple tabulation of data is very convenient and avoids confusion and mistakes.

Reading, x	***Deviation***	
	d	***d^2***
101.2	-0.1	0.01
101.7	0.4	0.16
101.3	0.0	0.00
101.0	-0.3	0.09
101.5	0.2	0.04
101.3	0.0	0.00
101.2	-0.1	0.01
101.4	0.1	0.01
101.3	0.0	0.00
101.1	-0.2	0.04

$\sum x = 1.013.0$	$\sum \lvert d \rvert = 1.4$	$\sum d^2 = 0.36$

a. Arithmetic mean, $\bar{x} = \frac{\sum x}{n} = \frac{1,013.0}{10} = 101.3\Omega$

b. Standard deviation, $\sigma = \sqrt{\frac{d^2}{n-1}} = \sqrt{\frac{0.36}{9}} = 0.2\Omega$

c. Probable error = 0.6745 σ = 0.6745 x 0.2 = 0.1349Ω

1-7 LIMITING ERRORS

In most indicating instruments the accuracy is guaranteed to a certain percentage of full-scale reading. Circuit components (such as capacitor, resistors, etc.) are guaranteed within a certain percentage of their rated value. The limits of these deviations from the specified values are as limiting errors or guarantee errors. For example, if the resistance of a resistor is given as 500Ω ± 10 percent, the manufacturer guarantees that the resistance falls between the limits 450Ω and 550Ω. The maker is not specifying a standard deviation or a probable error, but promises that the error is no greater than the limits set.

EXAMPLE 1-12

A 0-150-V voltmeter has a guaranteed accuracy of 1 percent full-scale reading. The voltage measured by this instrument is 83 V. Calculate the limiting error in percent.

SOLUTION

The magnitude of the limiting error is

$$0.01 \times 150V = 1.5V$$

The percentage error at a meter indication of 83 V is

$$\frac{1.5}{83} \times 100 \text{ percent} = 1.81 \text{ percent}$$

It is important to note in Example 1-12 that a meter is guaranteed to have an accuracy of better than 1 percent of the full-scale reading, but when the meter reads 83V the limiting error increases to 1.81 percent. Correspondingly, when a smaller voltage is measured, the limiting error will increase further. If the meter reads 60V, the percent limiting error is 1.5/60 x 100 = 2.5 percent; if the meter reads 30V, the limiting error is 1.5/30 x 100 = 5 percent. The increase in percent limiting error, as smaller voltages are measured, occurs because the magnitude of the limiting error is fixed quantity based on the full scale reading of the meter. Example 1-2 shows the importance of taking measurements as close to full scale as possible.

Measurements or computations, combining guarantee errors, are often made. Example 1-13 illustrates such a computation.

EXAMPLE: 1-13

The voltage generated by a circuit is equally dependent on the value of three resistors and is given by the following equation:

$$V_{out} = \frac{R_1 R_2}{R_3}$$

If the tolerance of each resistor is 0.1 percent, what is the maximum error of the generated voltage?

SOLUTION

The highest resulting voltage occurs when R_1 and R_2 are at the maximum value allowed by the tolerance, while R_3 is at the lowest value allowed by the tolerance. The actual value need not be known but only the relative value. For a variation of 0.1 percent the highest value of a resistor is 1.001 times the nominal value, while the lowest value is 0.999 times the nominal value. Using the maximum value of R_1, and R_2 and the minimum value for R_3 results in the greatest value for V_{out} of

$$V_{out} = \frac{(1.001R_1)(1.001R_2)}{0.999R_3} = 1.003$$

The lowest resulting voltage occurs when the value of R_3 is highest and R_1 and R_2 are the lowest. The resulting voltage is

$$V_{out} = \frac{(0.999R_1)(0.999R_2)}{1.003R_3} = 0.997$$

The total variation of the resultant voltage is ± 0.3 percent, which is the algebraic sum of the three tolerances. This is true in the first approximation. The maximum error is slightly different from the sum of the individual tolerances. On the other hand, it is highly unlikely that all three components of this example would have the maximum error and in such a fashion to produce the maximum or minimum voltage. Therefore, the statistical methods outlined in the previous sections must be used.

EXAMPLE: 1-14

The current passing through a resistor of $100 \pm 0.2\ \Omega$ is 2.00 ± 0.01 A. Using the relationship $P = I^2R$, calculate the limiting error in the computed value of power dissipation.

SOLUTION

Expressing the guaranteed limits of both current and resistance in percentages instead of units, we obtain

$$I = 2.00 \pm 0.01 \text{ A} = 2.00 \pm 0.5\%$$

$$R = 100 \pm 0.2\% = 100 \pm 0.2\%$$

It the worst possible combination of errors for the calculation of power, that is, the highest value of resistance and the highest value of current, is used, the power dissipation becomes

$$P = I^2 (1 + 0.005)^2 R(l.002) = l.012I^2R$$

For the lowest power dissipation,

$$P = I^2 (1 - 0.005)^2 R(l\text{-}0.002) = 0.988I^2R$$

The error is ± 1.2 percent, which is two times the 0.5 percent error of the current plus the 0.2 percent error of the resistor. This is because the I term of the equation essentially appears twice in the equation. This can be seen by rewriting the equation

$$P = I \times I \times R = I^2R$$

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1-2. Maloney, Timothy J., Electric Circuits: Principles and Applications, chap. I. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1984.

1-3. Young, Hugh D., Statistical Treatment of Experimental Data. New York: McGraw- Hill Book Company, 1962.

PROBLEMS

1-1. What is the difference between accuracy and precision?

1-2. List four sources of possible errors in instruments.

4-3. What are the three general classes of errors?

1-4. Define (a) instrumental error; (b) limiting error; (c) calibration error; (d) environmental error; (e) random error; (f) probable error.

1-5 A 0-I-mA milliammeter has 100 divisions which can easily be read to the nearest division. What is the resolution of the meter?

1-6. A digital voltmeter has .a read-out range from 0 to 9,999 counts. Determine the resolution of the instrument in volts when the full-scale reading is 9.999V.

1-7. State the number of significant figures in each of the following: (a) 542; (b) 0.65; (c) 27.25; (d) 0.00005; (e) 40 x 10^6; (f) 20,000.

1-8. Four capacitors are placed in parallel. The capacitor values are 36.3 μF, 3.85 μF, 34.002 μF, and 850 nF, with an uncertainty of one digit in the last place. What is the total capacitance? Give only the significant figures in the answer.

1-9. A voltage drop of 112.5V is measured across a resistor passing a current of 1:62 A. Calculate the power dissipation of the resistor. Give only significant figures in the answer.

1-10. What voltage would a 20,00041/V meter on a 0-1-V scale show in the circuit of Fig. P1-10?

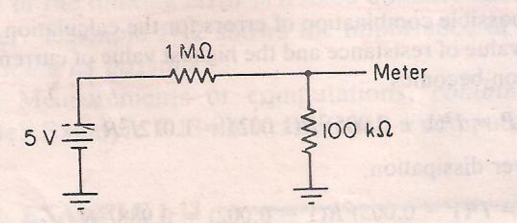

Figure P1-10

1-11 The voltage across a resistor is 200 V, with a probable error of ±2 percent, and the resistance is 42Ω with a probab1e error of ± 1.5 percent. Calculate (a) the power dissipated in the resistor; (b) the percentage error in the answer.

1-l2. The following values were obtained from the measurements of the value of a resistor: 147.2Ω, 147.4Ω, 147.9Ω, 148.1Ω, 147.1Ω. 147.5Ω, 147.6Ω, 147.4Ω, 147.6Ω, and 147.5Ω. Calculate (a) the arithmetic mean; (h) the average deviation; (c) the standard deviation; (d) the probable error of the average of the ten readings.

1-13. Six determinations of a quantity, as entered on the data sheet and presented to you for analysis, are 12.35, 12.71, 12.48, 10.24, 12.63, and 12.58. Examine the data and on the basis of your conclusions calculate (a) the arithmetic mean; (b) the standard deviation; (c) the probable error in percent of the average of the readings.

1-14. Two resistors have the following ratings:

$$R_1 = 36\ \Omega \pm 5\% \qquad \text{and} \qquad R_2 = 75\Omega \pm 5\%$$

Calculate (a) the magnitude of error in each resistor; (b) the limiting error in ohms and in percent when the resistors are connected in series; (c) the limiting error in ohms and in percent when the resistors are connected in parallel.

1-15 The resistance of an unknown resistor is determined by the Wheatstone bridge method. The solution for the unknown resistance is stated as $R_x = R_1R_2 / R_3$, where

$$R_1 = 500\Omega \pm 1\%$$

$$R_2 = 615\Omega \pm 1\%$$

$$R_3 = 100\Omega \pm 0.5\%$$

Calculate (a) the nominal value of the unknown resistor; (b) the limiting error in ohms of the unknown resistor; (c) the limiting error in percent of the unknown resistor.

1-16 A resistor is measured by the voltmeter-ammeter method. The voltmeter reading is 123.4 V on the 250-V scale and the ammeter reading is 283.5

mA on the 500-mA scale. Both meters are guaranteed to be accurate within ± 1 percent of full-scale reading. Calculate (a) the indicated value of the resistance; (b) the limits within which you can guarantee the result.

1-17 In a dc circuit, the voltage across a component is 64.3 V and the current is 2.53 A. Both current and voltage are given with an uncertainty of one unit in the last place. Calculate the power dissipation to the appropriate number of significant figures.

1-18 A power transformer was tested to determine tosses and efficiency. The input power was measured as 3,650 W and the delivered output power was 3,385 W, with each reading in doubt by ± 10W. Calculate the percentage uncertainty in the losses of the transformer; (b) the percentage uncertainty in the efficiency of the transformer, as determined by the difference in input and output power readings.

1-19 The power factor and phase angle in a circuit carrying a sinusoidal current are determined by measurements of current, voltage, and power. The current is read as 2.50 A on a 5-A ammeter, the voltage as 115 V on a 250-V voltmeter, and the power as 220 W on a 500-W wattmeter. The ammeter and voltmeter are guaranteed accurate to within ±0.5 percent of full-scale indication and the wattmeter to within ± 1 percent of full-scale reading. Calculate (a) the percentage accuracy to which the power factor can be guaranteed; (h) the possible error in the phase angle.

2. SYSTEM OF UNITS OF MEASUREMENT

2-1 FUNDAMENTAL AND DERIVED UNITS

To specify and perform calculations with physical quantities, the, physical quantities must be fined both in kind and magnitude. The standard measure of each kind of physical quantity is the unit the number of times the unit occurs in any given amount of the same quantity is the number of measure. For example, when we speak of a distance of 100 meters, we know that the meter is the unit of length and that the number of units of length is one hundred. The physical quantity, length, is therefore defined by the unit, meter. Without the unit, the number of measure has no physical meaning.

In science and engineering, two kinds of units are used: fundamental units and derived units. The fundamental units in mechanics are measures of length, mass, and time. The sizes of the fundamental units, whether foot or meter, pound or kilogram, second or hour, are arbitrary and can be selected to fit a certain set of circumstances.

units. Measures of certain physical quantities in the thermal, electrical, and illumination disciplines are also represented by fundamental units. These units are used only when these particular classes are involved, and they may therefore be defined as auxiliary .fundamental units.

All other units which can be expressed in terms of the fundamental units are called derived units. Every derived unit originates from some physical law defining that unit. For example, the area (A) of a rectangle is proportional to its length (l) and breadth (b), or A = lb. If the meter has been chosen as the unit of length, then the area of a rectangle of 3m by 4m is 12 m^2. Note that the number of measure multiplied (3 x 4 = 12) as well as the units (m x m = m^2) The derived unit for area (A) is then the square meter (m^2).

A derived unit is recognized by its dimensions, which can be defined as the complete algebraic formula for the derived unit. The dimensional symbols for the fundamental units of length, mass, and time are L, M, and T, respectively. The dimensional symbol for the derived unit of area is L^2 and that for volume, L^3. The dimensional symbol for the unit of force is LMT^{-2}, which follows from the defining equation for force. The dimensional formulas of the derived units are particularly useful for converting units from one system to another, as is shown in Sec. 2-6.

For convenience, some derived units have been given new names. For example, the derived unit of force in the SI system is called the newton (N), instead of the dimensionally correct name $kg.m/s^2$.

2-2 SYSTEM OF UNITS

In 1790 the French government issued a directive to the French Academy of Sciences to study and to submit proposals for a single system of weights and measures to replace all other existing systems. The French scientists decided, as a first principle, that a universal system of weight and measures should not depend on man-made reference standards, but instead be based on permanent measures provided by nature. As the unit of length, therefore, they chose the meter, defined as the ten-millionth part of the distance from the pole to the equator along the meridian passing through Paris. As the unit of mass they chose the mass of a cubic centimeter of distilled water at 4°C and normal atmospheric pressure (760mm Hg) and gave it the name gram. As the third unit, the unit of time, they decided to retain the traditional second, defining it as l/86,400 of the mean solar day.

As a second principle, they decided that all other units should be derived from the aforementioned the aforementioned three fundamental units of length, mass, and time. Next the third principle they proposed that all multiples of basic units be in the decimals system, and they devised the system of prefixes in use today. Table 2-1 lists the decimal multiples and submultiples.

The proposals of the French Academy were approved and introduced as the metric system of units in: France in 1795. The metric system aroused considerable interest elsewhere and finally, in 1875, 17 countries signed the so-called Meter Convention, making the metric system of units the legal system. Britain and the United States, although signatories of the convention, recognized its legality only in international transactions but did not accept the metric system for their own domestic use.

Table 2-1: Decimal Multiples and Submultiples

Name	**Symbol**	**Equivalent**
tera	T	10^{12}

giga	G	10^9
mega	M	10^6
kilo	k	10^3
hecto	h	10^2
deca	da	10
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	µ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}

Britain, in the meantime, had been working on a system of electrical units, and the British Association for the Advancement of Science decided on the centimeter and the gram as the fundamental units of length and mass. From this developed the centimeter-gram-second or CGS absolutes system of units, used by physicists all over the world. Complications arose when the CGS system was extended to electric and magnetic measurements because of the need to introduce at least one more unit in the system. In fact, two parallel systems were established. In the CGS electrostatic system, the unit of electric charge was derived from the centimeter, gram, and second by assigning the value 1 to the permittivity of free space in Coulomb's law for the force between electric charges. In the CGS electromagnetic system., the basic units are the same and the unit of magnetic pole strength is derived from them by assigning the value Ito the permeability of free space in the inverse square formula for the force between magnetic poles.

The derived units for electric current and electric potential in the electromagnetic system, the ampere and the volt, are used in practical measurements. These two units, and the corresponding ones, such as the Coulomb, ohm, henry, farad, etc., were incorporated in a third system, called the practical system. Further simplification in the establishment of a truly universal system cai1se result of pioneer work by the Italian engineer Giorgi, who pointed out that the practical units of current, voltage, energy, and power, used by electrical engineers, were compatible with the meter-kilogram-second system. He suggested that the metric system be expanded into a coherent system of units by including the practical electrical units. The Giorgi system, adopted by many countries in 1935, came to be known as the MKSA system, of units in which the ampere was selected as the fourth basic unit.

A more comprehensive system was adopted in 1954 and designated in 1960, by international agreement as the Système International d'Unités (SI). In the SI system, six basic units are used. namely, the meter, kilogram, second, and ampere

Table 2-2: Basic SI Quantities, Units, and Symbols

Quantity	Unit	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric current	Ampere	A
Thermodynamic temperature	Kelvin	K
Luminous intensity	Candela	cd

of the MKSA system and, in addition, the kelvin and the candela as the units of temperature and luminous intensity, respectively. The SI units are replacing other systems in science and technology; they have been adopted as the legal units in France, and will become obligatory in other metric countries.

The six basic SI quantities and units of measurement, with their unit symbols, are listed in Table 2-2.

2-3 ELECTRIC AND MAGNETIC UNITS

Before listing the SI units (sometimes called the International MKS system of units), the electrical and magnetic units seems appropriate. The practical electrical and magnetic units with which we are familiar, such as the volt, ampere, ohm, henry, etc., were first derived in the CGS systems of units.

The CGS electrostatic system (CGSe) is based on Coulomb's experimentally derived law for the force between two electric charges. Coulomb's law states that

$$F = k\frac{Q_1 Q_2}{r^2} \qquad (2\text{-}1)$$

where F = force between the charges, expressed in CGSe units of force (g cm/s2 = dyne)

k = proportionality constant

$Q_{1.2}$ = electric charges, expressed in (derived) CGSe units of. electric charge (statcoulomb)

r = separation between the charges, expressed in the fundamental CGSe unit of length (centimeter)

Coulomb also found that the proportionality factor k depended on the medium, varying inversely as its permittivity s. (Faraday called permittivity the dielectric constant.) Coulomb's law then takes the form

$$F = k\frac{Q_1 Q_2}{\varepsilon r^2} \qquad (2\text{-}2)$$

Since ε is a numerical value depending only on the medium, a value of 1 was assigned to the permittivity of free space, ε_0, thereby defining ε_0 as the fourth fundamental unit of the CGSe system. Coulomb's law then allowed the unit of electric charge *Q* to be determined in terms of these four fundamental units by the relation

$$\mathrm{dyne} = \frac{\mathrm{gcm}}{\mathrm{s}^2} = \frac{Q^2}{(\varepsilon_0 = 1)\mathrm{cm}^2}$$

and therefore, dimensionally,

$$Q = cm^{3/2} g^{1/2} s^{-1} \qquad (2\text{-}3)$$

The CGSe unit of electric charge was given the name statcoulomb.

The derived unit of electric charge in the CGSe system of units allowed other electrical units to be determined by their defining equations. For example, electric current (symbol *1*) is defined as the rate of flow of electric charge and is expressed as

$$I = \frac{Q}{t} \qquad \text{(statcoulomb/sec)} \qquad (2\text{-}4)$$

The unit for electric current in the CGSe system was given the name statampere. Electric field strength, E, potential difference, V, and capacitance, C, can similarly be derived from their defining equations.

The basis of the CGS electromagnetic system of units (CGSm) is Coulomb's experimentally determined law for the force between two magnetic poles, which states that

$$F = k\frac{m_1 m_2}{r^2} \qquad (2\text{-}5)$$

The proportionality factor, k, was found to depend on the medium in which the poles were placed, varying inversely with the magnetic permeability μ of the medium. The factor k was assigned the value 1 for the permeability of free space, μ_0, so that $k = l/\mu_0 = 1$. This established the permeability of free space, μ_0, as the fourth fundamental unit of the CGSm system. The derived electromagnetic unit of pole strength was then defined in terms of these four fundamental units by the relation:

$$\mathrm{dyne} = \frac{\mathrm{gcm}}{\mathrm{s}^2} = \frac{m^2}{(\mu_0 = 1)\mathrm{cm}^2}$$

and therefore, dimensionally,

$$m = cm^{3/2} g^{1/2} s^{-1}$$

The derived unit of magnetic polestrength in the COSm system led to the determination of other magnetic units, again by their defining equations. Magnetic flux density (symbol B), for example, is defined as the magnetic force per unit polestrength, where both force and polestrength are derived units in the CGSm system. Dimensionally, B is found to be equal to $\mathrm{cm}^{-1/2}\mathrm{g}^{1/2}\mathrm{s}^{-1}$ (dyne-second/abcoulomb-centimeter) and is given the name gauss. Similarly, other magnetic units can be derived from defining equations and we find that the unit for magnetic fluxis (symbol Φ) is given the name maxwell; the unit for magnetic field- strength (symbol H), the name oersted; and the unit for magnetic potential difference or magnetomotive force (symbol U), the name gilbert.

The two CGS systems were linked together by Faraday's discovery that a moving magnet could induce an electric current in a conductor, and conversely. that electricity in motion could produce magnetic effects. Ampere's law of the magnetic field relates electric current (I) to magnetic fieldstrength (H),* quantitatively connecting the magnetic units in the CGSm system to the electric units in the COSe system. The dimensions of the two systems did not agree exactly, and numerical conversion factors were introduced. The two systems finally formed one practual system of electrical units which was officially adopted by the international Electrical Congress.

These practical electrical units, det ved from the CGSm system, were later defined in terms of so-called international units . ltwas thought at the time (1908) that the establishment of the practical units from the definitions of the CGS system would be too difficult for most laboratories and it was therefore decided (unfortunately) to define the practical units in a way which would make it fairly simple to establish them. The ampere, therefore, was defined in terms of the rate of deposition of silver from a silver nitrate solution by passing current through that of a specified column of mercury. These units those derived from them were called international units. As measurement techniques improved, it was found that small differences existed between COSm derived practical units and the international units, which were then specified as follows.

1 int. ohm = 1.00049Ω (practical CGSm unit)

1 int. ampere = 0.99985 A

1 int. volt = 1.00034 V

1 int. coulomb = 0.99985 C

1 int. farad = 0.99951 F

1 int. henry = 1.00049 H

1 int. watt = 1.00019 W

1 int. joule = 1.00019 J

Particulars of the electric and magnetic units, and their defining relationships, are given in Table 2-3. Multiplication factors for conversion into SI units are given in the columns headed CGSm and CGSe.

* See a textbook on electromagnetic theory.

Table 2-3: Electric Magnetic Units

Quantity & Symbol	SI Unit			Conversion factors	
	Name & Symbol		Defining equation[a]	CGSm	CGSe[b]
Electric current, I	Ampere	A	$F_z = 10^{-7} I^2 \frac{dN}{dz}$	10	10/c
Electromotive force, E	Volt	V	P = IE	10^{-8}	10^{-8}c
Potential, V	Volt	V	P = IV	10^{-8}	10^{-8}c
Resistance, R	Ohm	Ω	R = V/I	10^{-9}	10^{-9}c
Electric charge, *Q*	Coulomb	C	*Q* = It	10	10/c
Capacitance, C	Farad	F	C = *Q*/V	10^{9}	$10^{9}/c^{2}$
Electric fieldstrength, E	-	V/m	E = V/*l*	10^{-6}	10^{-6}/c
Electric flux density, D	-	$C.m^2$	$D = Q/l^2$	10^{5}	10^{5}/c
Permittivity, ε	-	F/m	ε = D/E	-	$10^{11}/4\pi c^2$
Magnetic fieldstrength, H	-	A/m	ϕH dl = nl	-	-
Magnetic flux, Φ	Weber	Wb	E = dΦ/dt	10^{-8}	-
Magnetic flux density, B	tesla	T	$B = \Phi/I^2$	10^{-4}	-

Inductance, L. M	henry	H	$M = \Phi/1$	10^{-9}	-
Permeability, μ	-	H/m	$\mu = B/H$	$4\pi \times 10^{-7}$	-

a. N denotes Neumann's integral for two linear circuits each carrying the current I; F_z is the force between the two circuits in the direction defined by coordinate z, the circuits being in a vacuum; p denotes power; I^2 denotes area.

b. C=velocity of light in free space in cm/s = 2.997925×10^{10}

2-4 INTERNATIONAL SYSTEM OF UNITS

The international MKSA system of units was adopted in 1960 by the Eleventh General Conference of Weights and Measures under the name systeme international d' unites (SI). The SI system is replacing all other systems in the metric countries and its widespread acceptance dooms other systems to eventual obsolescence.

The six fundamental SI quantities are listed in Table 2-2. The derived units are expressed in terms of these six basic units by defining equations. Some examples of defining equations are given in Table 2-3 for the electric and magnetic quantities. Table 2-4 lists, together with the fundamental quantities which are repeated in this table, the supplementary and derived units in the SI which are recommended for use by the General Conference.

The first column in Table 2-4 shows the quantities (fundamental, supplementary, and derived). The second column gives the equation symbol for each quantity. The third column lists the dimension of each derived unit in terms of the six fundamental dimensions. The fourth column gives the name of each unit; the fifth, the unit symbol. The unit symbol should not be confused with the equation symbol; i.e., the equation symbol for resistance is R, but the unit abbreviation (symbol) for ohm is Ω.

Table 2 – 4: Fundamental, Supplementary, and Derived Units

TABLE 2-4 Fundamental, Supplementary, and Derived Units

Quantity	Equation symbol	Dimension	Unit	Unit symbol
Fundamental				
Length	l	L	meter	m
Mass	m	M	kilogram	kg
Time	t	T	second	s
Electric current	I	I	ampere	A
Thermodynamic temperature	T	Θ	kelvin	K
Luminous intensity			candela	cd
Supplementary[a]				
Plane angle	α, β, γ	$[L]^{\circ}$	radian	rad
Solid angle	Ω	$[L^2]^{\circ}$	steradian	sr
Derived				
Area	A	L^2	square meter	m^2
Volume	V	L^3	cubic meter	m^3
Frequency	f	T^{-1}	hertz	Hz (1/s)
Density	ρ	$L^{-3}M$	kilogram per cubic meter	kg/m^3
Velocity	v	LT^{-1}	meter per second	m/s
Angular velocity	ω	$[L]^{\circ}T$	radian per second	rad/s
Acceleration	a	LT^{-2}	meter per second squared	m/s^2
Angular acceleration	α	$[L]^{\circ}T^{-2}$	radian per second squared	rad/s^2
Force	F	LMT^{-2}	newton	N ($kg\ m/s^2$)
Pressure, stress	p	$L^{-1}MT^{-2}$	newton per square meter	N/m^2
Work, energy	W	L^2MT^{-2}	joule	J (N m)
Power	P	L^2MT^{-3}	watt	W (J/s)
Quantity of electricity	Q	TI	coulomb	C (A s)
Potential difference, electromotive force	V	$L^2MT^{-3}I^{-1}$	volt	V (W/A)
Electric fieldstrength	E, ε	$LMT^{-3}I^{-1}$	volt per meter	V/m
Electric resistance	R	$L^2MT^{-3}I^2$	ohm	Ω (V/A)
Electric capacitance	C	$L^{-2}M^{-1}T^4I^2$	farad	F (A s/V)
Magnetic flux	Φ	$L^2MT^{-2}I^{-1}$	weber	Wb (V s)
Magnetic fieldstrength	H	$L^{-1}I$	ampere per meter	A/m
Magnetic flux density	B	$MT^{-2}I^{-1}$	tesla	T (Wb/m^2)
Inductance	L	$L^2MT^{-2}I^2$	henry	H (V s/A)
Magnetomotive force	U	I	ampere	A
Luminous flux			lumen	lm (cd sr)
Luminance			candela per square meter	cd/m^2
Illumination			lux	lx (lm/m^2)

[a] The Eleventh General Conference designated these units as *supplementary*, although it could be argued that they are derived units.

2-5 OTHER SYSTEMS OF UNITS

The English system of units uses the foot (ft), the pound-mass (lb), and the second (s) as the three fundamental units of length; mass, and time, respectively. Although the measures of length and weight are legacies of the Roman occupation of Britain and therefore rather poorly defined, the inch (defined as one-twelfth of the foot) has since been fixed at exactly 25.4 mm. Similarly, the measure for the pound (lb) has been determined as exactly 0.45359237 kg. These two figures allow all units in the English system to be converted into SI units.

Starting with the fundamental units, foot, pound, and second, the mechanical units may be derived simply by substitution into the dimensional equations of Table 2-4. For example, the unit of density will be expressed in lb/ft^3 and the unit of acceleration in ft/s^2. The derived unit of force in the ft-lb-s system is called the poundal and is the force required to accelerate 1 pound-mass at the rate of 1 ft/s^2. As a result the unit for work or energy becomes the foot-poundal (ft pdl).

Various other systems have been devised and were used in various parts of the world. The MTS (meter-tonne-second) system was especially designed for engineering purposes in France and provided a replica of the CGS system except that the length and mass units (meter and tonne, respectively) were more suitable in practical engineering applications. Gravitational systems define the second fundamental unit as the weight of a mass measure; i.e., as the force by which that mass is attracted to the earth by gravity. In contrast to the gravitational systems, the so-called absolute systems, as the CGS and SI, use the mass measure as the second fundamental unit, but its value is independent of gravitational attraction.

Since English measures are still extensively used, both in Britain and on the North American continent, conversion into the SI becomes necessary if we wish to work in that system. Table 2-5 lists some of the common conversion factors for English into SI units.

TABLE 2-5 English into SI Conversion

Quantity	English unit	Symbol	Metric equivalent	Reciprocal
Length	1 foot	ft	30.48 cm	0.0328084
	1 inch	in.	25.4 mm	0.0393701
Area	1 square foot	ft^2	9.29030×10^2 cm^2	0.0107639×10^{-2}
	1 square inch	$in.^2$	6.4516×10^2 mm^2	0.155000×10^{-2}
Volume	1 cubic foot	ft^3	0.0283168 m^3	35.3147
Mass	1 pound (avdp)	lb	0.45359237 kg	2.20462
Density	1 pound per cubic foot	lb/ft^3	16.0185 kg/m^3	0.062428
Velocity	1 foot per second	ft/s	0.3048 m/s	3.28084
Force	1 poundal	pdl	0.138255 N	7.23301
Work, energy	1 foot-poundal	ft pdl	0.0421401 J	23.7304
Power	1 horsepower	hp	745.7 W	0.00134102
Temperature	degree F	°F	$5(t - 32)/9$°C	—

2-6 CONVERSION OF UNITS

It is often necessary to convert physical quantities from one system of units into another. Section 2- stated that a physical quantity is expressed in both unit and number of measure: it is the unit that must be converted, not the number of measure. Dimensional equations are very convenient for converting the numerical value of a dimensional quantity, when the units are transformed from one system to the other. The technique requires a knowledge of the numerical relation between the fundamental units and some dexterity in the manipulation of multiples and submultiples of the units.

The method used in converting from one system into the other is illustrated by a number of examples of progressively increasing difficulty.

EXAMPLE 2-1

The floor area of an office building is 5,000 m^2. Calculate the floor area in ft^2.

SOLUTION To convert the unit m^2 into the new unit ft^2, we must know the relation between them. In Table 2-5 the metric equivalent of 1 ft is 30.48 cm, or 1 ft = 0.3048 m. Therefore

$$A = 5{,}000 \text{ m}^2 \times \left(\frac{1 \text{ ft}}{0.3048 \text{ m}}\right)^2 = 53{,}820 \text{ ft}^2$$

EXAMPLE 2-2

A flux density in the CGS system is expressed as 20 maxwells/cm^2. Calculate the flux density in lines/in^2. (NOTE: 1 maxwell = 1 line.)

SOLUTION

$$B = \frac{20 \text{ maxwells}}{\text{cm}^2} \times \left(\frac{2.54 \text{ cm}}{\text{in.}}\right)^2 \times \frac{1 \text{ line}}{1 \text{ maxwell}} = 129 \text{ lines/in.}^2$$

EXAMPLE 2-3

The velocity of light in free space is given as 2.997925×10^8 m/s. Express the velocity of light in km/hr.

SOLUTION

$$c = 2.997925 \times 10^8 \frac{\text{m}}{\text{s}} \times \frac{1 \text{ km}}{10^3 \text{ m}} \times \frac{3.6 \times 10^3 \text{ s}}{1 \text{ hr}} = 10.79 \times 10^8 \text{ km/hr}$$

EXAMPLE 2-4

Express the density of water, 62.5 lb/ft^3, in (a) lb/$in.^3$; (b) g/cm^3.

SOLUTION

$$\text{(a) Density} = \frac{62.5 \text{ lb}}{\text{ft}^3} \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3 = 3.62 \times 10^{-2} \text{ lb/in.}^3$$

$$\text{(b) Density} = 3.62 \times 10^{-2} \frac{\text{lb}}{\text{in.}^3} \times \frac{453.6 \text{ g}}{1 \text{ lb}} \times \left(\frac{1 \text{ in.}}{2.54 \text{ cm}}\right)^3 = 1 \text{ g/cm}^3$$

EXAMPLE 2-5

The speed limit on a highway is 60 km/hr. Calculate the limit in (a) mi/hr; (b) ft/s.

SOLUTION

$$\text{(a) Speed limit} = \frac{60 \text{ km}}{\text{hr}} \times \frac{10^3 \text{ m}}{1 \text{ km}} \times \frac{10^2 \text{ cm}}{1 \text{ m}} \times \frac{1 \text{ in.}}{2.54 \text{ cm}} \times \frac{1 \text{ ft}}{12 \text{ in.}} \times \frac{1 \text{ mi}}{5{,}280 \text{ ft}} = 37.3 \text{ mi/hr}$$

$$\text{(b) Speed limit} = \frac{37.3 \text{ mi}}{\text{hr}} \times \frac{5{,}280 \text{ ft}}{1 \text{ mi}} \times \frac{1 \text{ hr}}{3.6 \times 10^3 \text{ s}} = 54.9 \text{ ft/s}$$

REFERENCES

2-1. Geczy. Steven, Basic Electrical Measurements, chap. 1 and Appendix. Englewood Cliffs, N.J.: Prentice-Half, Inc., 1984.

2-2. UTT Staff, Reference Data for Radio Engineers, 7th ed., chap.3. Indianapolis, Ind.: Howard W. Sams & Company. Inc., 1985.

PROBLEMS

2-1. Complete the following conversions:

1,500 MHz = GHz

12.5 kHz Hz

125 nH = μH

346.4 kV = V

5.3 mA = A

5 H = mH

4.6 pJ = J

1.4 μs = ms

3.2 ns = hr

14 = fs

2-2. What is the velocity of light in free space in feet per second?

2-3. The charge of an electron is 1.6 x 10^{-19} C. How many electrons pass by a point each microsecond if the current at that point is 4.56 A?

2-4. Typical “room” temperature is 25°C. What is this temperature in degrees Fahrenheit and kelvin?

2-5. Calculate the height in cm of a man 5 ft 11 in. tall.

2-6. Calculate the mass in kg of 1 yd^3 of iron when the density of iron is 7.86 g/cm^3.

2-7. Calculate the conversion factor to change mi/hr to ft/s.

2-8. An electrically charged body has an excess of 10^{15} electrons. Calculate its charge in C.

2-9. A train covers a distance of 220 mi in 2 hr and 45 min. Calculate; he average speed of the train in m/s.

2-10. Two electric charges are separated by a distance of 1 m. If one charge is + 10 C and the other charge -6C, calculate the force of attraction between the charges in N and in lb. Assume that the charges are placed in a vacuum.

2-11. The practical unit of electrical energy is the kWh. The unit of energy in the SI is the joule (J). Calculate the number of joules in 1 kWh.

2-12. A crane lifts a 100-kg mass a height of 20 m in 5s. Calculate (a) the work done by the crane, in SI units: (b) the increase of potential energy of the mass, in SI units; (c) the power, or rate of doing the work, in SI units.

2-13. Calculate the voltage of a battery if a charge of 3×10^{-4} C residing on the positive battery terminal possesses 6×10^{-2} J of energy.

2-14. An electric charge of 0.035 C flows through a copper conductor in 5 mm. Calculate the average current in mA.

2-15. An average current of 25 μA is passed through a wire for 30 s. Calculate the number of electrons transferred through the conductor.

2-16. The speed limit on a four-lane highway is 70 mi/hr. Calculate the speed limit in (a) km/hr; (b) ft/s.

2-17. The density of copper is 8.93 g/cm^3. Express the density in (a) kg/m^3; (b) lb/ft^3.

3. STANDARDS OF MEASUREMENT

3-1 CLASSIFICATION OF STANDARDS

A standard of measurements a physical representation of a unit of measurement. A unit is realized by reference to an arbitrary material standard or to natural phenomena including physical and atomic constants. For example, the fundamental unit of mass in the international system (SI) is the kilogram, defined as the mass of a cubic decimeter of water as its temperature of maximum density of 4°C (see Sec. 2-2). This unit of mass is represented by a material standard; the mass of the International Prototype Kilogram, consisting of a platinum-iridium alloy cylinder. This cylinder is preserved at the International Bureau of Weights and Measures at Sèvres, near Paris, and is the material representation of the kilogram. Similar standards have been developed for other units of measurement, including standards for the fundamental units as well as for some of the derived mechanical and electrical units.

Just as there are fundamental and derived units of measurement, we find different types of standards of measurement, classified by their function and application in the following categories:

a. International standards

b. Primary standards

c. Secondary standards

d. Working standards

The international standards are defined by international agreement. They represent certain units of measurement to the closest possibly accuracy that production and measurement technology allow. International standards are periodically evaluated and checked by absolute measurements in terms of the fundamental units (see Table 2-2). These standards are maintained at the International Bureau of Weights and Measures and are not available to the ordinary user of measuring instruments for purposes of comparison or calibration.

The primary (basic) standards are maintained by national standards laboratories in different parts of the world. The National Bureau of Standards (NBS) in Washington is responsible for maintenance of the primary standards in North America. Other national laboratories include the National Physical Laboratory (NPL) in Great Britain and, the oldest in the world, the Physikalisch-Technische Reichsanstalt in Germany. The primary standards, again representing the fundamental units and some of the derived mechanical and electrical units, are independently calibrated by absolute measurements at each of the national laboratories. The results of these measurements are compared against each other, leading to a world average figure for the primary standard. Primary standards are not available for use outside the national laboratories. One of the main functions of primary standards is the verification and calibration of secondary standards.

Secondary standards are the basic reference standards used in industrial measurement laboratories. These standards are maintained by the particular involved industry and are checked locally against other reference standards in the area. The responsibility for maintenance and calibration of secondary standards rests entirely with the industrial laboratory itself. Secondary standards are generally sent to the national standards laboratories on a periodic basis for calibration and comparison against the primary standards. They are then returned to the industrial user with a certification of their measured value in terms of the primary standard.

Working standards are the principal tools of a measurement laboratory. They are used to check and calibrate general laboratory instruments for accuracy and performance or to perform comparison measurements in industrial applications. A manufacturer of precision resistances, for example, may use a standard resistor (a working standard) in the quality control department of his plant to check his testing equipment. In this case, he verifies that his measurement setup performs within the required limits of accuracy.

In electrical and electronic measurement we are concerned with the electrical and magnetic standards of measurement. These are discussed in the following sections. We have seen, however, that electrical units can be traced back Lathe basic units of length, mass, and time (in fact, the national laboratories perform measurements to relate derived electrical units to fundamental units) and the deserve some investigation here.

3-2 STANDARDS FOR MASS, LENGTH, AND VOLUME

The metric unit of mass was originally defined as the mass of a cubic decimeter of water at its temperature of maximum density. The material representation of this unit is the International Prototype Kilogram, preserved at the International Bureau of Weights and Measures near Paris. The primary standard of mass in North America is the United States Prototype Kilogram, preserved by the NUS to an accuracy of 1 part in 10^8 and occasionally verified against the standard at the International Bureau. Secondary standards of mass, kept by the industrial laboratories, generally have an accuracy of 1 ppm (part per million) and may be verified against the NBS primary standard. Commercial working standards are available in a wide range of values to suit almost any application. Their accuracy is in the order of 5 ppm. The working standards, in turn, are checked against the secondary laboratory standards.

The pound (lb), established by the Weights and Measures Act of 1963 (which actually came into effect on January 31, 1964), is defined as equal to 0.45359237 kg exactly. All countries which retain the pound as the basic unit of measurement have now adopted the new definition, which supersedes the former imperial standard pound made of platinum.

The metric unit of length, the meter, was initially defined as 1/ l0 part of the meridional quadrant through Paris (Sec. 2-2). This was an outgrowth of a suggestion in 1790 by the well-known French astronomer Pierre-Simon Laplace that the right angle be divided into 100 degrees, rather than 90, and each degree into 100 minutes. rather than 60. The measure of one meter would be the distance on the surface of the earth covered by one second of arc, which would be one ten- thousandth of the meridional quadrant, or the line from the equator to the north geographical pole. This was materially represented by the distance between two lines engraved on a platinum-iridium bar preserved at the International Bureau of Weights and Measures near Paris. In 1960 the meter was redefined more accurately in terms of a number of wavelengths of light emitted from the krypton-86 atom. For over 20 years the international standard meter was 1,650,763.73 wavelengths of the orange-red radiation from a carefully specified and observed krypton discharge lamp. Because this standard did not prove as precise as originally thought, in 1983 a new standard for the meter was adopted. This standard is simply that one meter is the distance light that propagates in a vacuum in 1/299,792,458 seconds.

The yard is defined as 0.9144 meter exactly, or 1 inch is 25.4 mm exactly. This is because the standards for the English units of measurement are based on the metric standards. This definition of a yard and inch superseded the former definition in terms of a standard imperial yard. The few countries that have retained the yard and other English units of measurement have adopted this metric- based definition.

The most widely used industrial working standards of length are precision gage blocks, made of steel. These steel blocks have two plane parallel surfaces, a specified distance apart, with accuracy tolerances in the 0.5-0.25-micron range (1 micron = one millionth of 1 m). The development and use of precision gage blocks, low in cost and of high accuracy, have made it possible to manufacture interchangeable industrial components in a very economical application of precision measurement.

The unit of volume is a derived quantity and is not represented by an international standard. The NBS, however, has constructed a number of primary standards of volume, calibrated in terms of the absolute dimensions of length and mass. Secondary derived standards of volume are available and may be calibrated in terms of the NBS primary standards.

As the need for more accurate standards arises and the technology is developed to create and preserve these standards, the basis for international weights and measures will change to fill the needs of the scientific and commerce community. Additions and improvements will be added to the international standards to keep in pace with the needs of the world.

3.3 TIME AND FREQUENCY STANDARDS*

Since early times men have sought a reference standard for a uniform time scale together with means to interpolate from it a small time interval. For many centuries the time reference used was the rotation of the earth about its axis with respect to the sun. Precise astronomical observations have shown that the rotation of the earth about the sun is very irregular, owing to secular and irregular variations in the rotational speed of the earth. Since the time scale based on this apparent solar time does not represent a uniform time scale, other

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* Frequency and Time Standards, Application Note AN 52, published by Hewlett-Packard, Palo Alto, Calif., describes methods of frequency comparisons, time scales, and worldwide time standards broadcasts.

avenues were explored. Mean solar time was thought to give a more accurate time scale. A mean solar day is the average of all the apparent days in the year. A mean solar second is then equal to 1/86,400 of the mean solar day. The mean solar second, thus defined, is still inadequate as the fundamental unit of time, since it is tied to the rotation of the earth, which is now known to be non-uniform.

The system of universal time (UT), or mean solar time, is also based on the rotation of the earth about its axis. This system is known as UT_0 and is subject to periodic, long-term, and irregular variations. Correction of UT_0 has led to two subsequent universal time scales: UT_1 and UT_2. UT_1 recognizes the fact that the earth is subject to polar motion, and the UT1 time scale is based on the true angular rotation of the earth, corrected for polar motion. The UT_2 time scale is UT_1 with an additional correction for seasonal variations in the rotation of the earth. These variations are apparently caused by seasonal displacement of matter over the earth's surface, such as changes in the amount of ice in the polar regions as the sun moves from the southern hemisphere to the northern and back again through the year. This cyclic redistribution of mass acts on the earth's rotation since it produces changes in its moment of inertia. The epoch, or instant of time, of UT_2 can be established to an accuracy of a few milliseconds, but it is not usually distributed to this accuracy. The epoch indicated by the standard radio time signals may differ from the epoch of UT_2 by as much as 100 ms. The actual values of the differences are given in bulletins published by the national time services (NBS) and by the Bureau International de l'Heure (Paris Observatory).

The search for a truly universal time unit has led astronomers to define a time unit called ephemeris time (ET). ET is based on astronomical observations of the motion of the moon about the earth. Since 1956 the ephemeris second has been defined by the International Bureau of Weights and Measures as the fraction 1/31,556,925.9747 of the tropical year for 1900 January Oat 12 h ET, and adopted as the fundamental invariable unit of time. A disadvantage of the use of the ephemeris second is that it can be determined only several years in arrears and then only indirectly, by observations of the positions of the sun and the moon. For physical measurements, the unit of time interval has now been defined in terms of an atomic standard. The universal second and the ephemeris second, however, will continue to be used for navigation, geodetic surveys, and celestial mechanics.

Development and refinement of atomic resonators have made possible control of the frequency of an oscillator and, hence, by frequency conversion, atomic clocks. The transition between two energy levels, E_1 and E_2, of an atom is accompanied by the emission (or absorption) of radiation having a frequency given $h\eta = E_2 - E_1$, where h is Planck's constant. Provided that the energy states are not affected by external conditions, such as magnetic fields, the frequency, η is a physical constant, depending only on the internal structure of the atom. Since frequency is the inverse of time interval, such an atom provides a constant time interval. Atomic transitions of various metals were investigated, and the first atomic clock, based on the cesium atom, was put into operation in 1955. The time interval, provided by the cesium clock, is more accurate than that provided by a clock calibrated by astronomical measurements. The atomic unit of time was first related to UT but was later expressed in terms of ET. The International Committee of Weights and Measures has now defined the second in terms of the frequency of the cesium transition, assigning a value of 9,192,631,770 Hz to the hyperfine transition of the cesium atom unperturbed by external fields.

The atomic definition of the second realizes an accuracy much greater than that achieved by astronomical observations, resulting in a more uniform and much more convenient time base. Determinations of time intervals can now be made in a few minutes to greater accuracy than was possible before in astronsmical measurements that took many years to complete. An atomic clock with a precision exceeding 1 μs per day is in operation as a primary frequency standard at the NBS. An atomic time scale, designated NBS-A, is maintained with this clock.

Time and frequency standards are unique in that they may be transmitted from the primary standard at NBS to other locations via radio or television transmissions. Early standard time and frequency transmissions were in the high- frequency (HF) portion of the radio spectrum. but these transmissions suffered from. Doppler shifts due to the fact that radio propagation was primarily ionospheric. Transmission of time and frequency standards via low-frequency and very low frequency radio reduces this Doppler shift because the propagation is strictly ground wave. Two NBS-operated stations, WWVL and WWVB, operate at 20 and 60 kHz, respectively, providing precision time and frequency transmissions.

Another source of precision time and frequency information is the low-frequency navigation system called LORAN-C. This navigation system transmits shaped pulses at a carrier frequency of 100 kHz with a bandwidth of 20 kHz. The LORAN-C transmitters are controlled by cesium beam clocks and provide strong signals within most of the United States and in other parts of the world. Because LORAN-C is primarily a marine navigation system, coverage is not provided away from significant bodies of water.

Another source of accurate time and frequency dissemination is via television transmissions. The color burst frequency, which is nominally 3.579545 MHz, is phase locked to a cesium clock and is distributed over the television networks. Because television programming is distributed via terrestrial and satellite microwave links, there is no significant Doppler shift, and the color burst frequency can he transmitted accurately and is readily available for use as a precision standard.

3-4 ELECTRICAL STANDARDS

3-4.1 The Absolute Ampere

The international system of units (SI) defines the ampere (the fundamental unit of electric current) as the constant current which, if maintained in two straight parallel conductors of infinite length and negligible circular cross section placed I m apart in a vacuum, will produce between these conductors a force equal to 2 x 10, newton per meter length. Early measurements of the absolute value of the ampere were made with a current balance which measured the force between two parallel conductors. These measurements were rather crude and the need was felt to produce a more practical and reproducible standard for the national laboratories. By international agreement, the value of the International Ampere was based on the electrolytic deposition of silver from a silver nitrate solution. The International Ampere was then defined as that current which deposits silver at the rate of 1.118 mg/s from a standard silver nitrate solution. Difficulties were encountered in the exact measurement of the deposited silver and slight discrepancies existed between measurements made independently by the various national standards laboratories.

In 1948 the International Ampere was superseded by the Absolute Ampere. The determination of the Absolute Ampere is again made by means of a current balance, which weighs the force exerted between two current-carrying coils. Improvement in the techniques of force measurement yields a value for the ampere far superior to the early measurements. The relationship between the force and the current which produces the force can be calculated from fundamental electromagnetic theory concepts and reduces to a simple computation involving the geometric dimensions of the coils. The Absolute Ampere is now the fundamental unit of electric current in the SI and is universally accepted by international agreement.

Instruments manufactured before 1948 are calibrated in terms of the international Ampere but newer instruments are using the Absolute Ampere as the basis for calibration. Since both types of instruments may be found side by side in one laboratory, the NBS has established conversion factors to relate both units. These factors are given in Sec. 2-3.

Voltage, current, and resistance are related by Ohm's law of constant proportionality (E = IR). The specification of any two quantities automatically sets the third. Two types of material standards form a combination which conveniently serves to maintain the ampere with high precision over long periods of time: the standard resistor and the standard cell (for voltage). Each of these is described below.

3-4.2 Resistance Standards

The absolute value of the ohm in the SI system is defined in terms of the fundamental units of length, mass, and time. The absolute measurement of the ohm is carried out by the International Bureau of Weights and Measures in Sèvres and also by the national standards laboratories, which preserve a group of primary resistance standards. The NBS maintains a group of those primary standards (1-Ω standard resistors) which are periodically checked against each other and are occasionally verified by absolute measurements. The standard resistor is a coil of wire of some alloy like manganin which has a high electrical resistivity and a low temperature coefficient of resistance (almost constant temperature-resistance relationship). The resistance coil is mounted in a double-walled sealed container (Fig. 3-I) to prevent changes in resistance due to moisture conditions in the atmosphere. With a set of four or five I-fl resistors of this type, the unit of

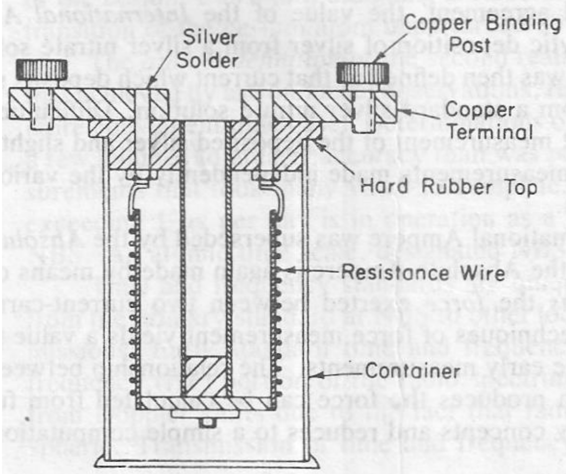

Figure 3-1 Cross-sectional view of a double-walled resistance standard. (Courtesy of Hewlett-Packard Co.)

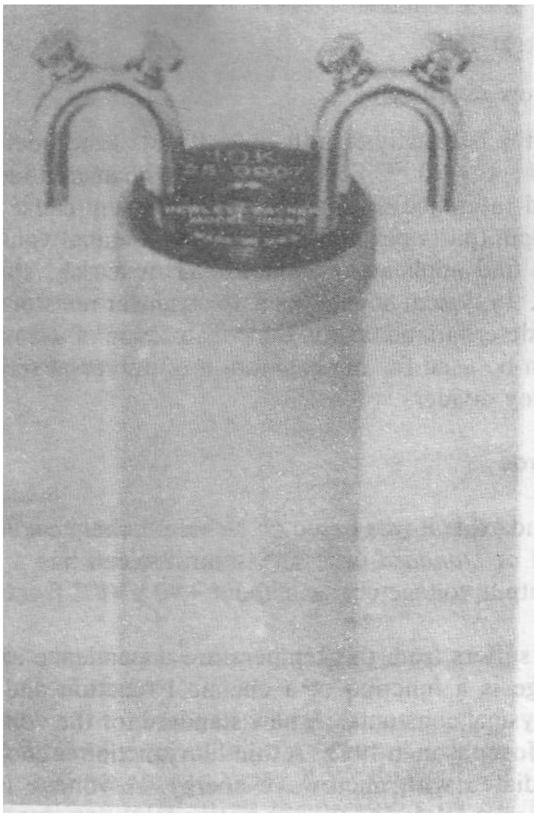

Figure 3-2 Ten-kilohm resistance standard. (Courtesy of Hewlett-Packard Company.)

resistance can be represented with a precision of a few parts in 10 over several years.

Secondary standards and working standards are available from some instrument manufacturers in a wide range of values, usually in multiples of 10Ω. These standard resistors are made of alloy resistance wire, such as manganin or Evanohm. Figure 3-2 is a photograph of a laboratory secondary standard, sometimes referred to as a transfer resistor. The resistance coil of the transfer resistor is supported between polyester film to reduce stresses on the wire and to improve the stability of the resistor. The coil is immersed in moisture-free oil and placed in a sealed can. The connections to the coil are silver soldered, and the terminal hooks are made of nickel-plated oxygen-free copper. The transfer resistor is checked for stability and temperature characteristics at its rated power and a specified operating temperature (usually 25°C). A calibration report accompanying the resistor specifies its traceability to NBS standards and includes the a and $ temperature coefficients. Although the selected resistance wire provides almost constant resistance over a fairly wide temperature range, the exact value of the resistance at any temperature can be calculated from the formula

$$R_t = R_{25}{}^oC + \alpha(t-25) + \beta(t-25)^2 \qquad (3\text{-}1)$$

where R_t = resistance at the ambient temperature

$R_{25}{}^oC$ R = resistance at 25°C

α, β = temperature coefficients

Temperature coefficient a is usually less than 10 x 10^{-6}, and coefficient β lies between - 3 x 10^{-7} and -6 x 10^{-7}. This means that a change in temperature of

10°C from the specified reference temperature of 25°C may cause a change in resistance of 30 to 60 ppm (parts per million) from the nominal value.

Transfer resistors find application in industrial, research, standards, and calibration laboratories. In typical applications, the transfer resistor may be used for resistance and ratio determinations or in the construction of ultra linear decade dividers which can then be used for the calibration of universal ratio sets, volt- boxes, and Kelvin-Varley dividers.

3-4.3 Voltage Standards

For many years the standard volt was based on an electrochemical cell called the saturated standard cell or standard cell. The saturated cell has a temperature dependence, and the output voltage changes about -40 μV/°C from the nominal of 1.01858 V.

The standard cell suffers from this temperature dependence and also from the fact that the voltage is a function of a chemical reaction and not related directly to any other physical constants. A new standard for the volt came about from the work of Brian Josephson in 1962. A thin-film junction is cooled to nearly absolute zero and irradiated with microwave energy. A voltage is developed across the junction, which is related to the irradiating frequency by the following relationship:

$$v = \frac{hf}{2e} \qquad (3\text{-}2)$$

where h = Planck’s constant (6.63×10^{-34} J-s) e = charge of an electron (1.602×10^{-19} C)

f = frequency of the microwave irradiation

Because only the irradiating frequency is a variable in the equation, the standard volt is related to the standard of time/frequency. When the microwave irradiating frequency is locked to an atomic clock or a broadcast frequency standard such as WWVB, the accuracy of the standard volt, including all of the system inaccuracies, is one part in 10^8.

The major method of transferring the volt from the standard based on the Josephson junction to secondary standards used for calibration is the standard cell. This device is called the normal or saturated Weston cell. The Weston cell has a positive electrode of mercury and a negative electrode of cadmium amalgam (10% cadmium). The electrolyte is a solution of cadmium sulfate. These components are placed in an H-shaped glass container, as shown in Fig. 3-3.

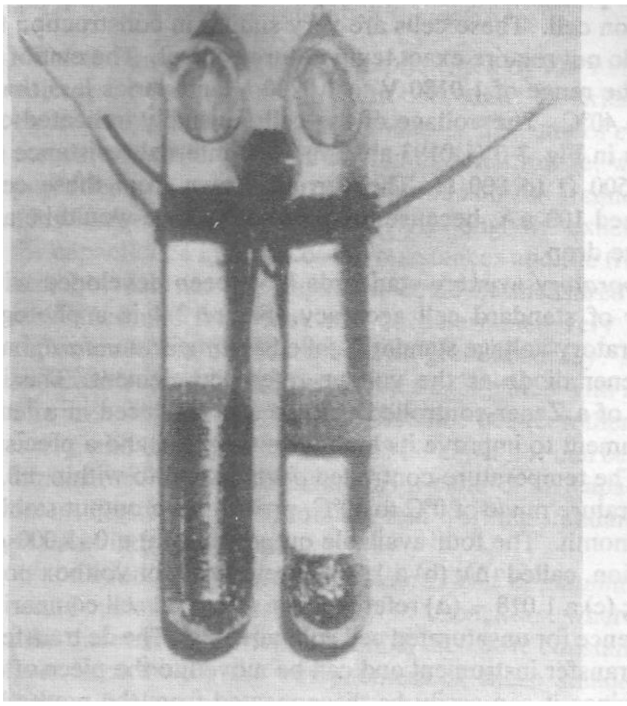

Figure 3-3 Weston cadmium cell: emf of 1.0193, accuracy of 0.1%. (Courtesy of The Eppley Laboratory, Inc.)

There are two types of Weston cell: the saturated cell, in which the electrolyte is saturated at all temperatures by cadmium sulfate crystals covering the electrodes, and the unsaturated cell, in which the concentration of cadmium sulfate is such that it produces saturation at 4°C. The unsaturated cell has a negligible temperature coefficient of voltage at normal room temperatures. The saturated cell has a voltage variation of approximately -40μV per 1°C rise, but is better reproducible and more stable than the unsaturated cell.

National standards laboratories, such as the NBS, maintain a number of saturated cells as the primary standard for voltage. The cells are kept in an oil bath to control their temperature to within 0.01°C. The voltage of the Weston saturated cell at 20°C is 1.01858 V (absolute), and the emf at other temperatures is given by the formula

$$e_t = e_{20^\circ C} - 0.000046(t-20) - 0.00000095(t-20)^2 + 0.00000001(t-20)^3 \qquad (3\text{-}3)$$

Saturated Weston cells remain satisfactory as voltage standards for periods of 10 to 20 years, provided that they are carefully treated. Their drift in voltage is on the order of 1 μV per year. Since saturated cells are temperature sensitive, they are unsuited for general laboratory use as secondary or working standards.

More rugged portable secondary and working standards are found in the unsaturated Weston cell. These cells are very similar in construction to the normal cell but they do not require exact temperature control. The emf of an unsaturated cell lies in the range of 1.0180 V to 1.0200 V and varies less than 0.01 percent from 10°C to 40°C. The voltage of the cell is usually indicated on the cell housing, as shown in Fig. 3-3 (1.0193 abs. V). The internal resistance of Weston cells range from 500Ω to 800 Ω. The current drawn from these cells should therefore not exceed 100 μA, because the nominal voltage would be affected by the internal voltage drop.

Versatile laboratory working standards have been developed with accuracies on the order of standard cell accuracy. Figure 3-4. is a photograph of a multipurpose laboratory voltage standard, called a transfer standard, based on the operation of a Zener diode as the voltage reference element. The instrument basically consists of a Zener-controlled voltage source placed in a temperature-controlled environment to improve its long-term stability, and a precision output voltage divider. The temperature-controlled oven is held to within ± 0.03°C over an ambient temperature range of 0°C to 50°C, providing an output stability on the order of 10 ppm/month. The four available outputs are (a) a 0-

1,OOO-μV source with 1-μV resolution, called (Δ); (b) a 1.000-V reference for voltbox potentiometeric measurements; (c) a 1.018 + (Δ) reference for saturated cell comparisons; (d) a 1.0190 ± (Δ) reference for unsaturated cell comparisons. The dc transfer standard can be used as a transfer instrument and can be moved to the piece of equipment to be calibrated, since it can easily be disconnected from the power line at one location and set up at a different location where it will recover to within ± 1 ppm in approximately 30 minutes warm-up time.

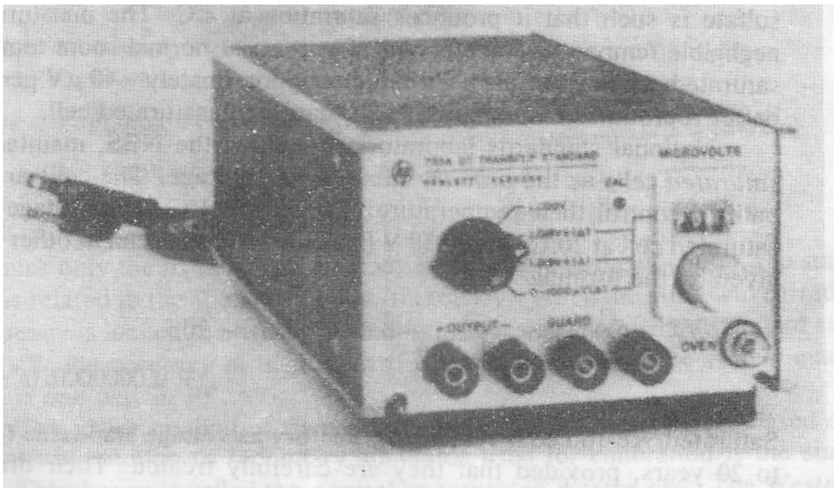

Figure 3-4 Dc transfer standard that can be used as a 1.000-V reference source, a standard cell comparison instrument, and a 0 to 1,000-μV dc source. (Courtesy of Hewlett-Packard Company.)

3-4.4 CAPACITANCE STANDARDS

Since the unit of resistance is represented by the standard resistor, and the unit of voltage by the standard Weston cell, many electrical and magnetic units may be expressed in terms of these standards. The unit of capacitance (the farad) can be measured with a Maxwell dc commutated bridge, where the capacitance is computed from the resistive bridge arms and the frequency of the dc commutation. This bridge is shown in Fig. 3-5. Although the exact derivation of the expression for capacitance in terms of the resistances and the frequency is rather involved, it may be seen that the capacitor could be measured by this method. Since both resistance and frequency can be determined vet y accurately, the value of the capacitance can be measured with great accuracy. Standard capacitors are usually constructed from interleaved metal plates with air as the dielectric material. The area of the plates and the distance between them must be known very accurately, and the capacitance of the air capacitor can be determined from these basic dimensions. The NBS maintains a bank of air capacitors as standards and uses them to calibrate the secondary and working standards of measurement laboratories and industrial users.

Capacitance working standards can be obtained in a range of suitable values. Smaller values are usually air capacitors, whereas the larger capacitors use solid dielectric materials. The high dielectric constant and the very thin dielectric layer account for the compactness of these standards. Silver-mica capacitors make excellent working standards; they are very stable and have a very low dissipation factor (Sec. 5-8), a very small temperature coefficient, and little or no aging effect. Mica capacitors are available in decade mounting, but decade capacitors are usually not guaranteed better than 1 percent. Fixed standards are generally used where accuracy is important.

3-4.5 Inductance Standards

The primary inductance standard is derived from the ohm and the farad, rather than from the large geometrically constructed inductors used in the determination of the absolute value of the ohm. The NBS selected a Campbell standard of

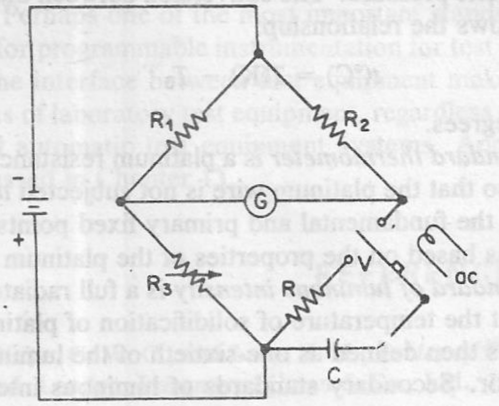

Figure 3-5 Commutated dc method for measuring capacitance. Capacitor C is alternately charged and discharged through the commutating contact and resistor R. Bridge balance is obtained by adjustment of R_3, allowing exact determination of the capacitance value in terms of bridge-arm constants and frequency commutation.

mutual inductance as the primary standard for both mutual and self-inductance. Inductance working standards are commercially available in a wide range of practical values, both fixed and variable. A typical set of fixed inductance standards includes values from approximately 100 μH to 10 H, with a guaranteed accuracy of 0.1 percent at a specified operating frequency. Variable inductors are also available. Typical mutual inductance accuracy is on the order of 2.5 percent and inductance values range from 0 to 200 mH. Distributed capacitance exists between the windings of these inductors, and the errors they introduce must be taken into account. These considerations are usually specified with commercial equipment.

3-5 STANDARDS OF TEMPERATURE AND LUMINOUS INTENSITY

Thermodynamic temperature is one of the basic SI quantities and its unit is the Kelvin (Sec. 2-2). The thermodynamic Kelvin scale is recognized as the fundamental scale to which all temperatures should be referred. The temperatures on this scale are designated as K and denoted by the symbol T. The magnitude of the Kelvin has been fixed by defining the thermodynamic temperature of the triple point of water at exactly 273.16 K. The triple point of water is the temperature of equilibrium between ice, liquid water, and its vapor.

Since temperature measurements on the thermodynamic scale are inherently difficult, the Seventh General Conference of Weights and Measures adopted in 1927 a practical scale which has been modified several times and is now called the International Practical Scale of Temperature. The temperatures on this scale are designated as °C (degree Celsius) and denoted by the symbol t. The Celsius scale has two fundamental fixed points: the boiling point of water as 100°C and the triple point of water as 0.01°C, both points established at atmospheric pressure. A number of primary fixed points have been established above and below the two fundamental points. These are the boiling point of oxygen (-182.97°C), the boiling point of sulfur (444.6°C), the freezing point of silver (960.8°C), and the freezing point of gold (1,063°C). The numerical values of all these points are reproducible quantities at atmospheric pressure. The conversion between the Kelvin scale and the Celsius scale follows the relationship:

$$T\ (°C) = 1(K) - T_0 \qquad (3\text{-}4)$$

where $T_0 = 273.15$ degrees.

The primary standard thermometer is a platinum resistance thermometer of special construction so that the platinum wire is not subjected to strain.

Interpolated values between the fundamental and primary fixed points on the scale are calculated by formulas based on the properties of the platinum resistance wire.

The primary standard of luminous intensity is a full radiator (black body or Planckian radiator), at the temperature of solidification of platinum (2,042 K approx.). The candela is then defined as one-sixtieth of the luminous intensity per cm^2 of the full radiator. Secondary standards of luminous intensity are special tungsten filament lamps, operated at a temperature whereby their spectral power distribution in the visible region matches that of the basic standard. These secondary standards are recalibrated against the basic standard at periodic intervals.

3-6 IEEE STANDARDS

A slightly different type of standard is published and maintained by the Institute of Electrical and Electronics Engineers, IEEE, an engineering society headquartered in New York City. These standards are not. physical items that are available for comparison and checking of secondary standards but are standard procedures, nomenclature, definitions, etc. These standards have been kept updated, and some of the early standards were in use before World War II. Many of the IEEE standards have been adopted by other agencies and societies as standards for their organization, such as the American National Standards Institute.

A large group of the IEEE standards is the standard test methods for testing and evaluating various electronics systems and components. As an example, there is a standard method for testing and evaluating attenuators. Although any test method should result in the same value for the attenuation, when certain factors are introduced, such as high frequency or high attenuation, measurement errors are possible. Specifying a methodology for the measurement decreases the chances for disparity between measurements.

Another useful standard is the specifying of test equipment. The common laboratory oscilloscope becomes difficult to use when each manufacturer adopts a different arrangement of knobs and functions and, worst of all, different names for the same function. An IEEE standard addresses the laboratory oscilloscope and specifies the controls, functions, etc., so that an oscilloscope operator does not have to reeducate himself for each oscilloscope he uses.

There are various standards concerning the safety of wiring for power plants, ships, industrial buildings, etc. Not only is safety a factor, but standard voltages, current ratings, etc., are specified so that components may be interchanged without damage or danger.

Standard schematic and logic symbols are defined so that engineering drawings can be understood by all engineers.

Perhaps one of the most important standards is the IEEE-488 digital interface for programmable instrumentation for test and other equipment. Standardizing the interface between test equipment makes it possible to interface various pieces of laboratory test equipment, regardless of manufacture, to create sophisticated automatic test equipment systems. Applications of this standard will be discussed in Chapter 13.

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3-4. Time and Frequency User's Manual, NBS Publication 559, November 1979.

PROBLEMS

3-1. What is the difference between a primary and secondary standard?

3-2. How is the standard meter defined?

3-3. What is atomic time? How does this differ from ephemeris time?

3-4. How can time and frequency standards be disseminated?

3-5. How is the Absolute Ampere determined?

3-6. A precision 1-Ω resistance standard has been calibrated at 25°C, and has an alpha factor of 0.6 x 10^{-6} and a beta factor of -4 x $l0^{-7}$ What would the resistance of the standard be at 30°C?

3-7. A Josephson junction is irradiated with 10.25 GHz of microwave radiation. What would be the potential across the junction?

3-8. What are the disadvantages of transmitting time and frequency standards by high- frequency, 3-30-MHz, radio? What are some of the methods used to improve the dissemination of these standards?

3-9. What are IEEE standards? How do these standards differ from those maintained by national standards laboratories?

3-10. What is the normal emf of a Weston cell at 20°C, and how does this emf change when the cell is used at 0°C?

4 ELECTROMECHANICAL INDICATING INSTRUMENTS

4-1 SUSPENSION GALVANOMETER

Early measurements of direct current re required a suspension galvanometer. This instrument was the forerunner of the moving-coil instrument, t dc indicating movements currently used.

A coil of fine wire is suspended in a magnetic field produced by a permanent magnet. According to the fundamental law of electromagnetic force, the coil will rotate in the magnetic field when it carries an electric current. The fine filament suspension of the if serves to carry current to and from it, and elasticity of the filament sets up a moderate torque in opposition to the rotation of the coil. The coil will continue to deflect until its electromagnetic torque balances the mechanical counter torque of the suspension. The coil deflection therefore is a measure of the magnitude of the current carried by the coil. A mirror attached to the coil deflects a beam of light, causing a magnified light spot to move on a scale at some distance from the instrument. The optical effect is that of a pointer of great length but zero mass.

4-2 TORQUE AND DEFLECTION OF THE GALVANOMETER

4-2.1 Steady-State Deflection

Although the suspension galvanometer is neither a practical no portable instrument, the principles governing its operation apply equally to its more modem version, the permanent-magnet moving-coil mechanism (PMMC). Figure 4-1 shows the construction of the PMMC mechanism. The different parts of the instrument are identified alongside the figure.

Here again we have a coil, suspended in the magnetic field of a permanent magnet, this time in the shape of a horseshoe. The coil is suspended so that it can rotate freely in the magnetic field. When current flows in the coil, the developed electromagnetic (EM) torque causes the coil to rotate. The EM torque is counterbalanced by the mechanical torque of control springs attached to the movable coil. The balance of torques, and therefore the angular position of the movable coil, is indicated by a pointer against a fixed reference, called a scale.

The equation for the developed torque, derived from the basic law for electromagnetic torque, is

$$T = B \times A \times I \times N \tag{4-1}$$

where T = torque [newton-meter (N-m)]

B = flux density in the air gap [webers/square meter (tesla)]

A = effective coil area [square meters (m^2)]

I = current in the movable coil [amperes (A)]

N = turns of wire on the coil

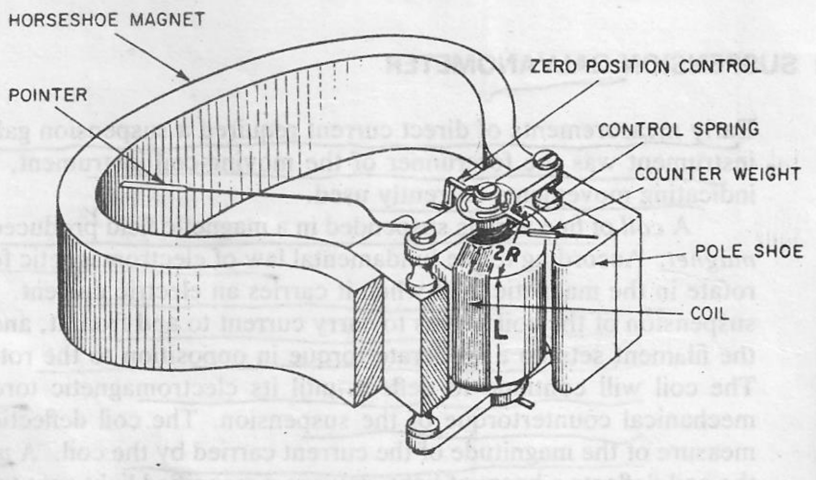

Figure 4-1 Construction details of the external magnet PMMC movement. (Courtesy of Weston Instruments, Inc.)

Equation (4-1) shows that the developed torque is directly proportional to the flux density of the field in which the coil rotates, the current in the coil, and the coil constants (area and turns). Since both flux density and coil area are fixed parameters for a given instrument, the developed torque is a direct indication of the current in the coil. This torque causes the pointer to deflect to a steady-state position where it is balanced by the opposing control-spring torque.

Equation (4-i) also shows that the designer may very only the value of the control torque and the number of turns on the moving coil to measure a given full scale current. The practical coil area generally ranges from approximately 0.5 to 2.5 cm^2. Flux densities for modern instruments usually range from 1,500 to 5 000 gauss (0 15 to 05 tesla). Thus a wide choice of mechanisms is available to the designer to meet many different measurement applications.

A typical panel PMMC instrument, with a 3k-in. case, a 1-mA range, and full-scale deflection of 100 degrees of arc, would have the following characteristics:

$A = 1.75\ cm^2$

$B = 2{,}000$ G (0.2 tesla)

$N = 84$ turns

$T = 2.92 \times 10^{-6}$ N-m

coil resistance = 88Ω

power dissipation = 88 μW

4-2.2 Dynamic Behavior

In Sec. 4-2.1 we considered the galvanometer as a simple indicating instrument in which the deflection of the pointer is directly proportional to the magnitude of the current applied to the coil. This is perfectly satisfactory when we are dealing with a steady-state condition in which we are mainly interested in obtaining a reliable reading of a direct current. In some applications, however, the dynamic behavior of the galvanometer (such as speed of response, damping, overshoot) can be important. For example, when an alternating or varying current is applied to a recording galvanometer, the written record produced by the motion of the moving coil includes the response characteristics of the moving element itself and it is therefore important to consider its dynamic behavior.

| The dynamic behavior of the galvanometer can be observed by suddenly interrupting the applied current, so that the coil swings back from its deflected position toward the zero position. It will be seen that as a result of inertia of the moving system the pointer swings past the zero mark in the opposite direction, and then oscillates back and forth around zero. These oscillations gradually die down as a result of the damping of the moving element, and the pointer will finally come to rest at zero.

The motion of a moving coil in a magnetic field is characterized by three quantities:

a. The moment of inertia (J) of the moving coil about its axis of rotation
b. The opposing torque (5) developed by the coil suspension
c. The damping constant (D)

The differential equation that relates these three factors yields three possible solution, each of which describes the dynamic behavior are shown in the curves Fig. 4-2 and are known as overdamped, underdamped, and critically damped. Curve of fig. 4-2 shows the overdamped case in which the coil returns slowly to its test position, without overshoot or oscillations. The pointer seems to approach the steady-state position in a sluggish manner. This case is of minor interest because we prefer to operate under the conditions of curve II or curve III for most applications. Curve II of Fig. 4-2 shows the underdamped case in which the motion of the coil is subject to damped sinusoidal oscillations. The rate at which these oscillations die away is determined by the damping constant (D), the moment of inertia (J), and the counrer torque (S) produced by the coil suspension. Curve Ill of Fig. 4-2 shows the critically damped case in which the pointer returns promptly to its steady-state position, without oscillations.

Ideally, the galvanometer response should be such that the pointer travels to its final position without overshoot; hence, the movement should be critically damped. In practice, the galvanometer is usually slightly underdamped, causing the pointer to overshoot a little before coming to rest. This method is perhaps less direct than critical damping, but it assures the user that the movement has hot been damaged because of rough handling, and it compensates for any additional friction that may develop in time because of dust or wear.

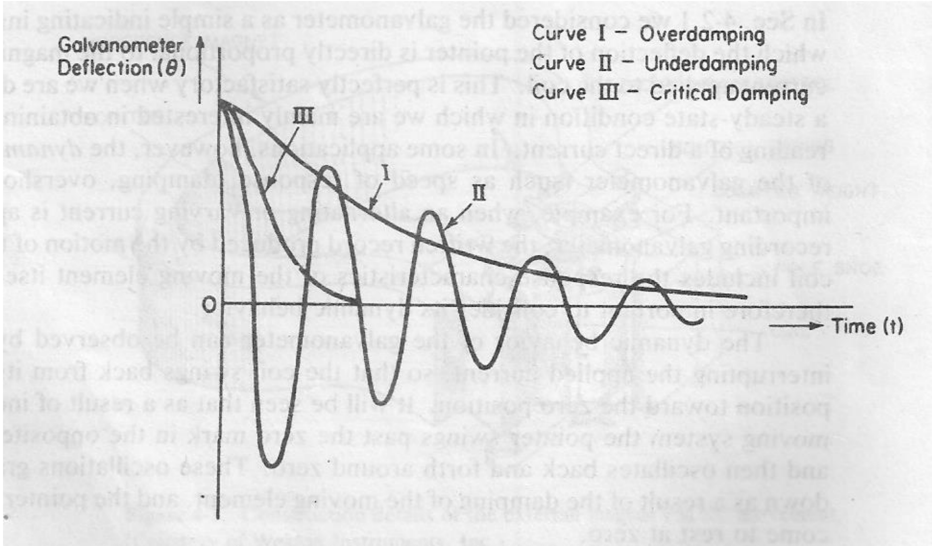

Figure 4-2 Dynamic behavior of a galvanometer.

4-2.3 Damping Mechanisms

Galvanometer damping is provided by two mechanisms: mechanical and electromagnetic. Mechanical damping is caused mainly by the motion of the coil through the air surrounding it; it is independent of any electrical current through the coil. Friction of the movement in its bearing and flexing of the suspension springs caused by the rotating coil also contribute to the mechanical damping effects. Electromagnetic damping is caused by induced effects in the moving coil as it rotates in the magnetic field, provided that the coil forms part of a closed electrical circuit.

PMMC instruments are generally constructed to produce as little viscous damping as possible and the required degree of damping is added. One of the simplest damping mechanisms is provided by an aluminum vane, attached to the shaft of the moving coil. As the coil rotates, the vane moves in an air chamber. The amount of clearance between the chamber walls and the air vane effectively controls the degree of damping.

Some instruments use the principle of electromagnetic damping (Lenz’s law), where the movable coil is wound on a light aluminum frame. The rotation of the coil in the magnetic field sets up circulating currents in the conductive metal frame, causing a retarding torque that opposes the motion of the coil. Indeed, the same principle is often used to protect PMMC instruments during shipment by placing a metal shorting strap across the coil terminals to reduce deflection.

A galvanometer may also be damped by connecting a resistor across the coil. When the coil rotates in the magnetic field, a voltage is generated in the coil which circulates a current through the coil and the external resistor. This produces an opposing, or retarding, torque that damps the motion of the movement. For any galvanometer, a value for the external resistor can be found that produces critical damping. This resistance is called the Critical Damping Resistance External (CRDX); it is an important galvanometer constant. The dynamic damping torque produced by the CDRX depends on the total circuit resistance: the smaller the total circuit resistance, the larger the damping torque.

One way to determine the CDRX consists of observing the galvanometer swing when a current is applied or removed from the coil. Beginning with the oscillating condition, decreasing values of external resistances are tried until a value is found for which the overshoot just disappears. A determination like this is not very precise, but it is adequate for most practical purposes. The value of the CDRX may also be computed from known galvanometer constants.

4-3 PERMANENT-MAGNET MOVING-COIL MECHANISM

4-3.1 D' Arsonval Movement

The basic PMMC movement of fig: 4-1 is often called the d' Arsonval movement, after its inventor. This design offers the largest magnet in a given space and is used when maximum flux in the air gap is required - It provides an instrument with very low power consumption and low current required for full-scale deflection (fsd).

Inspection of the diagram of Fig. 4-1 shows a permanent magnet of horseshoe form, with soft iron pole pieces attached to it. Between the pole pieces is a cylinder of soft iron, which serves to provide a uniform magnetic field in the air gap between the pole pieces and the cylinder. The coil is wound on a light metal frame and is mounted so that it can rotate freely in the air gap. The pointer, attached to the coil, moves over a graduated scale and indicates the angular deflection of the coil and therefore the current through the coil.

Two phosphor-bronze conductive springs, normally equal in strength, provide the calibrated force opposing the moving-coil torque. Constancy of spring performance is essential to maintain instrument accuracy. The spring thickness is accurately controlled in manufacture to avoid permanent set of the springs. Current is conducted to and from the coil by the control springs.

The entire moving system is statically balanced for all deflection positions by three balance weights, as shown in Fig. 4-3. The pointer, springs, and pivots are assembled to the coil structure by means of pivot bases, and the entire movable- coil element is supported by jewel bearings. Different bearing systems are shown in Fig. 4-4.

The V-jewel, shown in Fig. 4-4(a), is almost universally used in instrument bearings. The pivot, bearing in the pit in the jewel, may have a radius at its tip from 0.01 mm to 0.02 mm, depending on the weight of the mechanism and the vibration the instrument will encounter. The radius of the pit in the jewel is

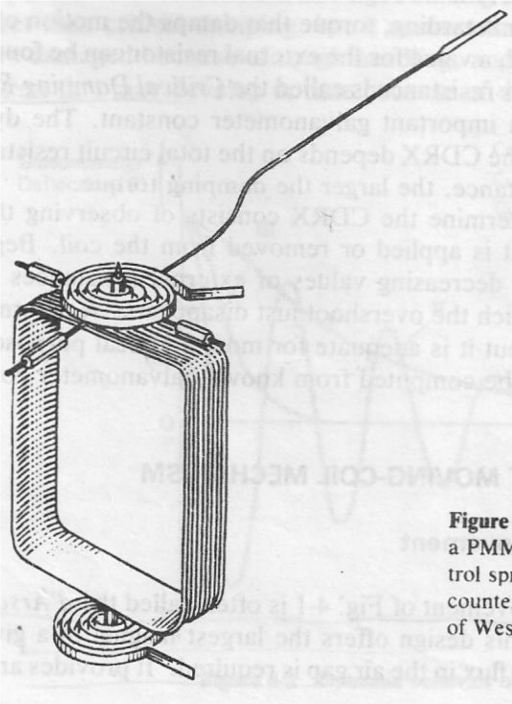

Figure 4-3 Details of a moving coil for a PMMC movement showing the control springs and the indicator with its counterbalance weights. (Courtesy of Weston Instruments, Inc.)

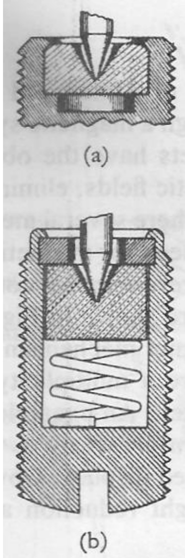

Figure 4-4 Details of instrument bearings: (a) V-jewel bearing; (b) spring-back jewel bearing. (Courtesy of Weston Instruments, Inc.)

slightly larger than the pivot radius, so that the contact area is circular, only a few microns across. The V-jewel design of Fig. 4-4(a) has the least friction of any practical type of instrument bearing. Although the moving elements of instruments are designed to have the smallest possible weight, the extremely minute area of contact between pivot and jewel results in stresses on the order of 10 kg/mm^2. If the weight of the moving element is further increased, the contact area does not increase in proportion so that the stress is even greater. Stresses set up by relatively moderate accelerations (like jarring or dropping an instrument) may consequently cause pivot damage. Specially protected (ruggedized) instruments use the spring-back (incabloc) jewel bearing, whose construction is shown in Fig. 4-4(b). It is located in its normal position by the spring and is free to move axially when the shock to the mechanism becomes severe.

The scale markings of the basic dc PMMC instrument are usually linearly spaced because the torque (and hence the pointer deflection) is directly proportional to the coil current. [See Eq. (4-1) for the developed torque] The basic PMMC instrument is therefore a linear-reading dc device. The power requirements of the d'Arsonval movement are surprisingly small: typical values range from 25μW to 200μW. Accuracy of the instrument is generally on the order of 2 to 5 percent of full-scale reading.

If low-frequency alternating current is applied to the movable coil, the deflection of the pointer would be up-scale for one half-cycle of the input waveform and down-scale (in the opposite direction) for the next half-cycle At power line frequencies (60 Hz) and above, the pointer could not follow the rapid variations in direction and would quiver slightly around the zero mark, seeking the average value of the alternating current (which equals zero). The PMMC instrument is therefore unsuitable for ac measurements, unless the current is rectified before application to the coil.

4-3.2 Core-Magnet Construction

In recent years, with the development of Alnico and other improved magnetic materials, it has become feasible to design a magnetic system in which the magnet itself serves as the core. These magnets have the obvious advantage of being relatively unaffected by external magnetic fields, eliminating the magnetic shunting effects in steel panel construction, where several meters operating side by side may affect each other's readings. The need for magnetic shielding, in the form of iron cases, is also eliminated by the core-magnet construction. Details of the core-magnet self-shielding movement are shown in Fig. 4-5.

Self-shielding makes the core-magnet mechanism particularly useful in aircraft and aerospace applications, where a multiplicity of instruments must be mounted in close proximity to each other. An example of this type of mounting may be found in the cross-pointer indicator, where as many as five mechanisms are mounted in one case to form a unified display. Obviously, the elimination of iron cases and the corresponding weight reduction are of great advantage in aircraft and aerospace instruments.

4-3.3 Taut-Band Suspension

The suspension-type galvanometer mechanism has been known for many years. Until recently the device was used only in the laboratory where high sensitivities were required and the torque was extremely low (because of small currents). It was desirable in such instruments to eliminate even the low friction of pivots and

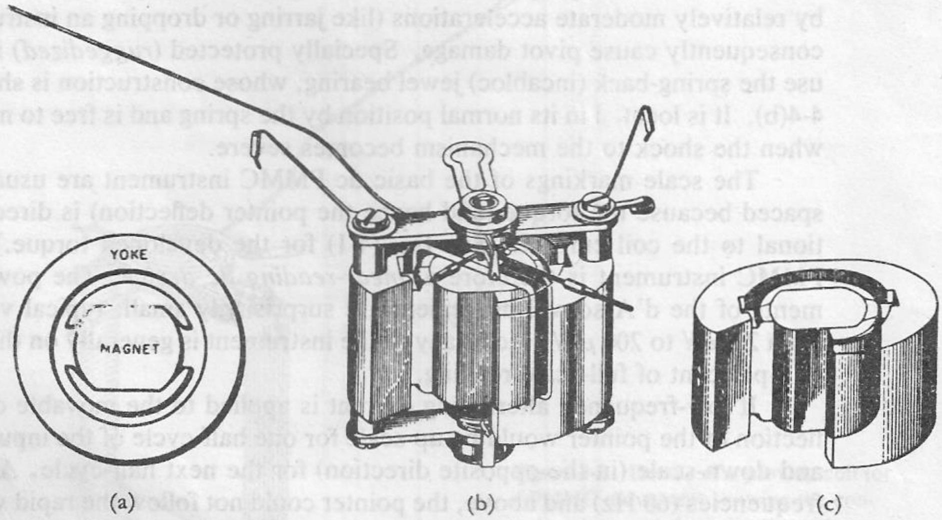

Figure 4-5 Construction of the core-magnet moving-coil mechanism: (a) magnet with its poleshoes is surrounded by the yoke, which acts as a magnetic shield; (b) assembled movement; (c) cutaway view of the yoke, the core, and the poleshoes. (Courtesy of Weston Instruments, Inc.)

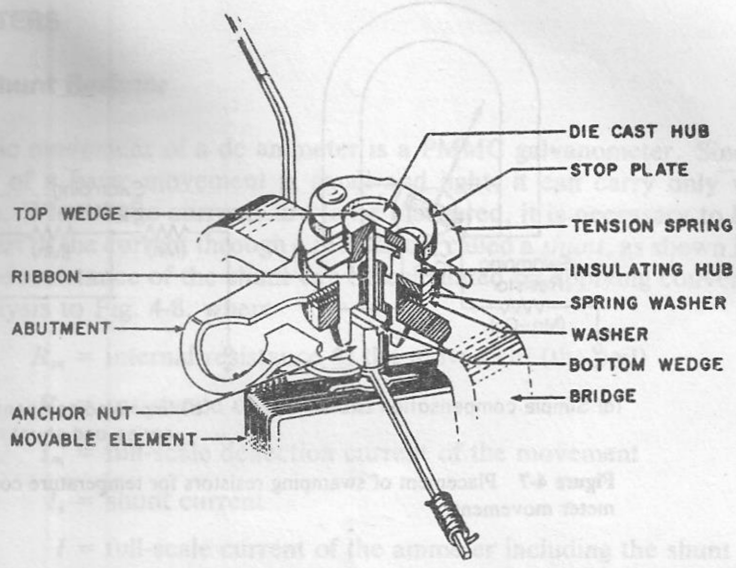

Figure 4-6 Taut-band suspension eliminates the friction of conventional pivot-and-jewel suspensions. This figure shows some construction details, in particular the torsion ribbon with its tension-spring mechanism. (Courtesy of Weston Instruments, Inc.)

jewels. The suspension galvanometer had to be used in the upright position, because sag in the low-torque ligaments caused the moving system to come in contact with stationary members of the mechanism in any other position. This increase in friction caused errors.

The taut-band instrument of Fig. 4-6 has the advantage of eliminating the friction of the jewel-pivot suspension. The movable coil is suspended by means of two torsion ribbons. The ribbons are placed under sufficient tension to eliminate any sag, as was the case in the suspension galvanometer. This tension is provided by a tension spring, so that the instrument can be used in any position. Generally speaking, taut-band suspension instruments can be made with higher sensitivities than those using pivots and jewels, and they can be used in almost every application served by pivoted instruments. Furthermore, taut-band instruments are relatively insensitive to shock and temperature and are capable of withstanding greater overloads then previous types described.

4-3.4 Temperature Compensation

The PMMC basic movement is not inherently insensitive to temperature, but it may be temperature-compensated by the appropriate use of series and shunt resistors of copper and manganin. Both the magnetic fieldstrength and spring tension decrease with an increase in temperature. The coil resistance increases with an increase in temperature. These changes tend to make the pointer read low for a given current with respect to magnetic fieldstrength and coil resistance. The

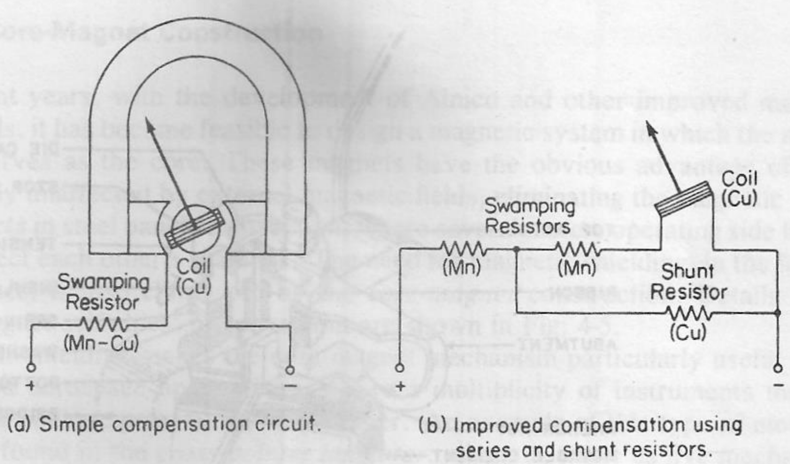

Figure 4-7 Placement of swamping resistors for temperature compensation of a meter movement.

spring change, conversely, tends to cause the pointer to read high with an increase in temperature. The effects are not identical, however; hence an uncompensated meter (ends to read low by approximately 0.2 percent per °C rise in temperature. For purposes of instrument specification, the movement is considered to be compensated when the change in accuracy, due to a 10°C-change in temperature, is not more than one-fourth of the total allowable error.*

Compensation may be accomplished by using swamping resistors in series with the movable coil, as shown in Fig. 4-7(a). The swamping resistor is made of manganin (which has a temperature coefficient of practically zero) combined with copper in the ratio of 2011 to 30/1 - The total resistance of coil and swamping resistor increases slightly with a rise in temperature, but only just enough to counteract the change of springs and magnet, so that the overall temperature effect is zero.

A more complete cancellation of temperature effects is obtained with the arrangement of Fig. 4-7(b). Here the total circuit resistance increases slightly with a rise in temperature, owing to the presence of the copper coil and the copper shunt resistor. For a fixed applied voltage, therefore, the total current decreases slightly with a rise in temperature. The resistance of the copper shunt resistor increases more than the series combination of coil and manganin resistor; hence a larger fraction of the total current passes through the coil circuit. By correct proportioning of the copper and manganin parts in the circuit, complete cancellation of temperature effects may be accomplished. One disadvantage of the use of swamping resistors is a reduction in the full-scale sensitivity of the movement, because a higher applied voltage is necessary to sustain the full-scale current.

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* PMMC Data Sheets, Weston Instruments, Inc., Newark, NJ.

4-4 DC AMMETERS

4-4.1 Shunt Resistor

The basic movement of a dc ammeter is a PMMC galvanometer. Since the coil winding of a basic movement is small and light, it can carry only very small currents. When large currents are to be measured, it is necessary to bypass the major part of the current through a resistance, called a shunt, as shown in Fig. 4-8.

The resistance of the shunt can be calculated by applying conventional circuit analysis to Fig. 4-8, where

R_m = internal resistance of the movement (the coil)

R_s= resistance of the shunt

I_m = full-scale deflection current of the movement

I = shunt current

I = full-scale current of the ammeter including the shunt

Since the shunt resistance is in parallel with the meter movement, the voltage drops across the shunt and movement must be the same and we can write

$$V_{shunt} = V_{movement}$$

Or

$$I_sR_s = I_m R_m \text{ and } \frac{I_m R_m}{I_s} \qquad (4\text{-}2)$$

Since $I_s = I - I_m$, we can write

$$R_s = \frac{I_m R_m}{I - I_m} \qquad (4\text{-}3)$$

For each required value of full-scale meter current we can then solve for the value of the shunt resistance required.

EXAMPLE 4-1

A 1-mA meter movement with an internal resistance of 100Ω is to be converted into a 0-100-mA ammeter. Calculate the value of the shunt resistance required.

SOLUTION

$$I_s = I - I_m = 100 - 1 = 99 \text{ mA}$$

$$R_s = \frac{I_m R_m}{I_s} = \frac{1\text{mA} \times 100\Omega}{99\text{mA}} = 1.01\Omega$$

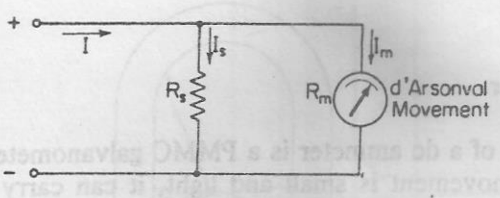


Figure 4-8 Basic dc ammeter circuit.

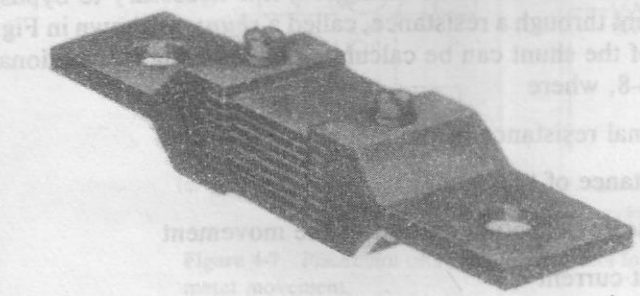


Figure 4-9 High-current shunt for a switchboard instrument. (Courtesy of Weston Instruments, Inc.)

The shunt resistance used with a basic movement may consist of a length of constant-temperature resistance wire within the case of the instrument or it may be an external (manganin or constantan) shunt having a very low resistance. Figure 4-9 shows an external shunt. It consists of evenly spaced sheets of resistive material welded into a large block of heavy copper on each end of the sheets. The resistance material has a very low temperature coefficient, and a low thermoelectric effect exists between the resistance material and the copper. External shunts of this type are normally used for measuring very large currents.

4-4.2 Ayrton Shunt

The current range of the dc ammeter may be further extended by a number of shunts, selected by a range switch. Such a meter is called a multirange ammeter. Figure 4-10 shows the schematic diagram of a multirange ammeter. The circuit has four shunts, RQ, Rb, R, and Rd. which can be placed in parallel with the movement to give four different current ranges. Switch S is a multi-position, make-before-break type switch, so that the movement will not be damaged, unprotected in the circuit, without a shunt as the range is changed.

The universal, or Ayrton, shunt of Fig. 4-11 eliminates the possibility of having the meter in the circuit without a shunt. This advantage is pined at the price of a slightly higher overall meter resistance. The Ayrton shunt provides an excellent opportunity to apply basic network theory to a practical circuit.

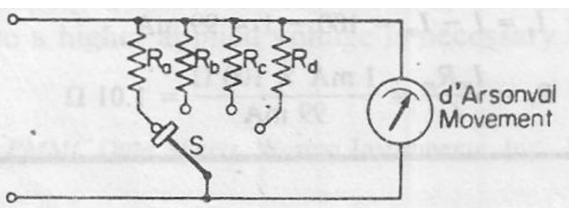


Figure 4-10 Schematic diagram of a simple multirange ammeter.

EXAMPLE 4-2

Design an Ayrton shunt to provide an ammeter with current ranges of 1 A, 5 A, and 10 A. A d'Arsonval movement with an interval resistance $R_m = 50\ \Omega$ and full-scale deflection current of 1 mA is used in the configuration of Fig. 4-11.

SOLUTION *On the 1-A range:* $R_a + R_b + R_c$ are in parallel with the 50-Ω movement. Since the movement requires 1 mA for full-scale deflection, the shunt will be required to pass a current of 1 A − 1 mA = 999 mA. Using Eq. (4-2), we get

$$R_a + R_b + R_c = \frac{1 \times 50}{999} = 0.05005\ \Omega \qquad \text{(I)}$$

On the 5-A range: $R_a + R_b$ are in parallel with $R_c + R_m$ (50 Ω). In this case there will be a 1-mA current through the movement and R_c in series, and 4,999 mA through $R_a + R_b$. Again using Eq. (4-2), we get

$$R_a + R_b = \frac{1 \times (R_c + 50\ \Omega)}{4{,}999} \qquad \text{(II)}$$

On the 10-A range: R_a now serves as the shunt and $R_b + R_c$ are in series with the movement. The current through the movement again is 1 mA, and the shunt passes the remaining 9,999 mA. Using Eq. (4-2) again, we get

$$R_a = \frac{1 \times (R_b + R_c + 50\ \Omega)}{9{,}999} \qquad \text{(III)}$$

Solving the three simultaneous equations (I), (II), and (III), we obtain

$$4{,}999 \times \text{(I)}: 4{,}999R_a + 4{,}999R_b + 4{,}999R_c = 250.2$$

$$\text{(II)}: 4{,}999R_a + 4{,}999R_b - R_c = 50$$

Subtracting (II) from (I), we obtain

$$5{,}000R_c = 200.2$$

$$R_c = 0.04004\ \Omega$$

Similarly,

$$9{,}999 \times \text{(I)}: 9{,}999R_a + 9{,}999R_b + 9{,}999R_c = 500.45$$

$$\text{(III)}: 9{,}999R_a - R_b - R_c = 50$$

Subtracting (III) from (I), we obtain

$$10{,}000R_b + 10{,}000R_c = 450.45$$

Substituting the previously calculated value for R_c into this expression yields

$$10{,}000R_b = 450.45 - 400.4$$

$$R_b = 0.005005\ \Omega$$

$$R_a = 0.005005\ \Omega$$

This calculation indicates that for larger currents the value of the shunt resistor may become very small.

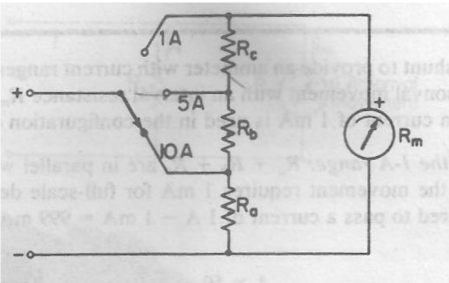


Figure 4-11 Universal or Ayrton shunt.

Direct-current ammeters are commercially available in a large number of ranges, from 20 μA to 50 A full-scale for a self-contained meter and to 500 A for a meter with external shunt. Laboratory-type precision ammeters are provided with a calibration chart, so that the user may correct his readings for any scale errors.

The following precautions should be observed when using an ammeter in measurement work:

a. Never connect an ammeter across a source of emf. Because of its low resistance it would draw damaging high currents and destroy the delicate movement. Always connect an ammeter in series with a load capable of limiting the current.

b. Observe the correct polarity. Reverse polarity causes the meter to deflect against the mechanical stop and this may damage the pointer.

c. When using a multirange meter, first use the highest current range; then decrease the current range until substantial deflection is obtained. To increase accuracy of the observation (see Chapter I), use the range that will give a reading as near to full-scale as possible.

4-5 DC VOLTMETERS

4-5.1 Multiplier Resistor

The addition of a series resistor, or multiplier, converts the basic d’Arsonval movement into a de voltmeter, as shown in Fig. 4-12. The multiplier limits the current through the movement so as not to exceed the value of the full-scale deflection current (I_{fsd}). A dc voltmeter measures the potential difference between

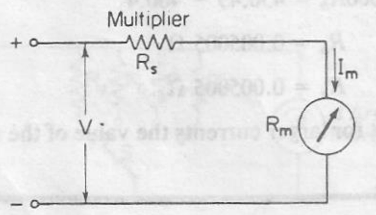


Figure 4-12 Basic dc voltmeter circuit.

two points in a dc circuit and is therefore connected across a source of emf or a circuit component. The meter terminals are generally marked “pos” and “neg,” since polarity must be observed.

The value of a multiplier, required to extend the voltage range, is calculated from Fig. 4-12, where

I_m = deflection current of the movement (I)

R_m = internal resistance of the movement

R_s = multiplier resistance

V = full-range voltage of the instrument

For the circuit of Fig. 4-12,

Solving for R_s gives

$$R_s = \frac{V - I_m R_m}{I_m} = \frac{V}{I_m} - R_m \qquad (4\text{-}4)$$

The multiplier is usually mounted inside the case of the voltmeter for moderate ranges up to 500 V. For higher voltages, the multiplier may be mounted separately outside the case on a pair of binding posts to avoid excessive heating inside the case.

4-5.2 Multirange Voltmeter

The addition of a number of multipliers, together with a range switch, provides the instrument with a workable number of voltage ranges. Figure 4-13 shows a multirange voltmeter using a four-position switch and four multipliers, R_1, R_2, R_3, and R_4, for the voltage ranges V_1, V_2, V_3, and V_4, respectively. The values of the multipliers can be calculated using the method shown earlier or, alternatively, by the sensitivity method. The sensitivity method is illustrated by Example 4-4 in Sec. 4-6, where sensitivity is discussed.

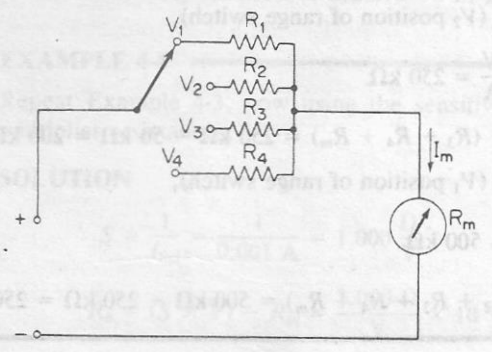


Figure 4-13 Multirange voltmeter.

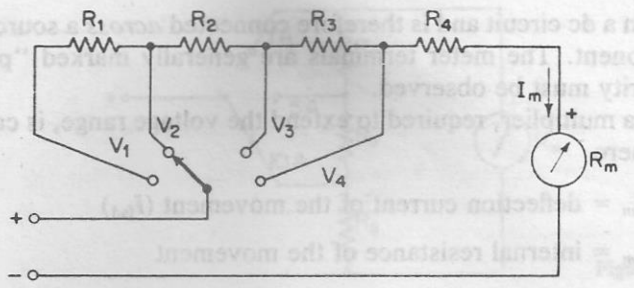


Figure 4-14 More practical arrangement of multiplier resistors in the multirange voltmeter.

A variation of the circuit of Fig. 4-13 is shown in Fig. 4-14, where the multipliers are connected in a series string and the range selector switches the appropriate amount of resistance in series with the movement. This system has the advantage that all multipliers except the first have standard resistance values and can be obtained commercially in precision tolerances. The low-range multiplier, R_4, is the only special resistor that must be manufactured to meet the specific circuit requirements.

EXAMPLE 4-3

A basic d'Arsonval movement with internal resistance, $R_m = 100\Omega$, and full- scale current, $I_{fsd} = 1$ mA, is to be converted into a multirange dc voltmeter with voltage ranges of 0-10 V, 0-50 V, 0-250 V. and 0-500 V. The circuit arrangement of Fig. 4-16 is to be used for this voltmeter.

SOLUTION

For the 10-V range (V_4 position of range switch), the total circuit resistance is

$$R_T \qquad = \frac{10V}{1mA} = 10k\Omega$$

$R_4 = R_T - R_m = 10\ k\Omega – 100\ \Omega = 9{,}900\ \Omega$

For the 50-V range (V_3 position of range switch),

$$R_T = \frac{50V}{1mA} = 50k\Omega$$

$$R_3 = R_T - (R_4 + R_m) = 50k\Omega - 10k\Omega = 40k\Omega$$

For the 250-V range (V_2 position of range switch),

$$R_T = \frac{250V}{1mA} = 250k\Omega$$

$$R_2 = R_T - (R_3 + R_4 + R_m) = 250k\Omega - 50k\Omega = 200k\Omega$$

For the 500-V range (V_1 position of range switch),

$$R_T = \frac{250V}{1mA} = 500k\Omega$$

$$R_2 = R_T - (R_2 + R_3 + R_4 + R_m) = 500k\Omega - 250k\Omega = 250k\Omega$$

Notice in Example 4-3 that only the low-range multiplier R_4 has a nonstandard value.

4-6 VOLTMETER SENSITIVITY

4-6.1 Ohms-per-Volt Rating

In Sec. 4-5 it was shown that the full-scale deflection current I_{fsd} was reached on all voltage ranges when the corresponding full-scale voltage was applied. As shown in Example 4-3, a current of 1 mA is obtained for voltages of 10 V, 50 V, 250 V, and 500 V across the meter terminals. For each voltage range, the quotient of the total circuit resistance R_T and the range voltage V is always 1,000 Ω/V. This figure is often referred to as the sensitivity, or the ohms-per-volt rating, of the voltmeter. Note that the sensitivity, S, is essentially the reciprocal of the full- scale deflection current of the basic movement, or

$$S = \frac{1}{I_{fsd}} \frac{\Omega}{V} \qquad (4\text{-}5)$$

The sensitivity S of the voltmeter may be used to advantage in the sensitivity method of calculating the resistance of the multiplier in a dc voltmeter. Consider the circuit of Fig. 4-14, where

S = sensitivity of the voltmeter (Ω/V)

V = the voltage range, as set by the range switch

R_m = internal resistance of the movement (plus the previous series resistors)

R_s= resistance of the multiplier

For the circuit of Fig. 4-14,

$$R_T = S \times V \qquad (4\text{-}6)$$

and

$$R_S = (S \times V) - R_m \qquad (4\text{-}7)$$

Use of the sensitivity method is illustrated in Example 4-4.

EXAMPLE 4-4

Repeat Example 4-3, now using the sensitivity method for calculating the multiplier resistances.

SOLUTION

$$S = \frac{1}{I_{fsd}} = \frac{1}{0.001\ \text{A}} = 1{,}000\ \frac{\Omega}{\text{V}}$$

$$R_4 = (S \times V) - R_m = \frac{1{,}000\ \Omega}{\text{V}} \times 10\ \text{V} - 100\ \Omega = 9{,}900\ \Omega$$

$$R_3 = (S \times V) - R_m = \frac{1{,}000\ \Omega}{\text{V}} \times 50\ \text{V} - 10{,}000\ \Omega = 40\ \text{k}\Omega$$

$$R_2 = (S \times V) - R_m = \frac{1{,}000\ \Omega}{\text{V}} \times 250\ \text{V} - 50\ \text{k}\Omega = 200\ \text{k}\Omega$$

$$R_1 = (S \times V) - R_m \frac{1{,}000\ \Omega}{\text{V}} \times 500\ \text{V} - 250\ \text{k}\Omega = 250\ \text{k}\Omega$$

4-6.2 Loading Effect

The sensitivity of a dc voltmeter is an important factor when selecting a meter for a certain voltage measurement. A low-sensitivity meter may give correct readings when measuring voltages in low-resistance circuits, but it is certain to produce very unreliable readings in high resistance circuits. A voltmeter connected across two points in a highly resistive circuit, acts as a shunt for that portion of the circuit and thus reduces the equivalent resistance in that portion of the circuit. The meter will then give a lower indication of the voltage drop than actually existed before the meter was connected. This effect is called the loading effect of an instrument; it is caused principally. low-sensitivity instruments. The loading effect of a voltmeter is illustrated in Example 4-5.

EXAMPLE 4-5

It is desired to measure the voltage across the 50-kΩ resistor in the circuit of Fig. 4-15. Two voltmeters are available for this measurement: voltmeter 1 with a sensitivity of 1,000 Ω/V and voltmeter 2 with a sensitivity of 20,000 Ω/V. Both meters are used on their 50-V range. Calculate (a) the reading of each meter; (b) the error in each reading, expressed as a percentage of the true value.

SOLUTION Inspection of the circuit indicates that the voltage across the 50-kΩ resistor is

$$\frac{50\ \text{k}\Omega}{150\ \text{k}\Omega} \times 150\ \text{V} = 50\ \text{V}$$

This is the *true* value of voltage across the 50-kΩ resistor.

(a) *Voltmeter 1* ($S = 1{,}000\ \Omega/\text{V}$) has a resistance of $50\ \text{V} \times 1{,}000\ \Omega/\text{V} = 50\ \text{k}\Omega$ on its 50-V range. Connecting the meter across the 50-kΩ resistor *causes the equivalent parallel resistance to be decreased to 25 kΩ and the total* circuit resistance to 125 kΩ. The potential difference across the combination of meter and 50-kΩ resistor is

$$V_1 = \frac{25\ \text{k}\Omega}{125\ \text{k}\Omega} \times 150\ \text{V} = 30\ \text{V}$$

Hence the voltmeter indicates a voltage of 30 V. *Voltmeter 2* ($S = 20\ \text{k}\Omega/\text{V}$) has a resistance of $50\ \text{V} \times 20\ \text{k}\Omega/\text{V} = 1$ megohm on its 50-V range. When this meter is connected across the 50-kΩ resistor, the equivalent parallel resistance equals 47.6 kΩ. This combination produces a voltage of

$$V_2 = \frac{47.6\ \text{k}\Omega}{147.6\ \text{k}\Omega} \times 150\ \text{V} = 48.36\ \text{V}$$

which is indicated on the voltmeter.

(b) The error in the reading of voltmeter 1 is

$$\%\ \text{error} = \frac{\text{true voltage} - \text{apparent voltage}}{\text{true voltage}} \times 100\%$$

$$= \frac{50\ \text{V} - 30\ \text{V}}{50\ \text{V}} \times 100\% = 40\%$$

The error in the reading of voltmeter 2 is

$$\%\ \text{error} = \frac{50\ \text{V} - 48.36\ \text{V}}{50\ \text{V}} \times 100\% = 3.28\%$$

The calculation of Example 4-5 indicates that the meter with the higher sensitivity or ohms-per-volt rating gives the most reliable result. It is important to realize the factor of sensitivity, particularly when voltage measurements are made in high-resistance circuits.

The matter of reliability and accuracy of the test result raises an interesting point. When an insensitive, yet highly accurate, dc voltmeter is placed across the terminals of a high resistance, the meter accurately reflects the voltage condition produced by loading. The error is a human oLg sixr (Sec. 1-4), because the proper instrument was not selected. TI1èThiter “disturbs” the circuit, and the ideal of instrumentation, at all times, is to measure a condition without affecting it in any way. The human investigator has the responsibility to select an instrument which is precise, reliable, and sufficiently sensitive not to disturb that which is being measured. The fault lies not with the highly accurate instrument but with the investigator, who is using it incorrectly. In fact, the sophisticated instrument user could calculate the true voltage by using an insensitive yet accurate meter. Therefore accuracy is always required in instruments; sensitivity is needed only in special applications where loading disturbs that which is being measured. Example 4-6 illustrates how an insensitive yet accurate instrument is used to perform an entirely valid measurement.

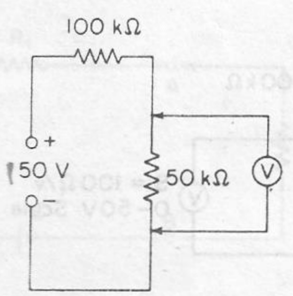


Figure 4-15 Voltmeter loading effect.

EXAMPLE 4-6

The only voltmeter available in a laboratory has a sensitivity of 100 Ω/V and three scales, 50 V, 150 V, and 300 V. When connected in the circuit of Fig. 4-16, the meter reads 4.65 V on its lowest (50-V) scale. Calculate the value of R_x, where R_V is the voltmeter resistance.

SOLUTION The equivalent resistance of the voltmeter on its 50-V scale is

$$R_V = 100\ \frac{\Omega}{\text{V}} \times 10\ \text{V} = 5\ \text{k}\Omega$$

Let R_p = the parallel resistance of R_x and R_V.

$$R_p = \frac{V_p}{V_s} \times R_s = \frac{4.65}{95.35} \times 100\ \Omega = 4.878\ \text{k}\Omega$$

Then

$$R_x = \frac{R_p \times R_V}{R_V - R_p} = \frac{4.878\ \text{k}\Omega \times 5\ \text{k}\Omega}{0.122\ \text{k}\Omega} = 200\ \text{k}\Omega$$

Example 4-6 shows that when the instrument user is aware of the limitations of his instrument, he can still make allowances provided that the voltmeter is accurate.

The following general precautions should be observed when using a voltmeter:

a. Observe the correct polarity. Wrong polarity causes the meter to deflect against the mechanical stop and this may damage the pointer.

b. Place the voltmeter across the circuit or component whose voltage is so be measured.

c. When using a multirange voltmeter, always use the highest voltage range and then decrease the range until a good up-scale reading is obtained.

d. Always be aware of the loading effect. The effect can be minimized by using as high a voltage range (and highest sensitivity) as possible. The precision of measurement decreases if the indication is at the low end of the scale (Sec. 1-4).

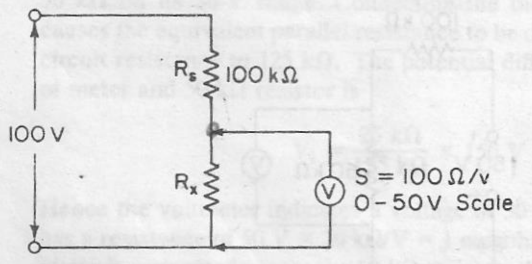


Figure 4-16 Use of an accurate but insensitive voltmeter to determine resistance of R_x.

4-7 SERIES-TYPE OHMMETER

The series-type ohmmeter essentially consists of a d'Arsonval movement connected in series with a resistance and a battery to a pair of terminals to which the unknown is connected. The current through the movement then depends on the magnitude of the unknown resistor, and the meter indication is proportional to the value of the unknown, provided that calibration problems are taken into account. Figure 4-17 shows the elements of a simple single-range series ohmmeter. In Fig. 4-17,

R_1 = current-limiting resistor

R_2 = zero adjust resistor

E = internal battery

R_m = internal resistance of the d'Arsonval movement

R_x = unknown resistor

When the unknown resistor R_x = 0 (terminals A and B shorted), maximum current flows in the circuit. Under this condition, shunt resistor R_2 is adjusted until the movement indicates full-scale current (I_{fsd}). The full-scale current position of the pointer is marked "0Ω" on the scale. Similarly, when R_x= α (terminals A and B open), the current in the circuit drops to zero and the movement indicates zero current, which is then marked "α" on the scale. Intermediate markings may be placed on the scale by connecting different known values of R_x to the instrument. The accuracy of these scale markings depends on the repeating accuracy of the movement and the tolerances of the calibrating resistors.

Although the series-type ohmmeter is a popular design and is used extensively in portable instruments for general-service work, it has certain disadvantages. Important among these is the internal battery whose voltage decreases gradually with time, so that the full-scale current drops and the meter does not read "0" when A and B are shorted. The variable shunt resistor R_2 in Fig. 4-17 provides an adjustment to counteract the effect of battery change. Without R_2, it would be possible to bring the pointer back to full scale by adjusting R_1. but this would change the calibration all along the scale. Adjustment by R_2 is a superior solution, since the parallel resistance of R_2 and the coil R_m is always low compared to R_1 and therefore the change in R_2 needed for adjustment does not change the

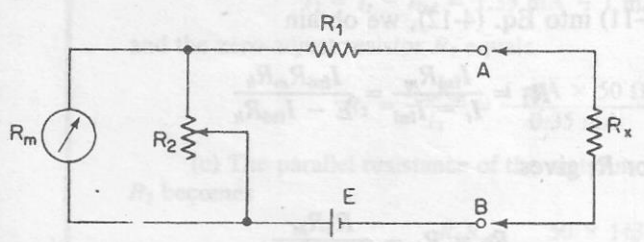


Figure 4-17 Series-type ohmmeter.

calibration very much. The circuit of Fig. 4-17 does not compensate completely for aging of the battery, but it does a reasonably good job within the expected limits of accuracy of the instrument.

A convenient quantity to use in the design of a series-type ohmmeter is the value of R_x which causes half-scale deflection of the meter. At this position, the resistance across terminals A and B is defined as the half-scale position resistance R_h. Given the full-scale current I_{fsd} and the internal resistance of the movement R_m, the battery voltage E, and the desired value of the half-scale resistance R_h, the circuit can be analyzed; i.e., values can be found for R_1 and R_2.

The design can be approached by recognizing that, if introducing R_h reduces the meter current to ½ I_{fsd}, the unknown resistance must be equal to the total internal resistance of the ohmmeter. Therefore

$$R_h = R_1 + \frac{R_2 R_m}{R_2 + R_m} \qquad (4\text{-}8)$$

The total resistance presented to the battery then equals $2R_h$, and the battery current needed to supply the half-scale defection is

$$I_h = \frac{E}{2R_h} \qquad (4\text{-}9)$$

To produce full-scale deflection, the battery current must be doubled, and therefore

$$I_t = 2I_h = \frac{E}{R_h} \qquad (4\text{-}10)$$

The shunt current through R_2 is

$$I_2 = I_t - I_{fsd} \qquad (4\text{-}11)$$

The voltage across the shunt (E_{sh}) is equal to the voltage across the movement and

$$E_{sh} = E_m \text{ or } I_2R_2 = I_{fsd}R_m$$

and

$$R_2 = \frac{I_{fsd}R_m}{I_2} \qquad (4\text{-}12)$$

substituting Eq. (4-11) into Eq. (4-12), we obtain

$$R_2 = \frac{I_{fsd}R_m}{I_t - I_{fsd}} = \frac{I_{fsd}R_mR_h}{E - I_{fsd}R_h} \qquad (4\text{-}14)$$

Solving Eq. (4-8) for R_1 gives

$$R_1 = R_h - \frac{R_2 R_m}{R_2 + R_m} \tag{4-14}$$

Substituting Eq. (4-13) into Eq. (4-14) and solving for R_1 yields

$$R_1 = R_h - \frac{I_{fsd} R_m R_h}{E} \tag{4-15}$$

A typical calculation for the series-type ohmmeter is given in Example 4-7.

EXAMPLE 4-7

The ohmmeter of Fig. 4-17 uses a 50-Ω basic movement requiring a full-scale current of 1 mA. The internal battery voltage is 3 V. The desired scale marking for half-scale deflection is 2,000 Ω. Calculate (a) the values of R_1 and R_2; (b) the maximum value of R_2 to compensate for a 10% drop in battery voltage; (c) the scale error at the half-scale mark (2,000 Ω) when R_2 is set as in (b).

SOLUTION (a) The total battery current at full-scale deflection is

$$I_t = \frac{E}{R_h} = \frac{3\text{ V}}{2{,}000\ \Omega} = 1.5\text{ mA} \qquad (4\text{-}16)$$

The current through the zero-adjust resistor R_2 then is

$$I_2 = I_t - I_{fsd} = 1.5\text{ mA} - 1\text{ mA} = 0.5\text{ mA} \qquad (4\text{-}17)$$

The value of the zero-adjust resistor R_2 is

$$R_2 = \frac{I_{fsd}R_m}{I_2} = \frac{1\text{ mA} \times 50\ \Omega}{0.5\text{ mA}} = 100\ \Omega \qquad (4\text{-}18)$$

The parallel resistance of the movement and the shunt (R_p) is

$$R_p = \frac{R_2R_m}{R_2 + R_m} = \frac{50 \times 100}{150} = 33.3\ \Omega$$

The value of the current-limiting resistor R_1 is

$$R_1 = R_h - R_p = 2{,}000 - 33.3 = 1{,}966.7\ \Omega$$

(b) At a 10% drop in battery voltage,

$$E = 3\text{ V} - 0.3\text{ V} = 2.7\text{ V}$$

The total battery current I_t then becomes

$$I_t = \frac{E}{R_h} = \frac{2.7\text{ V}}{2{,}000\ \Omega} = 1.35\text{ mA}$$

The shunt current I_2 is

$$I_2 = I_t - I_{fsd} = 1.35\text{ mA} - 1\text{ mA} = 0.35\text{ mA}$$

and the zero-adjust resistor R_2 equals

$$R_2 = \frac{I_{fsd}R_m}{I_2} = \frac{1\text{ mA} \times 50\ \Omega}{0.35\text{ mA}} = 143\ \Omega$$

(c) The parallel resistance of the meter movement and the new value of R_2 becomes

$$R_p = \frac{R_2R_m}{R_2 + R_m} = \frac{50 \times 143}{193} = 37\ \Omega$$

Since the half-scale resistance R_h is equal to the total internal circuit resistance, R_h will increase to

$$R_h = R_1 + R_p = 1{,}966.7\ \Omega + 37\ \Omega = 2{,}003.7\ \Omega$$

Therefore the true value of the half-scale mark on the meter is 2,003.7 Ω whereas the actual scale mark is 2,000 Ω. The percentage error is then

$$\%\text{ error} = \frac{2{,}000 - 2{,}003.7}{2{,}003.7} \times 100\% = -0.185\%$$

The negative sign indicates that the meter reading is low.

The ohmmeter of Example 4-7 could be designed for other values of R_h, within limits. If R_h = 3,000 Ω, the battery current would be 1 mA, which is required for the full-scale deflection current. If the battery voltage would decrease owing to aging, the total battery current would fall below 1 mA and there would then be no provision for adjustment.

4-8 SHUNT-TYPE OHMMETER

The circuit diagram of a shunt-type ohmmeter is shown in Fig. 4-18. It consists of a battery in series with an adjustable resistor R_1 and a d'Arsonval movement. The unknown resistance is connected across terminals A and B, in parallel with the meter. In this circuit it is necessary to have an off-on switch to disconnect the battery from the circuit when the instrument is not used. When the unknown resistor R_x = 0 Ω (A and B shorted), the meter current is zero. If the unknown resistor R_x = (A and B open), the current finds a path only through the meter, and by appropriate selection of the value of R1, the pointer can be made to read full scale. The ohmmeter therefore has the "zero" mark at the left-hand side of the scale (no current) and the "infinite" mark at the right-hand side of the scale (full-scale deflection current).

The shunt-type ohmmeter is particularly suited to the measurement of low- value resistors. It is not a commonly used test instrument, but it is found in laboratories or for special low-resistance applications.

The analysis of the shunt-type ohmmeter is similar to that of the series type ohmmeter (Sec. 4-7). In Fig. 4-18, when $R_x = \infty$, the full-scale meter current will

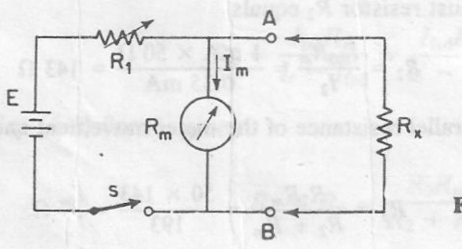


Figure 4-18 Shunt-type ohmmeter.

be

$$I_{fsd} = \frac{E}{R_1 + R_m} \qquad (4\text{-}19)$$

Where E = internal battery voltage

R_1 = current-limiting resistor

R_m = internal resistance of the movement

Solving for R_1, we find

$$R_1 = \frac{E}{I_{fsd}} - R_m \qquad (4\text{-}20)$$

For any value of R_x connected across the meter terminals, the meter current decreases and is given by

$$I_m = \frac{E}{R_1 + [R_m R_x/(R_m + R_x)]} \times \frac{R_x}{R_m + R_x}$$

or

$$I_m = \frac{ER_x}{R_1 R_m + R_x (R_1 + R_m)} \qquad (4\text{-}21)$$

The meter current for any value of R1, expressed as a fraction of the full-scale current, is

$$s = \frac{I_m}{I_{fsd}} = \frac{R_x (R_1 + R_m)}{R_1 (R_m + R_x) + R_m R_x}$$

Or

$$s = \frac{R_x (R_1 + R_m)}{R_x (R_1 + R_m) + R_1 R_m} \qquad (4\text{-}22)$$

Defining

$$\frac{R_1 R_m}{R_1 + R_m} = R_p \qquad (4\text{-}23)$$

and substituting Eq. (4-23) into Eq. (4-22), we obtain

$$s = \frac{R_x}{R_x + R_P} \qquad (4\text{-}24)$$

If Eq. (4-24) is used, the meter can be calibrated by calculating s in terms of R_x and R_p

At half-scale reading of the meter ($I_m = 0.5\ I_{fsd}$), Eq. (4-21) reduces to

$$0.51_{fsd} = \frac{ER_h}{R_1 R_m + R_h (R_1 + R_m)} \qquad (4\text{-}25)$$

where R_h = external resistance causing half-scale deflection. To determine the relative scale values for a given value of R1, the half-scale reading may be found by dividing Eq. (4-19) by Eq. (4-25) and solving for R_h:

$$R_h = \frac{R_1 R_m}{R_1 + R_m} \qquad (4\text{-}26)$$

The analysis shows that the half-scale resistance is determined by limiting resistor R_1 and the internal resistance of the movement, R_m. The limiting resistance, R_1, is in turn determined by the meter resistance R_m, and the full-scale deflection current, I_{fsd}.

To illustrate that the shunt-type ohmmeter is particularly suited to the measurement of very low resistances, consider Example 4-8.

EXAMPLE 4-8

The circuit of Fig. 4-18 uses a 10-mA basic d'Arsonval movement with an internal resistance of 5 Ω. The battery voltage $E = 3$ V. It is desired to modify the circuit by adding an appropriate resistor R_{sh} across the movement, so that the instrument will indicate 0.5 Ω at the midpoint on its scale. Calculate (a) the value of the shunt resistor, R_{sh}; (b) the value of the current-limiting resistor, R_1.

SOLUTION (a) For half-scale deflection of the movement,

$$I_m = 0.5 I_{fsd} = 5 \text{ mA}$$

The voltage across the movement is

$$E_m = 5 \text{ mA} \times 5\ \Omega = 25 \text{ mA}$$

Since this voltage also appears across the unknown resistor, R_x, the current through R_x is

$$I_x = \frac{25 \text{ mV}}{0.5\ \Omega} = 50 \text{ mA}$$

The current through the movement (I_m) plus the current through the shunt (I_{sh}) must be equal to the current through the unknown (I_x). Therefore

$$I_{sh} = I_x - I_m = 50 \text{ mA} - 5 \text{ mA} = 45 \text{ mA}$$

The shunt resistance then is

$$R_{sh} = \frac{E_m}{I_{sh}} = \frac{25 \text{ mV}}{45 \text{ mA}} = \frac{5}{9}\ \Omega$$

(b) The total battery current is

$$I_t = I_m + I_{sh} + I_x = 5 \text{ mA} + 45 \text{ mA} + 50 \text{ mA} = 100 \text{ mA}$$

The voltage drop across limiting resistor R_1 equals 3 V − 25 mV = 2.975 V. Therefore

$$R_1 = \frac{2.975 \text{ V}}{100 \text{ mA}} = 29.75\ \Omega$$

4-9 MULTIMETER OR VOM

The ammeter, the voltmeter, and the ohmmeter all use a d'Arsonval movement. The difference between these instruments is the circuit in which the basic movement is used. It is therefore obvious that a single instrument can be designed to perform the three measurement functions. This instrument, which contains a function switch to connect the appropriate circuits to the d'Arsonval movement, is often called a multimeter or volt-ohm-milliammeter (VOM).

A representative example of a commercial multirneter is shown in Fig. 4-19. The circuit diagram of this meter is given in Fig. 4-20. The meter is a combination of a dc milliammeter, a dc voltmeter, an ac voltmeter, a multirange ohmmeter, and an output meter. (The circuits of the ac voltmeter and the output meter are discussed in Sec. 4-11.2.)

Figure 4-21 shows the circuit for the dc voltmeter section, where the common input terminals are used for voltage ranges of 0-1.5 to 0-1,000 V. An external voltage jack, marked "DC 5,000 V," is used for dc voltage measurements to 5,000 V. The operation of this circuit is similar to the circuit of Fig. 4-12, which was discussed in Sec. 4-5.

The basic movement of the multimeter of Fig. 4-19 has a full-scale current of 50 μA and an internal resistance of 2,000Ω. The values of the multipliers are given in Fig. 4-21. Notice that on the 5,000-V range, the range switch should be set to the 1,000-V position, but the test lead should be connected to the external

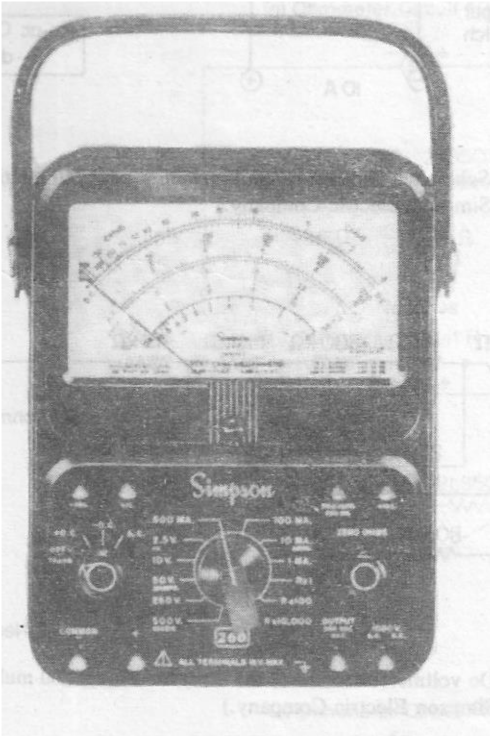


Figure 4-19 General-purpose multimeter. This instrument has been a familiar sight in electronics laboratories for many years. (Courtesy of Simpson Electric Company.)

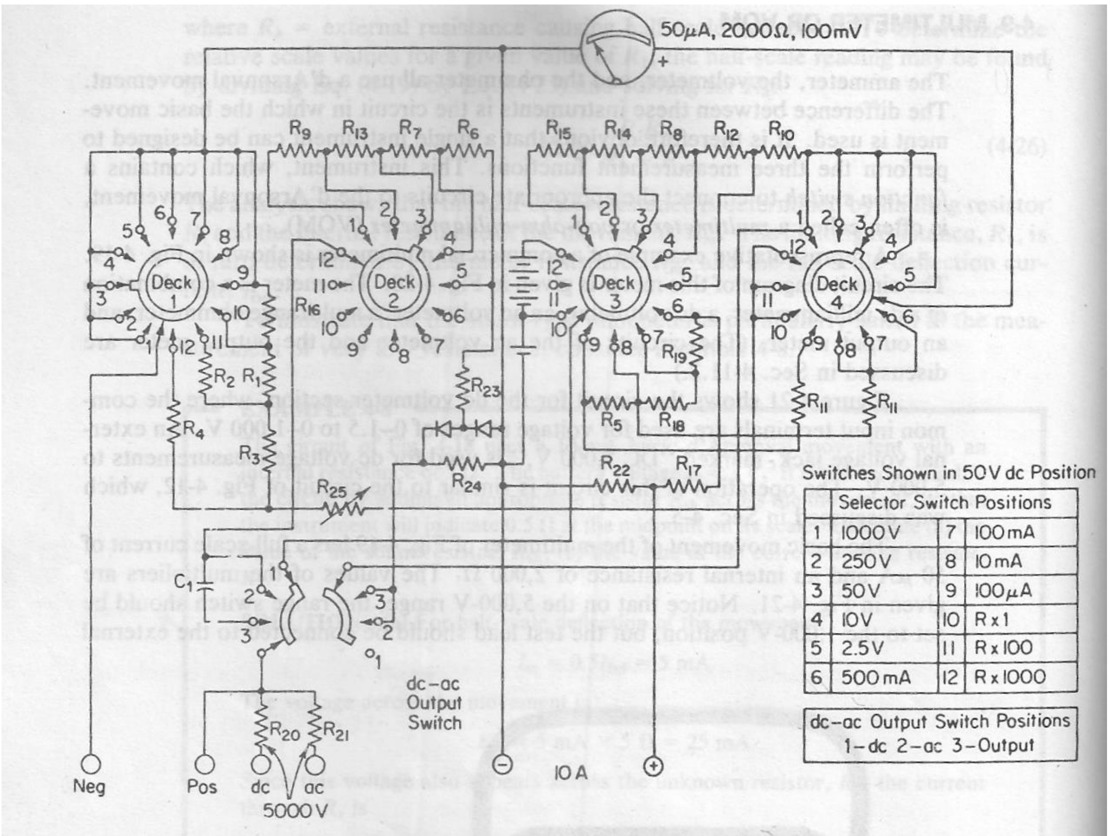


Figure 4-20 Schematic diagram of the Simpson Model 260 multimeter. (Courtesy of Simpson Electric Company.)

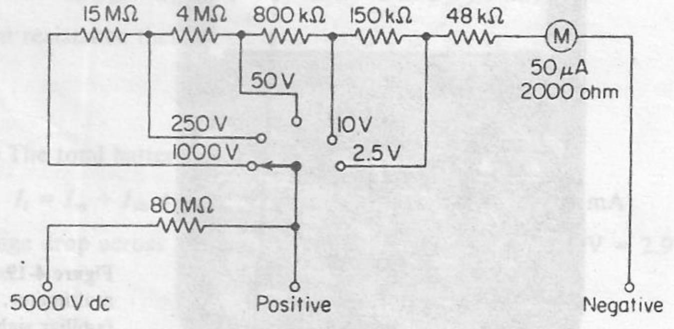


Figure 4-21 Dc voltmeter section of the Simpson Model 260 multimeter. (Courtesy of Simpson Electric Company.)

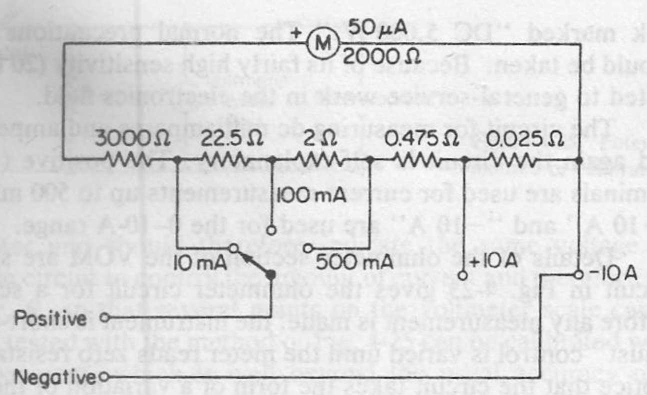
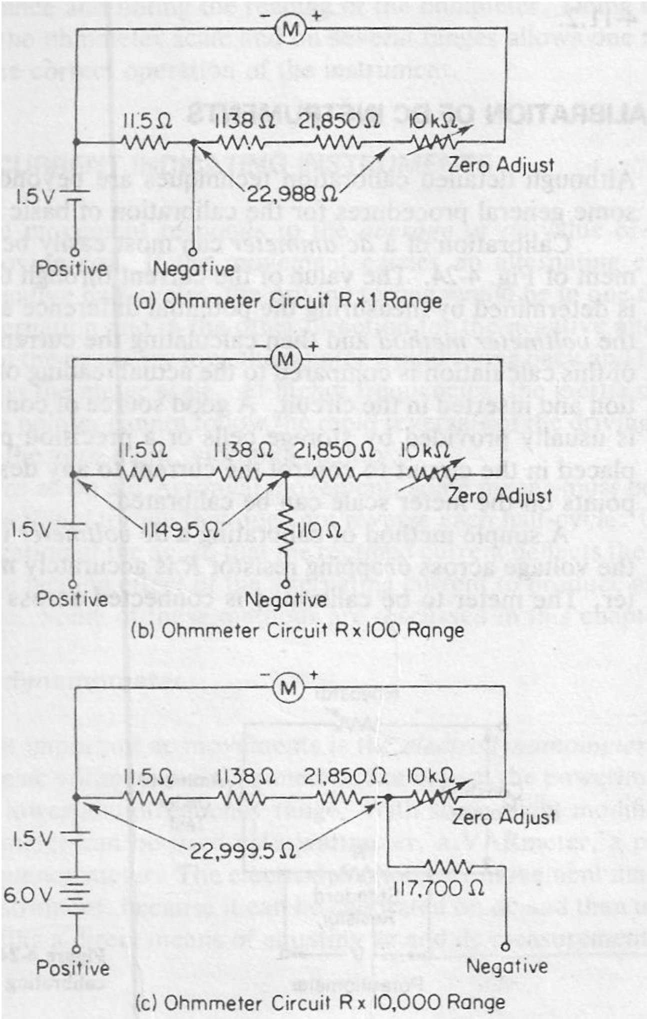


Figure 4-22 Dc ammeter section of the Simpson Model 260 multimeter. (Courtesy of Simpson Electric Company.)



4-23 Ohmmeter section of the Simpson Model 260 multimeter. (…esy of Simpson Electric Company.)

jack marked “DC 5,000 V.’ The normal precautions for measuring voltage & should be taken. Because of its fairly high sensitivity (20 kΩ/V), the instrument is suited to general-service work in the electronics field.

The circuit for measuring dc milliamperes and amperes is given in Fig. 4-22 and again the circuit is self-explanatory. The positive (+) and “negative” (-) terminals are used for current measurements up to 500 mA and the jacks marked “+10 A” and “-10 A” are used for the 0-10-A range.

Details of the ohmmeter section of the VOM are shown in Fig. 4-23. The circuit in Fig. 4-23 gives the ohmmeter circuit for a scale multiplication of 1. Before any measurement is made, the instrument is short-circuited and the "zero- adjust" control is varied until the meter reads zero resistance (full-scale current). Notice that the circuit takes the form of a variation of the shunt-type ohmmeter. Scale multiplications of 100 and 10,000 are shown in Fig. 4-23(b) and (c).

The ac voltmeter section of the meter is selected by setting the "ac-dc" switch to the "ac" position. The operation of this circuit is discussed in Sec. 4-11.2.

4-10 CALIBRATION OF DC INSTRUMENTS

Although detailed calibration techniques are beyond the scope of this chapter, some general procedures for the calibration of basic dc instruments are given.

Calibration of a dc ammeter can most easily be carried out by the arrangement of Fig. 4-24. The value of the current through the ammeter to be calibrated is determined by measuring the potential difference across a standard resistor by the voltmeter method and then calculating the current by Ohm's law. The result of this calculation is compared to the actual reading of the ammeter under calibration and inserted in the circuit. A good source of constant current is required and is usually provided by storage cells or a precision power supply. A rheostat is placed in the circuit to control the current to any desired value, so that different points on the meter scale can be calibrated.

A simple method of calibrating a dc voltmeter is shown in Fig. 4-25, where the voltage across dropping resistor R is accurately measured with a potentiometer. The meter to be calibrated is connected across the same two points as the

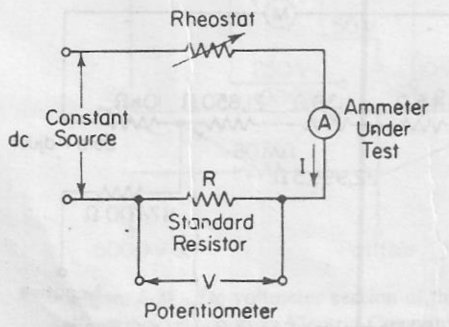


Figure 4-24 Potentiometer method of calibrating a dc ammeter.

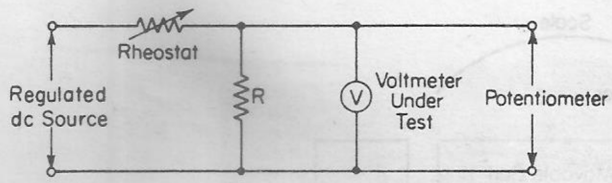


Figure 4-25 Potentiometer method of calibrating a dc voltmeter.

potentiometer and should therefore indicate the same voltage. A rheostat is placed in the circuit to control the amount of current and therefore the drop across the resistor, R, so that several points on the voltmeter scale can be calibrated. Voltmeters tested with the method of Fig. 4-25 can be calibrated with an accuracy of ±0.01 percent, which is well beyond the usual accuracy of a d’Arsonval movement.

The ohmmeter is generally considered to be an instrument of moderate accuracy and low precision. A rough calibration may be done by measuring a standard resistance and noting the reading of the ohmmeter. Doing this for several points on the ohmmeter scale and on several ranges allows one to obtain an indication of the correct operation of the instrument.

4-11 ALTERNATING-CURRENT INDICATING INSTRUMENTS

The d'Arsonval movement responds to the average or de value of the current through the moving coil. If the movement carries an alternating current with positive and negative half-cycles, the driving torque would be in one direction for the positive alternation and in the other direction for the negative alternation. If the frequency of the ac is very low, the pointer would swing back and forth around the zero point on the meter scale. At higher frequencies, the inertia of the coil is so great that the pointer cannot follow the rapid reversals of the driving torque and hovers around the zero mark, vibrating slightly.

To measure ac on a d'Arsonval movement, some means must be devised to obtain a unidirectional torque that does not reverse each half-cycle. One method involves rectification of the ac. so that the rectified current deflects the coil. Other methods use the heating effect of the alternating current to produce an indication of its magnitude. Some of these methods are discussed in this chapter.

4-11.1 Electrodynamometer

One of the most important ac movements is the electrodynamometer. It is often used in accurate ac voltmeters and ammeters, not only at the powerline frequency but also in the lower audio frequency range. With some slight modifications, the electrodynamometer can he used as a wattmeter, a VARmeter, a power-factor meter, or a frequency meter. The electrodynamometer movement may also serve as a transfer instrument, because it can be calibrated on dc and then used directly on ac, establishing a direct means of equating ac and dc measurements of voltage and current.

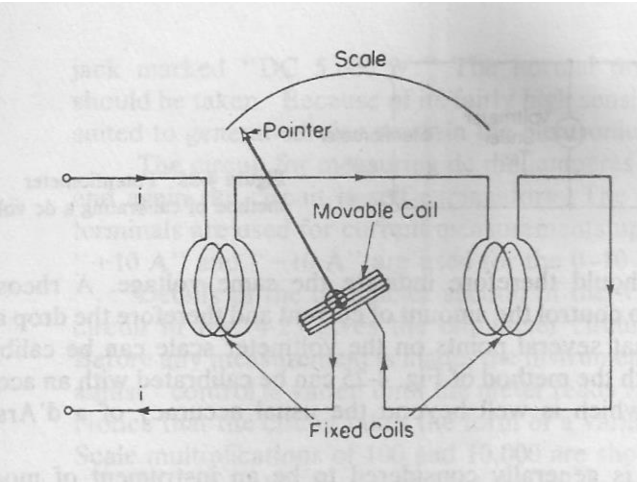


Figure 4-26 Schematic diagram of an electrodynamometer movement.

Where the d'Arsonval movement uses a permanent magnet to provide the magnetic field in which the movable coil rotates, the electrodynamometer uses the current under measurement to produce the necessary field flux. Figure 4-26 shows a schematic arrangement of the parts of this movement. A fixed coil, split into two equal halves, provides the magnetic field in which the movable coil rotates. The two coil halves are connected in series with the moving coil and are fed by current under measurement. The fixed coils are spaced far enough apart to allow passage of the shaft of the movable coil. The movable coil carries a pointer, which is balanced by counterweights. Its rotation is controlled by springs, similar to the d'Arsonval movement construction. The complete assembly is surrounded by a laminated shield to protect the instrument from stray magnetic fields which may affect its operation. Damping is provided by aluminum air vanes, moving in sector-shaped chambers. The entire movement is very solid and rigidly constructed in order to keep its mechanical dimensions stable and its calibration intact. A cutaway view of the electrodynamometer is shown in Fig. 4-27.

The operation of the instrument may be understood by returning to the expression for the torque developed by a coil suspended in a magnetic field. We previously stated [Eq. (4-1)] that

$$T = B \times A \times I \times N$$

indicating that the torque, which deflects the movable coil, is directly proportional to the coil constants (A and N), the strength of the magnetic field in which the coil moves (B), and the current through the coil (I). In the electrodynamometer the flux density (B) depends on the current through the fixed coil and is therefore directly proportional to the deflection current (I). Since the coil dimensions and the number of turns on the coil frame are fixed quantities for any given meter, the developed torque becomes a function of the current squared (I^2).

Af the electrodynamometer is exclusively designed for dc use, its square-law scale is easily noticed, with crowded scale markings at the very low current values, progressively spreading out at the higher current values. For ac use, the developed torque at any instant is proportional to the instantaneous current squared (I^2). The instantaneous value of P is always positive and torque pulsation

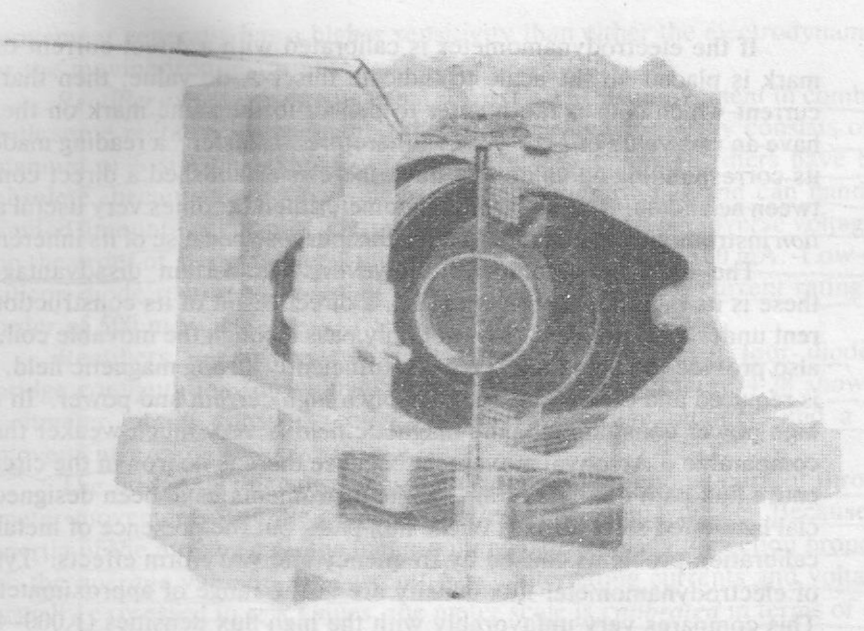


Figure 4-27 Phantom photograph of an electrodynamometer, showing the arrangement of fixed and movable coils. The rigidly constructed mechanism is surrounded by a laminated shield to minimize the effect of external magnetic fields on the meter indication. (Courtesy of Weston Instruments, Inc.)

are therefore produced. The movement, however, cannot follow the rapid variations of the torque and takes up a position in which the average torque is balanced by the torque or the control springs. The meter deflection is therefore a function of the mean of the squared current. The scale of the electrodynamometer is usually calibrated in terms of the square root of the average current squared, and the meter therefore reads the rms or effective value of the ac.

The transfer properties of the electrodynamometer become apparent when we compare the effective value of alternating current in terms of their heating effect or transfer of power. An alternating current that produces heat in a given resistance at the same average rate as a direct current (I) has by definition, a value of I amperes. The average rate of producing heat by a dc of I amperes in a resistance R is I^2R watts. The average rate of producing heat by an ac of i amperes during one cycle in the same resistance R is $\frac{I}{T}\int_0^T i^2 R\,dt$.

By definition, therefore.

$$I^2 R = \frac{I}{T}\int_0^T i^2 R dt$$

and

$$I = \sqrt{\frac{1}{T}\int_0^T i^2 dt} = \sqrt{\text{average } i^2}$$

This current, I, is then called the root-mean-square (rms) or effective value of the alternating current and is often referred to as the equivalent dc value.

If the electrodynamometer is calibrated with a direct current of I A and a mark is placed on the scale to indicate this I-A dc value, then that alternating current which causes the pointer to deflect to the same mark on the scale must have an rms value of I A. We can therefore “transfer” a reading made with dc to its corresponding ac value and have thereby established a direct connection between ac and dc. The electrodynamometer then becomes very useful as a calibration instrument and is often used for this purpose because of its inherent accuracy.

The electrodynamometer, however, has certain disadvantages. One of these is its high power consumption, a direct result of its construction. The current under measurement must not only pass through the movable coil, but it must also provide the field flux. To get a sufficiently strong magnetic field, a high mmf is required and the source must supply a high current and power. In spite of this high power consumption, the magnetic field is very much weaker than that of a comparable d'Arsonval movement because there is no iron in the circuit, i.e., the entire flux path consists of air. Some instruments have been designed using special laminated steel for part of the flux path, but the presence of metal introduces calibration problems caused by frequency and waveform effects. Typical values of electrodynamometer flux density are in the range of approximately 60 gauss. This compares very unfavorably with the high flux densities (1,000-4,000 gauss) of a good d'Arsonval movement. The low flux density of the electrodynamometer immediately affects the developed torque and therefore the sensitivity of the instrument is typically very low.

The addition of a series resistor converts the electro-dynamometer into a voltmeter, which again can be used to measure dc and ac voltages. For reasons previously mentioned, the sensitivity of the electrodynamometer voltmeter is low, approximately 10 to 30 Ω/V (compare this to the20 kΩ/V of a d'Arsonval meter). The reactance and resistance of the coils also increase with increasing frequency, limiting the application of the electro-dynamometer voltmeter to the lower frequency ranges. It is, however, very accurate at the power-line frequencies antis therefore often used as a secondary standard.

The electrodynamometer movement (even unshunted) may be regarded as an ammeter, but it becomes rather difficult to design a moving coil which can carry more than approximately 100 mA. Larger current would have to be carried to the moving coil through heavy lead-in wires, which would lose their flexibility. A shunt, when used, is usually placed across the movable coil only. The fixed coils are then made of heavy wire which can carry the large total current and it is feasible to build ammeters for currents up to 20 A. Larger values of ac currents are usually measured by using a current transformer and a standard 5-A ac ammeter (Sec. 4-16).

4-11.2 Rectifier-Type Instruments

One obvious answer to the question of ac measurement is found by using a rectifier to convert ac into a unidirectional dc and then to use a dc movement to indicate the value of the rectified ac. This method is very attractive, because a dc movement generally has a higher sensitivity than either the electrodynamometer or the moving-iron instrument.

Rectifier type instruments generally use a PMMC movement in combination with some rectifier arrangement. The rectifier element usually consist germanium or a silicon diode. Copper orxide and selenium rectifiers have become obsolete, because they have small inverse voltage ratings and can handle only limited amounts of current. Germanium diodes have a speak iverse voltage (PIV) on the order of 300 V and a current rating of approximately 100 mA. Low-current silicon diode rectifier have a PIV of up to 1,000 V and a current rating on the order of 500 mA.

Rectifier instrument work sometimes consist of four diodes in a bridge configuration. Providing full wave rectification. Figure 4-28 shows an ac voltmeter circuit consisting of a multiplier, a bridge rectifier, and a PMMC movement.

The bridge rectifier produces a pulsating unidirectional current through the meter movement over the com let the input voltage. Because of the inertia of the moving coil, the meter will indicate a steady deflection proportional to the average value of the current. Since alternating currents and voltages are usually expressed in rms values, the meter scale is calibrated in terms of the rms value of a sinusoidal waveform.

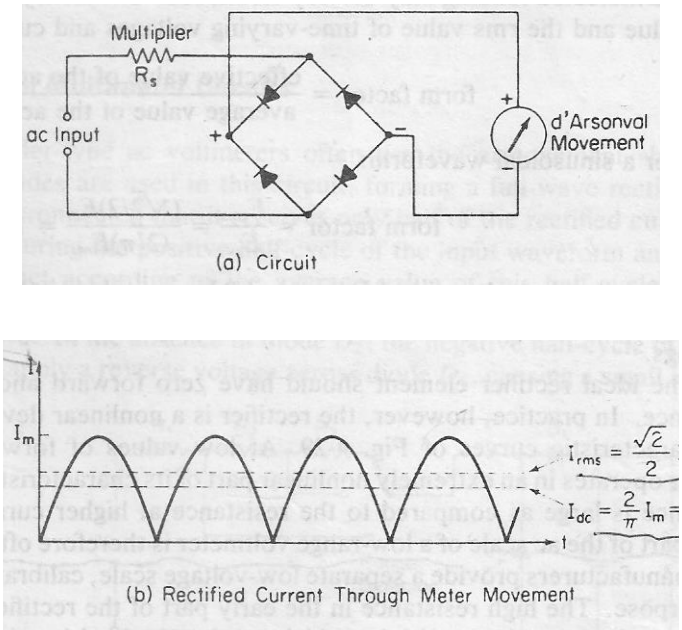


Figure 4-28 Full-wave rectifier ac voltmeter.

EXAMPLE 4-9

An experimental ac voltmeter uses the circuit of Fig. 4-28(a), where the PMMC movement has an internal resistance of 50 Ω and requires a dc current of 1 mA for full-scale deflection. Assuming ideal diodes (zero forward resistance and infinite reverse resistance), calculate the value of the multiplier R_s to obtain full-scale meter deflection with 10 V ac (rms) applied to the input terminals.

SOLUTION For full-wave rectification,

$$E_{dc} = \frac{2}{\pi} E_m = \frac{2\sqrt{2}}{\pi} E_{rms} = 0.9 E_{rms}$$

and

$$E_{dc} = 0.9 \times 10 \text{ V} = 9 \text{ V}$$

The total circuit resistance, neglecting the forward diode resistance, is

$$R_t = R_s + R_m = \frac{9 \text{ V}}{1 \text{ mA}} = 9 \text{ k}\Omega$$

$$R_s = 9{,}000\ \Omega - 50\ \Omega = 8{,}950\ \Omega$$

A nonsinusoidal waveform has an average value that may differ considerably from the average value of a pure sine wave (for which the meter is calibrated) and the indicated reading may be very erroneous. The form factor relates the average value and the rms value of time-varying voltages and currents:

$$\text{form factor} = \frac{\text{effective value of the ac wave}}{\text{average value of the ac wave}}$$

For a sinusoidal waveform:

$$\text{form factor} = \frac{E_{rms}}{E_{av}} = \frac{(\sqrt{2}/2)E_m}{(2/\pi)E_m} = 1.11 \qquad (4\text{-}27)$$

Note that the voltmeter of Example 4-9 has a scale suitable only for sinusoidal ac measurements. The form factor of Eq. (4-27) is therefore also the factor by which the actual (average) dc current is multiplied to obtain the equivalent rms scale markings.

The ideal rectifier element should have zero forward and infinite reverse resistance. In practice, however, the rectifier is a nonlinear device, indicated by the characteristic curves of Fig. 4-29. At low values of forward current, the rectifier operates in an extremely nonlinear part of its characteristic curve, and the resistance is large as compared to the resistance at higher current values. The lower part of the ac scale of a low-range voltmeter is therefore often crowded, and most manufacturers provide low-voltage scale, calibrated specially for this purpose. The high resistance in the early part of the rectifier characteristics also sets a limit on the sensitivity which can be obtained in micro-ammeters and voltmeters.

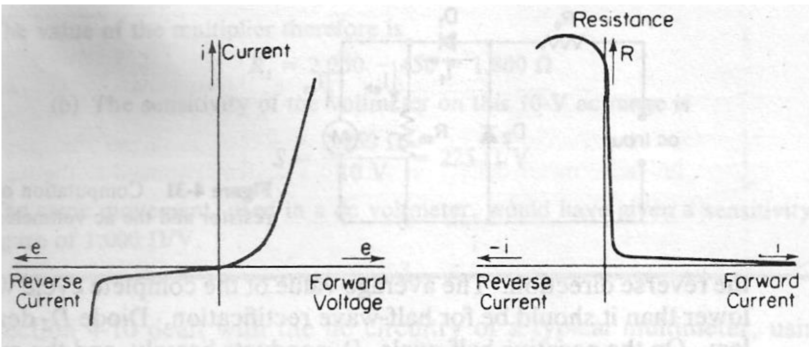


Figure 4-29 Characteristic curves of a solid-state rectifier.

The resistance of the rectifying element changes with varying temperature, one of the major drawbacks of rectifier-type ac instruments. The meter accuracy is usually satisfactory under normal operating conditions at room temperature and is generally on the order of ±5 percent of full-scale reading for sinusoidal waveforms. At very much higher or lower temperatures, the resistance of the rectifier changes the total resistance of the measuring circuit sufficiently to cause the meter to be gravely in error. If large temperature variations are expected, the meter should be enclosed in a temperature-controlled box.

Frequency also affects the operation of the rectifier elements. The rectifier exhibits capacitive properties and tends to bypass the higher frequencies. Meter readings may be in error by as much as 0.5 percent decrease for every l-kHz rise in frequency.

4-11.3 Typical Multimeter Circuits

General rectifier-type ac voltmeters often use the arrangement shown in Fig. 4-30. Two diodes are used in this circuit, forming a full-wave rectifier with the movement so connected that it receives only half of the rectified current. Diode D_1 conducts during the positive half-cycle of the input waveform and causes the meter to deflect according to the average value of this half-cycle. The meter movement is shunted by a resistance R_h, in order to draw more current through the diode D_1 and move its operating point into the linear portion of the characteristic curve. In the absence of diode D_2, the negative half-cycle of the input voltage would apply a reverse voltage across diode D_2, causing a small leakage current in

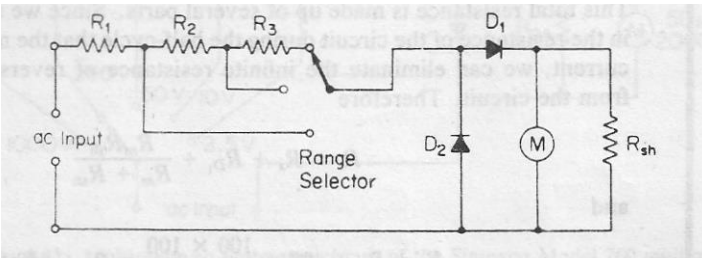


Figure 4-30 Typical ac voltmeter section of a commercial multimeter.

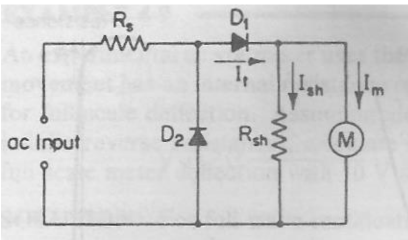


Figure 4-31 Computation of the multiplier resistor and the ac voltmeter sensitivity.

the reverse direction. The average value of the complete cycle would therefore be lower than it should be for half-wave rectification. Diode D_2 deals with this problem. On the negative half-cycle, D_2 conducts heavily, and the current through the measuring circuit, which is now in the opposite direction, bypasses the meter movement.

The commercial multimeter often uses the same scale markings for both its dc and ac voltage ranges. Since the dc component of a sine wave for half-wave rectification equals 0.45 times the rms value, a problem arises immediately. In order to obtain the same deflection on corresponding dc and ac voltage ranges, the multiplier for the ac range must be lowered proportionately. The circuit of Fig. 4-31 illustrates a solution to the problem and is discussed in some detail in Example 440.

Example 4-10

A meter movement has an internal resistance of 100 Ω and requires 1 mA dc for full-scale deflection. Shunting resistor R_{sh}, placed across the movement, has a value of 100 Ω. Diodes D_1 and D_2 have an average forward resistance of 400 Ω each and are assumed to have infinite resistance in the reverse direction. For a 10-V ac range, calculate (a) the value of multiplier R_s; (b) the voltmeter sensitivity on the ac range.

SOLUTION (a) Since R_m and R_{sh} are both 100 Ω, the total current the source must supply for full-scale deflection is I_t = 2 mA. For half-wave rectification the equivalent dc value of the rectified ac voltage will be

$$E_{dc} = 0.45E_{rms} = 0.45 \times 10 \text{ V} = 4.5 \text{ V}$$

The total resistance of the instrument circuit then is

$$R_t = \frac{E_{dc}}{I_t} = \frac{4.5 \text{ V}}{2 \text{ mA}} = 2{,}250 \ \Omega$$

This total resistance is made up of several parts. Since we are interested only in the resistance of the circuit during the half-cycle that the movement receives current, we can eliminate the infinite resistance of reverse-biased diode D_2 from the circuit. Therefore

$$R_t = R_s + R_{D_1} + \frac{R_m R_{sh}}{R_m + R_{sh}}$$

and

$$R_t = R_s + 400 + \frac{100 \times 100}{200} = R_s + 450 \ \Omega$$

The value of the multiplier therefore is

$$R_s = 2{,}250 - 450 = 1{,}800 \ \Omega$$

(b) The sensitivity of the voltmeter on this 10-V ac range is

$$S = \frac{2{,}250 \ \Omega}{10 \text{ V}} = 225 \ \Omega/\text{V}$$

The same movement, used in a dc voltmeter, would have given a sensitivity figure of 1,000 Ω/V.

Section 4-10 dealt with the dc circuitry of a typical multimeter, using the simplified circuit diagram of Fig. 4-20. The circuit for measuring ac volts (subtracted from Fig. 4-20) is reproduced in Fig. 4-32. Resistances R_9, R_{13}, R_7, and R_6 form a chain of multipliers for the voltage ranges of 1,000V, 250V, 50V, and 10V, respectively, and their values are indicated in the diagram of Fig. 4-32. On the 2.5-V ac range, resistor R_{23} acts as the multiplier and corresponds to the multiplier R_{24} of Example 4-10 shown in Fig. 4-31. Resistor R_{24} is the meter shunt and again acts to improve the rectifier operation. Both values are unspecified in the diagram and are factory selected. A little thought, however, will convince us that the shunt resistance could be 2,000 (1, equal to the meter resistance. If the average forward resistance of the rectifier elements is 500Ω (a reasonable assumption), then resistance R_{23} must have a value of 1,000Ω. This follows because the meter sensitivity on the ac ranges is given as 1,000Ω/V; on the 2.5-V ac range, the circuit must therefore have a total resistance of 2,500Ω. This value is made up of the sum of R_{13}, the diode forward resistance, and the combination of movement and-shunt resistance, as shown in Example 4-10.

4-12 THERMOINSTRUMENTS

Figure 4-33 shows a combination of a thermocouple and a PMMC movement that can be used to measure both ac and dc. This combination is called a thermocouple instrument, since its operation is based on the action of the thermocouple element.

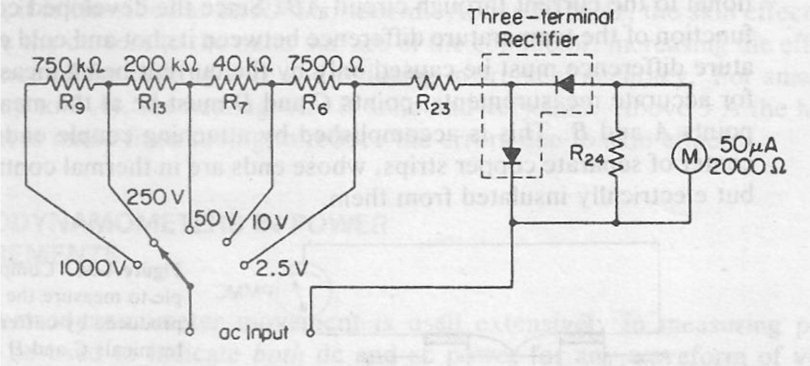


Figure 4-32 Multirange ac voltmeter circuit of the Simpson Model 260 multimeter. (Courtesy of Simpson Electric Company.)

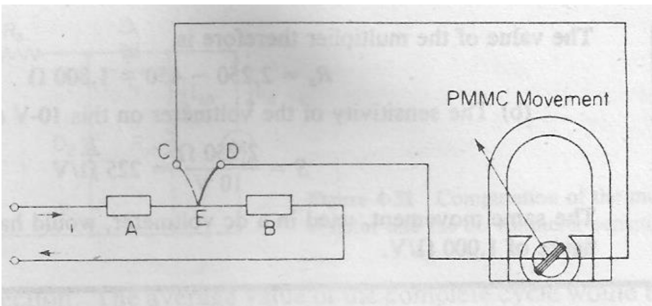


Figure 4-33 Schematic representation of a basic thermocouple instrument using thermocouple CDE and a PMMC movement.

When two dissimilar metals are mutually in contact, a voltage is generated at the junction of the two dissimilar metals. This voltage rises in proportion to the temperature of the junction. In Fig. 4-33, CE and DE represent the two dissimilar metals, joined at point E. and are drawn as a light and a heavy line, to indicate dissimilarity. The potential difference between C and D depends on the temperature of the so-called cold junction, E. A rise in temperature causes an increase in the voltage and this is used to advantage in the thermocouple. Heating element AB, which is in mechanical contact with the junction of the two metals at point E, forms part of the circuit in which the current is to be measured. AEB is called the hot junction. Heat energy generated by the current in the heating element raises the temperature of the cold junction and causes an increase in the voltage generated across terminals C and D. This potential difference causes a dc current through the PMMC-indicating instrument. The heat generated by the current is directly proportional to the current squared (12R), and the temperature rise (and hence the generated dc voltage) is proportional to the square of the rms current. The deflection of the indicating instrument will therefore follow a square-Law relationship, causing crowding at the lower end of the scale and spreading at the high end.

The arrangement of Fig. 4-33 does not provide compensation for ambient temperature changes.

The compensated thermo-element, shown schematically in Fig. 4-34, produces a thermoelectric voltage in thermocouple CED, which is directly proportional to the current through circuit AB. Since the developed couple voltage is a function of the temperature difference between its hot and cold ends, this temperature difference must be caused only by the current being measured. Therefore, for accurate measurements, points C and D must be at the mean temperature of points A and B. This is accomplished by attaching couple ends C and D to the center of separate copper strips, whose ends are in thermal contact with A and B, but electrically insulated from them.

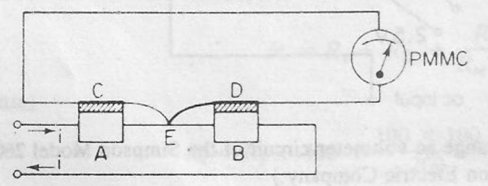


Figure 4-34 Compensated thermocouple to measure the thermo voltage produced by current *i* alone. Couple terminals *C* and *D* are in thermal contact with heater terminals *A* and *B*, but are electrically insulated from them.

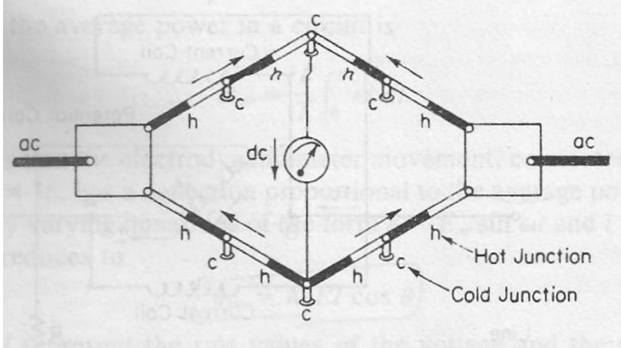


Figure 4-35 Bridge-type thermocouple instrument.

Self-contained thermoelectric instruments of the compensated type are available in the 0.5-20-A range. Higher current ranges are available, but in this case the heating element is external to the indicator. Thermo-elements used for current ranges over 60 A are generally provided with air cooling fins.

Current measurements in the lower ranges, from approximately 0.1-0.75 A, use a bridge-type thermo-element, shown schematically in Fig. 4-35. This arrangement does not use a separate heater: the current to be measured passes directly through the thermo-elements and raises their temperature in proportion to I^2R. The cold junctions (marked c) are at the pins which are embedded in the insulating frame, and the hot junctions (marked h) are at splices midway between the pins. The couples are arranged as shown in Fig. 4-35, and the resultant thermal voltage generates a dc potential difference across the indicating instrument. Since the bridge arms have equal resistances, the ac voltage across the meter is 0 V, and no ac passes through the meter. The use of several thermocouples in series provides a greater output voltage and deflection than is possible with a single element, resulting in an instrument with increased sensitivity.

Thermo-instruments may be converted into voltmeters using low-current thermocouples and suitable series resistors. Thermocouple voltmeters are available in ranges of up to 500 V and sensitivities of approximately 100 to 500Ω/V.

A major advantage of a thermocouple instrument is that its accuracy can be as high as 1 percent, up to frequencies of approximately 50 MHz. For this reason, it is classified as an RF instrument. Above 50 MHz, the skin effect tends to force the current to the outer surface of the conductor, increasing the effective resistance of the heating wire and reducing instrument accuracy. For small currents (up to 3 A), the heating wire is solid and very thin. Above 3 A the heating element is made from tubing to reduce the errors due to skin effect.

4-13 ELECTRODYNAMOMETERS IN POWER MEASUREMENTS

The electrodynamometer movement is used extensively in measuring power. It may be used to indicate both dc and ac power for any waveform of voltage and current and it is not restricted to sinusoidal waveforms. As described in

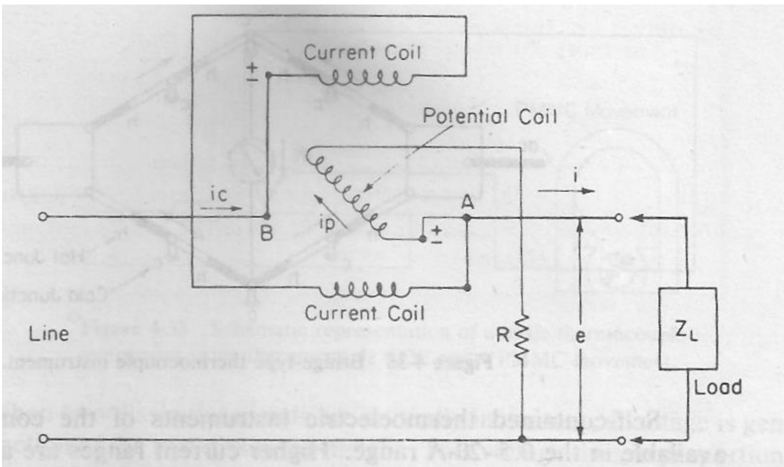


Figure 4-36 Diagram of an electrodynamometer wattmeter connected to measure the power of a single-phase load.

Sec. 4-11.1, the electrodynamometer used as a voltmeter or an ammeter has the fixed coils and the movable coil connected in series, thereby reacting to the effect of the current squared. When used as a single-phase power meter, the coils are connected in a different arrangement (see Fig. 4-36).

The fixed coils, or field coils, shown here as two separate elements, are connected in series and carry the total line current (i_c). The movable coil, located in the magnetic field of the fixed coils, is connected in series with a current- limiting resistor across the power line and carries a small current (i_p). The instantaneous value of the current in the movable coil is $i_p = e/R_p$, where e is the instantaneous voltage across the power line, and R_p is the total resistance of the movable coil and its series resistor. The deflection of the movable coil is proportional to the product of these two currents, i_c and i_p, and we can write for the average deflection over one period:

$$\theta_{av} = K\frac{1}{T}\int_0^T i_c i_p dt \qquad (4\text{-}28)$$

where θ_{av} = average angular deflection of the coil

K = instrument constant

i_c = instantaneous current in the field coils

i_p = instantaneous current in the potential coil

Assuming for the moment that I is equal to the load current, i (actually, $i_c = i_p + i$), and using the value for $i_p = e/R_p$, we see that Eq. (4-28) reduces to

$$\theta_{av} = K\frac{1}{T}\int_0^T i\frac{e}{R_p}dt = K_2\frac{1}{T}\int_0^T ei\ dt \qquad (4\text{-}29)$$

By definition, the avenge power in a circuit is

$$P_{av} = \frac{1}{T}\int_0^T ei\ dt \qquad (4\text{-}30)$$

which indicates that the electrodynamometer movement, connected in the configuration of Fig. 4-36, has a deflection proportional to the average power.

If e and i are sinusoidally varying quantities-of the form $e = E_m \sin wt$ and $i = I_m, \sin (wt \pm \theta)$, Eq. (4-29) reduces to

$$\theta_{av} = K_3 EI \cos \theta \qquad (4\text{-}31)$$

where E and I represent the rms values of the voltage and the current, and θ represents the phase angle between voltage and current. Equations (4-29) and (4-30) show that the electrodynamometer indicates the average power delivered to the load.

Wattmeters have one voltage terminal and one current terminal marked "±." When the marked current terminal is connected to the incoming line, and the marked voltage terminal is connected to the line side in which the current coil is connected, the meter will always read up-scale when power is connected to the load. If for any reason (as in the two-wattmeter method of measuring three-phase power), the meter should read backward, the current connections (not the voltage connections) should be reversed.

The electrodynamometer wattmeter consumes some power for maintenance of its magnetic field, but this is usually so small, compared to the load power, that it may be neglected. If a correct reading of the load power is required, the current coil should carry exactly the load current, and the potential coil should be connected across the load terminals. With the potential coil connected to point A, as in Fig. 4-36, the load voltage is properly metered, but the current through the field coils is greater by the amount i_p. The wattmeter therefore reads high by the amount of additional power loss in the potential circuit. If. however, the potential coil is connected to point B in Fig. 4-38, the field coils meter the correct load current, but the voltage across the potential coil is higher by the amount of the drop across the field coils. The wattmeter will again read high, but now by the amount of the I^2R losses in the field windings. Choice of the correct connection depends on the situation. Generally, connection of the potential coil at point A is preferred for high-current, low-voltage loads; connection at B is preferred for low current, high-voltage loads.

The difficulty in placing the connection of the potential coil is overcome in the compensated wattmeter, shown schematically in Fig. 4-37. The current coil consists of two windings, each winding having the same number of turns. One winding uses heavy wire that carries the load current plus the current for the potential coil. The other winding uses thin wire and carries only the current to the voltage coil. This current, however, is in a direction opposite to the current in the heavy winding, causing a flux that opposes the main flux. The effect of i_p is therefore canceled out, and the wattmeter indicates the correct power.

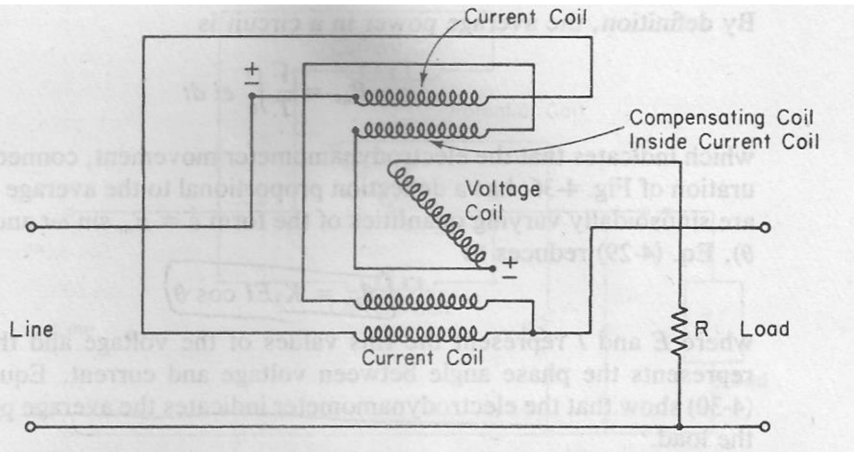


Figure 4-37 Diagram of a compensated wattmeter in which the effect of the current in the potential coil is canceled by the current in the compensating winding.

4-14 WATTHOUR METER

The watthour meter is not often found in a laboratory situation but it is widely used for the commercial measurement of electrical energy. In fact, it is evident wherever a power company supplies the industrial or domestic consumer with electrical energy. Figure 4-38 shows the elements of a single-phase watthour meter in schematic form.

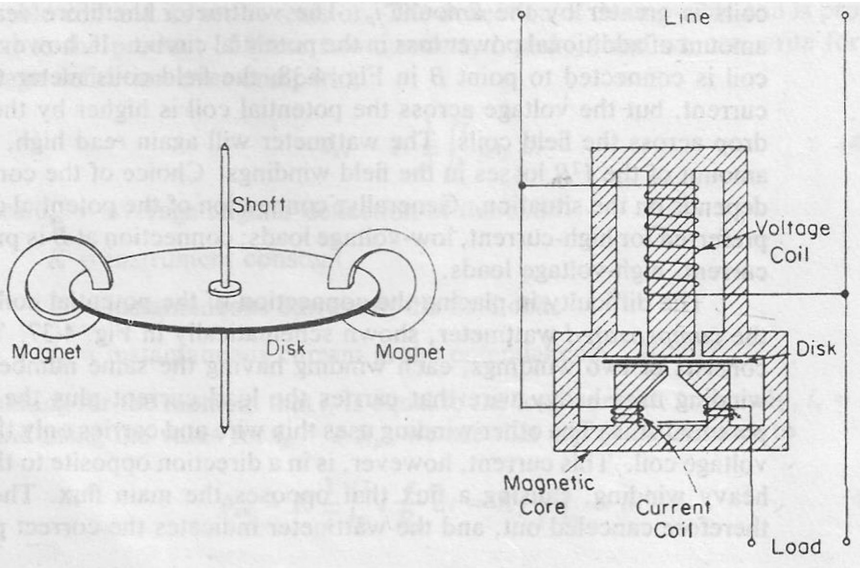


Figure 4-38 Elements of a single-phase watthour meter.

The current coil is connected in series with the line, and the voltage coil is connected across the line. Both coils are wound on a metal frame of special design, providing two magnetic circuits. A light aluminum disk is suspended in the air gap of the current-coil field, which causes eddy currents to flow in the disk. The reaction of the eddy currents and the field of the voltage coil creates a torque (motor action) on the disk, causing it to rotate. The developed torque is proportional to the fieldstrength of the voltage coil and the eddy currents in the disk which are in turn a function of the fieldstrength of the current coil. The number of rotations of the disk is therefore proportional to the energy consumed by the load in a certain time interval, and is measured in terms of kilowatthours (kWh). The shaft that supports the aluminum disk is connected by a gear arrangement o the clock mechanism on the front of the meter, providing a decimally calibrated readout of the number of kWh.

Damping of the disk is provided by two small permanent magnets located opposite each other at the rim of the disk. Whenever the disk rotates, the permanent magnets induce eddy currents in it. These eddy currents react with the magnetic fields of the small permanent magnets, damping the motion of the disk. A typical single-phase watthour meter is shown in Fig. 4-39.

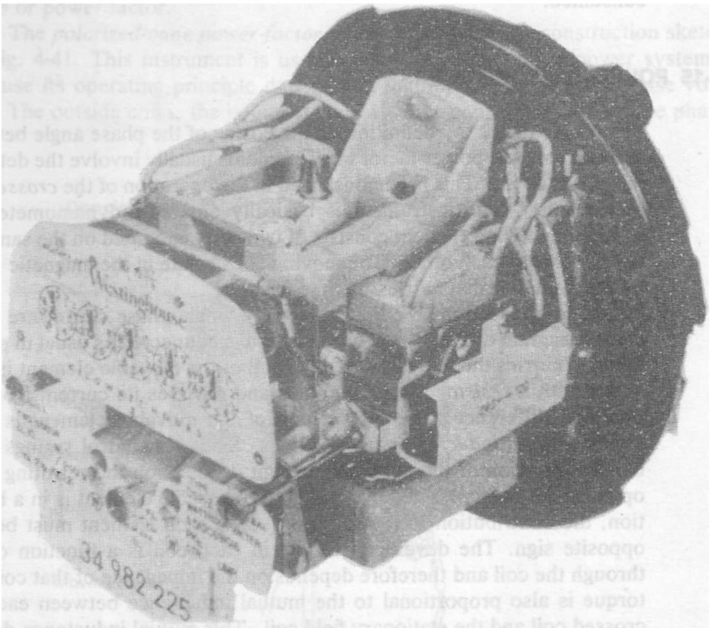


Figure 4-39 Watthour meter for industrial or domestic application. (Courtesy of Westinghouse Electric Corporation.)

Calibration of the watthour meter is performed under conditions of full rated load and 10 percent of rated load. At full load, the calibration consists pf adjustment of the position of the small permanent magnets until the meter reads correctly. At very light loads, the voltage component of the field produces a torque that is not directly proportional to the load. Compensation for the error is provided by inserting a shading coil or plate over a portion of the voltage coil, with the meter operating at 10 percent of rated load. Calibration of the meter at these two positions usually provides satisfactory readings at all other loads.

The floating-shaft watthour meter uses a unique design to suspend the disk. The rotating shaft has a small magnet at each end. The upper magnet of the shaft is attracted to a magnet in the upper bearing, and the lower magnet of the shaft is attracted to a magnet in the lower bearing. The movement thus floats without touching either bearing surface, and the only contact with the movement is that of the gear connecting the shaft with the gear train.

Measurements of energy in three-phase systems are performed with poly phase watthour meters. Each phase of the watthour meter has its own magnetic circuit and its own disk, but all the disks are mounted on a common shaft. The developed torque on each disk is mechanically summed and the total number of revolutions per minute of the shaft is proportional to the total three-phase energy consumed.

4-15 POWER-FACTOR METERS

The power factor, by definition, is the cosine of the phase angle between voltage and current, and power-factor measurements usually involve the determination of this phase angle. This is demonstrated in the operation of the crossed-coil power- factor meter. The instrument is basically an electrodynamometer movement, where the moving element consists of two coils, mounted on the same shaft but at right angles to each other. The moving coils rotate in the magnetic field provided by the field coil that carries the line current.

The connections for this meter in a single-phase circuit are shown in the circuit diagram of Fig. 4-40. The field coil is connected as usual in series with the line and carries the line current. One coil of the movable element is connected in series with a resistor across the lines and receives its current from the applied potential difference. The second coil of the movable element is connected in series with an inductor across the lines. Since no control springs are used, the balance position of the movable element depends on the resulting torque developed by the two crossed coils. When the movable element is in a balanced position, the contribution to the total torque by each element must be equal but of opposite sign. The developed torque in each coil is a function of the current through the coil and therefore depends on the impedance of that coil circuit. The torque is also proportional to the mutual inductance between each part of the crossed coil and the stationary field coil. This mutual inductance depends on the angular position of the crossed-coil elements with respect to the position of the stationary field coil. When the movable element is at balance, it can be shown that

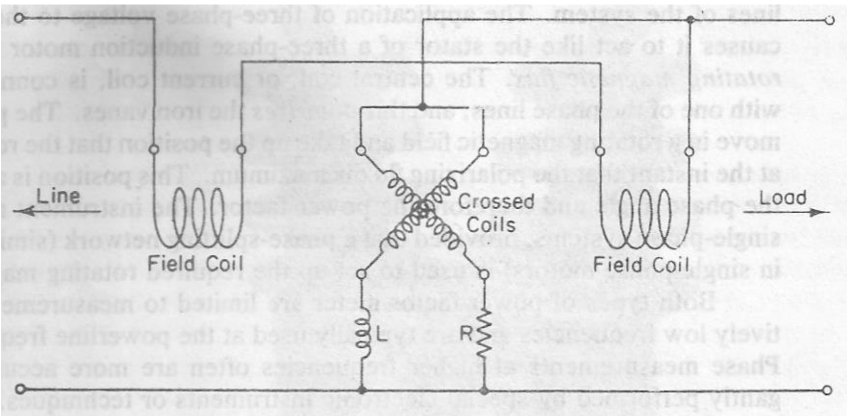


Figure 4-40 Connections for a single-phase crossed-coil power-factor meter.

its angular displacement is a function of the phase angle between line current (field coil) and line voltage (crossed coils). The indication of the pointer, which is connected to the movable element, is calibrated directly in terms of the phase angle or power factor.

The polarized-vane power-factor meter is shown in the construction sketch of Fig. 4-41. This instrument is used primarily in three-phase power systems, because its operating principle depends on the application of three-phase voltage. The outside coil is the potential coil, which is connected to the three phase

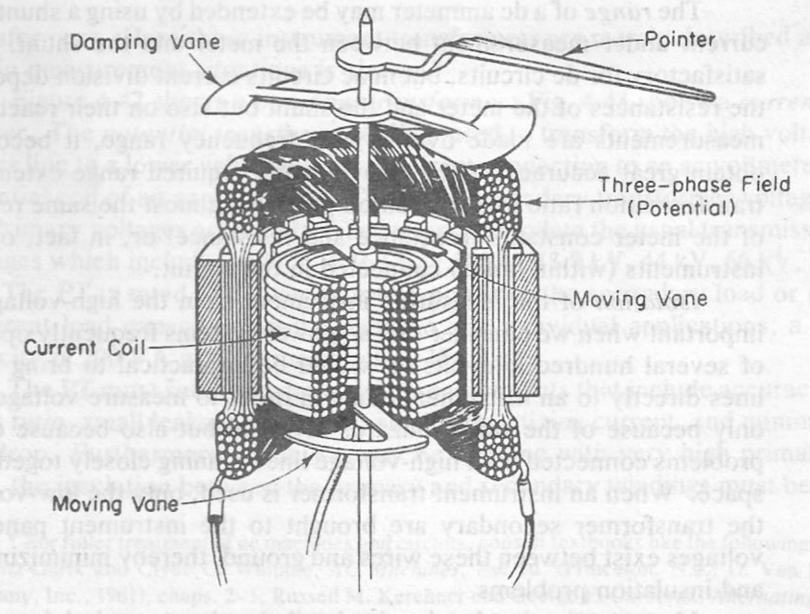


Figure 4-41 Polarized-vane power-factor meter.

lines of the system. The application of three-phase voltage to the potential coil causes it to act like the stator of a three-phase induction motor in setting up a rotating magnetic flux. The central coil, or current coil, is connected in series with one of the phase lines, and this polarizes the iron vanes. The polarized vanes move in a rotating magnetic field and take up the position that the rotating field has at the instant that the polarizing flux is maximum. This position is an indication of the phase angle and therefore the power factor. The instrument may be used in single-phase systems, provided that a phase-splitting network (similar to that used in single-phase motors) is used to set up the required rotating magnetic field.

Both types of power-factor meter are limited to measurement at comparatively low frequencies and are typically used at the powerline frequency (60 Hz). Phase measurements at higher frequencies often are more accurately and elegantly performed by special electronic instruments or techniques.

4-16 INSTRUMENT TRANSFORMERS

Instrument transformers are used to measure ac at generating stations, transformer stations, and at transmission lines, in conjunction with ac measuring instruments (voltmeters, ammeters, wattmeters, VARmeters, etc.). Instrument transformers are classified according to their use and are referred to as current transformers (CT) and potential transformers (PT).

Instrument transformers perform two important functions: They serve to extend the range of the ac measuring instrument, much as the shunt or the multiplier extends the range of a dc meter; they also serve to isolate the measuring instrument from the high-voltage power line.

The range of a dc ammeter may be extended by using a shunt that divides the current under measurement between the meter and the shunt. This method is satisfactory for dc circuits, but in ac circuits current division depends not only the resistances of the meter and the shunt but also on their reactances. Since ac measurements are made over a wide frequency range, it becomes difficult to obtain great accuracy. A CT provides the required range extension through its transformation ratio and in addition produces almost the same reading regardless of the meter constants (reactance and resistance) or, in fact, of the number of instruments (within limits) connected in the circuit.

Isolation of the measuring instrument from the high-voltage power line is important when we consider that ac power systems frequently operate at voltages of several hundred kilovolts. It would be impractical to bring the high-voltage lines directly to an instrument panel in order to measure voltage or current, not only because of the safety hazards involved but also because of the insulation problems connected with high-voltage lines running closely together in a confined space. When an instrument transformer is used, only the low-voltage wires from the transformer secondary are brought to the instrument panel and only low voltages exist between these wires and ground, thereby minimizing safety hazards and insulation problems:

Many textbooks develop in detail the theory underlying the operation of

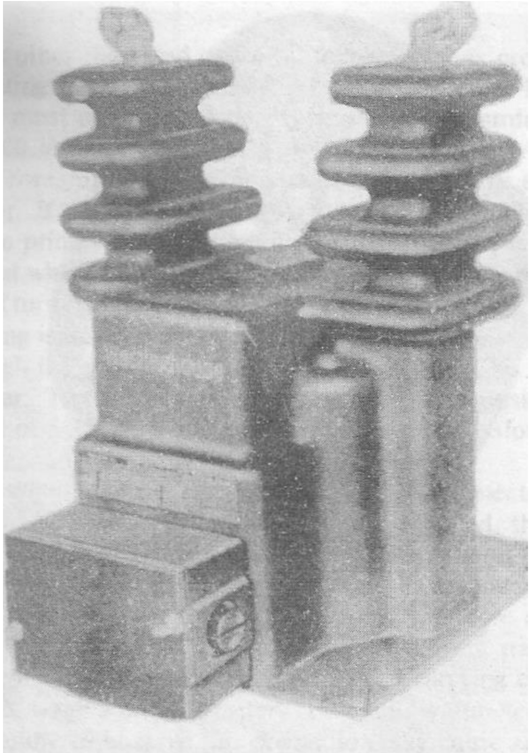


Figure 4-42 High-voltage potential transformer. (Courtesy of Westinghouse Electric Corporation.)

transformers. Here these instrument transformers are merely described and their use in measurement situations is shown.*

Figure 4-42 shows a potential transformer; Fig. 4-43 shows a current transformer. The potential transformer (PT) is used to transform the high voltage of a power line to a lower value suitable for direct connection to an ac voltmeter or the potential coil of an ac wattmeter. The usual secondary transformer voltage is 120 V. Primary voltages are standardized to accommodate the usual transmission line voltages which include 2,400 V, 4,160 V, 7,200 V. 13.8 kV, 44kv, 66kv, and 220 kV. The PT is rated to deliver a certain power to the secondary load or burden. Different load capacities are available to suit individual applications; a general capacity is 200 VA at a frequency of 60 Hz.

The PT must satisfy certain design requirements that include accuracy of the turns ratio, small leakage reactance, small magnetizing current, and minimal voltage drop. Furthermore, since we may be working with very high primary voltages, the insulation between the primary a secondary windings must be able to

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* For fuller treatment of ac machines and circuits, consult textbooks like the following; Michael Liwshitz-Garik and Clyde C. Whipple, AC Machines, 2nd ed. (Princeton, N.J.: D. Van Nostrand Company, Inc., 1961), chaps. 2-5; Russell M. Kerchper and George F. Corcoran, Alternating Current Circuits, 4th ed. (New York: John Wiley & Sons, Inc., 1961). pp. 291-317.

Figure 4-43 Current transformer. (Courtesy of Westinghouse Electric Corporation.)

withstand large potential differences, and the dielectric requirements are very high. In the usual case, the high-voltage coil is of a circular pancake construction, shielded to avoid localized dielectric stresses. The low-voltage coil or coils are wound on a paper form and assembled inside the high-voltage coil. The assembly is thoroughly dried and oil impregnated. The core and coil assembly is then mounted inside a steel case, which supports the high-voltage terminals or porcelain bushings. The case is then filled with an insulating oil.

Developments in the synthetic rubber industry have introduced the molded rubber potential transformer, replacing the insulating oil and porcelain bushings in some applications. Figure 4-42 shows a rubber-molded 25-kV potential transformer suitable for outdoor use. This unit is less expensive than the conventional oil-filled PT, and since the bushings are made of molded rubber, porcelain breakage is eliminated. A white polarity dot is placed on the proper bushing on the front of the transformer. Two stud-type secondary terminals are enclosed in a removable conduit box. The power rating of a potential transformer is based on considerations other than load capacity, for the reasons previously outlined. A typical load rating is 200 VA at 60 Hz for a transformer having a ratio of 2,400/120 V. For most metering purposes, however, the burden will be significantly less than 200 VA.

The current transformer (CD sometimes has a primary and always has a secondary winding. If there is a primary winding, it has a small number of turns. In most cases, the primary is only one turn or a single conductor connected in series with the load whose current is to be measured. The secondary winding has a larger number of turns and is connected to a current meter or a relay coil. Often the primary winding is a single conductor in the form of a heavy copper or brass bar running through the core of the transformer. Such a CT is called a bar-type current transformer. The CT secondary winding is usually designed to deliver a secondary current of 5 A. An 800/5-A bar-type current transformer would have 160 turns on the secondary coil.

The primary winding of the current transformer is connected directly in the load circuit. When the secondary winding is open-circuited, the voltage developed across the open terminals may be very high (because of the step-up ratio) and could easily break down the insulation between the secondary windings. The secondary winding of a current transformer should therefore always be short- circuited, or connected to a meter or relay coil. A current transformer should never have its secondary open while the primary is carrying current; it should always be closed through a current meter, relay coil, wattmeter current coil, or simply a short. Failure to observe this precaution may cause serious damage to either equipment or operating personnel.

The current transformer shown in Fig. 4-43 consists of a core with the secondary winding encased in molded-rubber insulation. The window in the core allows for the insertion of one or more turns of the current-carrying high-voltage conductor. A single conductor constitutes a one-turn primary winding. The nominal ratio of the transformer is given on the nameplate; this i not the turns ratio (since more than one turn can be used as the primary) but only indicates that a primary current of 500 A will cause a secondary current of 5 A when the secondary coil is connected to a 5-A ammeter. Within practical limits, the current in the secondary winding is determined by the primary excitation current and not by the secondary circuit impedance. Since the primary current is determined by the load in the ac system, the secondary current is related to the primary current by approximately the inverse of the turns-ratio. This is true within rather wide limits of the nature of the secondary burden.

Figure 4-44 indicates the use of instrument transformers in a typical measurement application. This diagram illustrates the connection of instrument transforms in a three-wire three-phase circuit, including two wattmeters, two voltmeters, and two ammeters. The potential transformers are connected across phase lines A and B, and phase lines C and B; the current transformers are in phase lines A. and D. The secondary windings of the potential transformers are connected to the voltmeter coils and the potential coils of the waltmeters; the current transformer secondaries feed the ammeters and the current coil of the wattmeters.

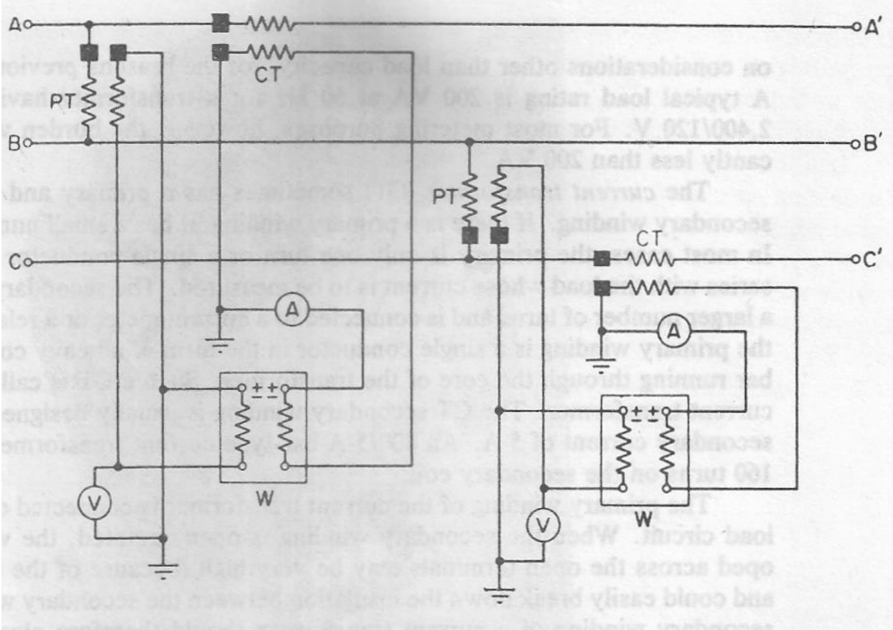

Figure 4-44 Instrument transformers in a three-phase measurement application. Polarity markings of the potential and current transformers are indicated by black squares.

The polarity markings on the transformers, indicated by a dot at the transformer leads, aid in making the correct polarity connections to the measuring instruments. At any given instant of the ac cycle, the dot-marked terminals have the same polarity and the marked wattmeter terminals must be connected to these transformer leads as shown.

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PROBLEMS

4-1. Determine the resistor value required to use a 0-1mA meter with an internal resistance of 1250 for a 0-1-V meter.

4-2. What value of shunt resistance is required for using a 50-µA meter movement, with an internal resistance of 250 Ω, for measuring 0-500 mA?

4-3. What series resistance must be used to extend the 0-200-V range of a 20,000-Ω/V meter to 0-2000 V? What power rating must this resistor have?

4-4. What will a 5,000-Ω/V meter read on a 0-5-V scale when connected to the circuit of Fig. P4-4?

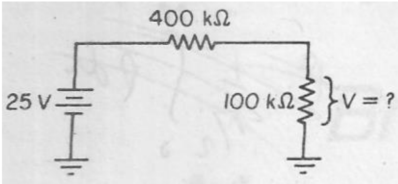


Figure P4-4

4-5. Draw the schematic, including Values, for an Ayrton shunt for a meter movement having a full-scale deflection of 1mA and an internal resistance of 500Ω to cover the current ranges of 10, 50, 100, and 500 mA.

4-6. Many electronic voltage measuring instruments have a fixed input resistance of 1 MΩ. Which settings of the range switch of the multimeter shown in Figs. 4-21 and 4-22 would have a higher input resistance than the typical electronic instrument for dc measurements?

4-7. The resistance of a 50-kΩ resistor is measured using the multimeter shown in Figs. 4- 21. 4-22, and 4-25. (a) How much power is dissipated in the resistor if the R x 10,000 range is used? (b) How much power is dissipated in the resistor if the R x 100 range is used? Assume that the zero control is set to its midpoint.

4-8. A series-type ohmmeter, designed to operate with a 6-V battery, has a circuit diagram as shown in Fig. 4-19. The meter movement has an internal resistance of 2,000Ω and requires a current of 100 μA for full-scale deflection. The value of R_1 is 49kΩ. (a) Assuming the battery voltage has fallen to 5.9 V, calculate the value of R_2 required to zero the meter. (b) Under the condition? mentioned in part (a), an unknown resistor s connected to the meter causing a 60 percent meter deflection. Calculate the value of the unknown resistance.

4-9. How low must the battery voltage of the 1.5-V cell in the multimeter ohmmeter section shown in Fig. 4-25(a) fall before it is impossible to zero the meter?

4-10. What is a transfer instrument? Why is an electrodynamometer a transfer instrument?

4-11. Why is sensitivity (ohms per volt) of the ac scales of a multimeter less than the dc section?

4-12. What is meant by a waveform error? Which ac meters are most likely to be affected by this form of error?

4-13. What are the advantages of a thermocouple meter?

4-14. What is the midscale point of a 10-A full-scale thermocouple meter?

4-15. The circuit diagram of Fig. 4-30 shows a full-wave rectifier ac voltmeter. The meter movement has an internal resistance of 250 Ω and requires 1 mA for full deflection. The diodes each have a forward resistance of 50 Ω and infinite reverse resistance. Calculate (a) the series resistance required for full-scale meter deflection when 25 V rms is applied to the meter terminals; (b) the ohms-per-volt rating of this ac voltmeter.

4-16. Calculate the indication of the Meter in Problem 4-15 when a triangular waveform with a peak value of 20 V is applied to the meter terminals.

4-17. If an electrodynamometer is used to measure power with a full-scale reading of 100 W, what is the one-quarter scale reading?

BRIDGE MEASUREMENTS

5-1 INTRODUCTION

Precision measurements of component values have been made for many years using various forms of bridges. The simplest form of bridge is for the purpose of and is called the Wheatstone bridge. There are variations of the Wheatstone bridge for measure very high and very low resistances. There is an entire group of ac bridges for measuring inductance, capacitance, admittance, conductance, and any of the impedance parameters.

General-purpose bridges are hardly used any more. Some specialized measurements, such as impedance at high frequencies are still made with a bridge.

The bridge circuit still forms the backbone of some measurements and for the interfacing of transducers; as an example, there are fully automatic bridges that electronically null a bridge to make precision component measurements. For this reason, a chapter is devoted to bridge measurements. Also, in this chapter, the concept of guarded measurements and three-terminal resistance measurement is covered.

5-2 WHEATSTONE BRIDGE

5.2.1 Basic Operation

Figure 5-1 shows the schematic of a Wheatstone bridge. The bridge has four resistive arms, together with a source of emf (a battery) and a null detector, usually a galvanometer or other sensitive current meter. The current through the galvanometer depends on the potential difference between point's c and d. The bridge is said to be balanced when the potential difference across the galvanometer is 0 V so that there is no current through the galvanometer. This condition occurs when the voltage from point c to point a equals the voltage from point d to point a; or by referring to the other battery terminal, when the voltage from point c to point b equals the voltage from point d to point 1'. Hence the bridge is balanced when

$$I_1 R_1 = I_2 R_2 \tag{5.1}$$

If the galvanometer current is zero, the following condition also exist:

$$I_1 = I_3 = \frac{E}{R_1 + R_3} \tag{5.2}$$

and

$$I_2 = I_4 = \frac{E}{R_1 + R_4} \tag{5.3}$$

Combining Eqs. (5-1), (5-2) and (5-3) and simplifying, we obtain

$$\frac{R_1}{R_1 + R_3} = \frac{R_2}{R_2 + R_4} \tag{5.4}$$

from which

$$R_1R_4 = R_2R_3 \qquad (5.4)$$

Equation (5-5) is the well-known expression for balance of the Wheatstone bridge. If three of the resistances have known values, the fourth may be determined from Eq. (5-5). Hence, if R4 is the unknown resistor, its resistance R_x can

Figure 5-I Wheatstone Bridge used for the precision measurement of. Resistances ranging from fractions of an ohm to several me ohms. The ratio control switches the ratio arms in decade steps. The remaining four step switches set the resistance of the standard arm.

Be expressed in term of the remaining resistors as follows:

$$R_x = R_3 \frac{R_1}{R_1} \qquad (5.6)$$

Resistor R_3 is called the standard arm of the bridge, and resistor R_2 and R_1 are called the ratio arms.

The measurement of the unknown resistance R_x is independent of the characteristics of the calibration of the null-detecting galvanometer, provided that the null detector has sufficient sensitivity to indicate the balance position of the bridge with the required degree of precision.

5-2.2 Measurement Errors

Wheatstone bridge is widely used for precision measurement of resistance from approximately I Ω to the low megohm range. The main source of measurement error is found in the limiting errors of the three known resistors. Other errors may include the following:

- Insufficient sensitivity of the null detector. This problem is discussed more fully in Sec 5-2.3.
- Changes resistance of the bridge arms due to the heating effect of the current through the resistors. He effect (I^2 R) of the bridge arm currents may change the resistance of the resist in question. The rise in tempera ture not only affects the resistance during the actual measurement, but ex-cessive currents may cause a permanent change in resistance values. This may not be discovered in time and subsequent measurements could well be erroneous. The power dissipation in the bridge arms must therefore be cornuted in advance, particularly when low-resistance

values are to be measured, and the current must be limited to a safe value.

- Thermal emfs in the bridge circuit or the galvanometer circuit can also cause problems when low value resistors are being measured. To prevent thermal emfs, the more sensitive galvanometers sometimes have copper coils and copper suspension systems to avoid having dissimilar metals in contact with one another and generating thermal emfs.
- Errors due to the resistance of leads and contacts exterior to the actual bridge circuit play a role in the measurement of very low-resistance values. These errors may be reduced by using a Kelvin bridge (see Sec. 5-3).

5-2.3 Thevenin Equivalent Circuit

To determine whether or not the galvanometer has the required sensitivity to detect an unbalance condition, it is necessary to calculate the galvanometer current. Different galvanometers not only may require different currents per unit deflection (current sensitivity), but they also may have a different internal resistance. It is impossible to say, without prior computation, which galvanometer with the required degree of precision.

Sec. 5-2 Wheatstone Bridge will make the bridge circuit more sensitive to an unbalance condition. This sensitivity can be calculated by "solving" the bridge circuit for a small unbalance. The solution is approached by converting the Wheatstone bridge of Fig. 5-1 to its Thevenin equivalent.

Since we are interested in the current through the galvanometer, the Thevenin equivalent circuit is determined by looking into galvanometer terminals c and din Fig. 5.1 Two steps must be taken to find the Thévenin equivalent; the first step involves finding the equivalent voltage appearing at terminals c and d when the galvanometer is removed from the circuit. The second step involves finding the equivalent resistance looking into terminals c and d, with the battery replaced by its internal resistance. For convenience, the circuit of Fig. 5-1(b) is redrawn in Fig. 5-2(a).

The Thevenin, or open-circuit, voltage is found by referring to Fig. 5-2(a), and we can write

$$E_{cd} = E_{ac} — 1_1R1 — I_2R_2$$

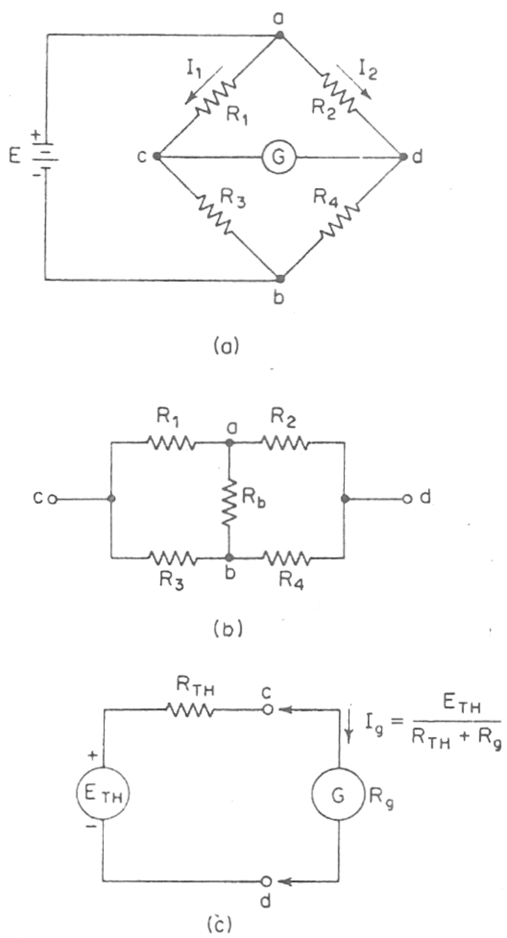


Figure 5-2 Application of Thévenin's theorem to the Wheatstone bridge. (a) Wheatstone bridge configuration. (b) Thévenin resistance looking into terminals *c* and *d*. (c) Complete Thévenin circuit, with the galvanometer connected to terminals *c* and *d*.

Where

$$I_1 = \frac{E}{R_1 + R_3} \text{ and } I_2 = \frac{E}{R_2 + R_4} \qquad (5\text{-}7)$$

This is the voltage of the Thévenin generator.

The resistance of the Thevenin equivalent circuit is found by looking back into terminals c and d and replacing the battery by its internal resistance. The circuit of Fig. 5-2(b) represents the Thévenin resistance. Notice that the internal resistance, R_b. of the battery has been included in Fig. 5-2(b). Converting this circuit into a more convenient form requires use of the delta-wye transformation theorem. Readers interested in this approach should consult texts on circuit analysis where this theorem is derived and applied.* In most cases, however, the extremely low internal resistance of the battery can be neglected and this simplifies the reduction of Fig. 5-2(a) to its Thévenin equivalent considerably.

Referring to Fig. 5-2(b), we see that a short circuit exists between points a and *b* when the internal resistance of the battery is assumed to be 0 Ω. The Thevenin resistance, looking into terminals c and d, then becomes

$$R_{\text{TH}} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4} \qquad (5\text{-}8)$$

The Thévenin equivalent of the Wheatstone bridge circuit therefore reduces to a Thevenin generator with an emf described by Eq. (5-7) and an internal resistance given by Eq. (5-8). This is shown in the circuit of Fig. 5-2(c).

When the null detector is now connected to the output terminals of the Thévenin equivalent circuit, the galvanometer current is found to be

$$I_g = \frac{E_{TH}}{R_{TH} + R_g} \qquad (5\text{-}9)$$

where I_g is the galvanometer current and R_g its resistance.

EXAMPLE 5-1

Figure 5-3(a) shows the schematic diagram of a Wheatstone bridge with values of the bridge elements as shown. The battery voltage is 5 V and its internal resistance negligible. The galvanometer has a current sensitivity of 10' mm/μA and an internal resistance of 100 Ω. Calculate the deflection of the galvanometer caused by the 5-fl unbalance in arm *BC*.

SOLUTION Bridge balance occurs if arm BC has a resistance of 2,000 Ω. The diagram shows arm BC as a resistance of 2.005 Ω, representing a small unbalance (42,000(I). The first step in the solution consists of converting the

- ❖ Herbert W. Jackson. Introduction to Electric Circuits, 5th ed. (Englewood Cliffs, N.J.: Pren-tice-Hall, Inc., 1981), pp. 448ff.

bridge circuit into its Thevenin equivalent circuit. Since we are interested in finding the current in the galvanometer, the Thévenin equivalent is determined with respect to galvanometer terminals B and D. The potential difference from B to D, with the galvanometer removed from the circuit, is the Thevenin voltage. Using Eq. (5.7), we obtain

$$E_{TH} = E_{AD} = 5 \text{ V x } \left(\frac{100}{100+200} - \frac{1{,}000}{1{,}000+2{,}005}\right)$$

$$= 2.77 \text{ mV}$$

The second step of the solution involves finding the equivalent Thévenin resistance, looking into terminals *B* and *D*, and replacing the battery with its internal resistance. Since the battery resistance is $0\,\Omega$, the circuit is represented by the configuration of Fig. 5-3(b) from which we find

$$R_{TH} = \frac{100\,x\,200}{300} + \frac{1{,}000\,x\,2{,}005}{3{,}005} = 734\,\Omega$$

The Thévenin equivalent circuit is given in Fig. 5-2(c). When the galvanometer is now connected to the output terminals of the equivalent circuit, the current through the galvanometer is

$$I_g = \frac{E_{TH}}{R_{TH} + R_g} = \frac{3.77\,\mathrm{mV}}{734\,\Omega + 100\,\Omega} = 3.32\ \mu\,\mathrm{A}$$

The galvanometer deflection is

$$\mathrm{d} = 3.34\ \mu\,\mathrm{A}\ \mathrm{x}\ \frac{10\,mm}{\mu\mathrm{A}} = 33.2\ \mathrm{mm}$$

At this point the merit of the Thevenin equivalent circuit for the solution of an unbalanced bridge becomes evident. If a different galvanometer is used (with a different current sensitivity and internal resistance), the computation of its deflection is very simple, as is clear from Fig. 5-3(c). Conversely, if the galvanometer sensitivity is given, we can solve for the unbalance voltage needed to give a unit deflection (say 1 mm). This value is of interest when we want to determine the sensitivity of the bridge to unbalance, or in response to the question: “Is the galvanometer selected capable of detecting a certain small unbalance?” The thevenin method is used to find the galvanometer response, which in most cases is of prime interest.

EXAMPLE 5-2

The galvanometer of Example 5-1 is replaced by one with an internal resistance of 500 Ω and a current sensitivity of 1 mm/μA. Assuming that a deflection of 1 mm can be observed on the galvanometer scale, determine if this new galvanometer is capable of detecting the 5-11 unbalance in arm *BC* of Fig. 5-3(a).

SOLUTION Since the bridge constants have not been changed, the equivalent circuit is again represented by a Thévenin generator of 2.77 mV and a

Thevenin resistance of 734 Ω. The new galvanometer is now connected to the output terminals resulting in a galvanometer current

$$I_g = \frac{E_{TH}}{R_{TH} + R_g} = \frac{2.77\,mA}{734\,\Omega + 500\,\Omega} = 2.24\,\mu A$$

The galvanometer deflection therefore equals 2.24 μA x 1 mm/μA = 2.24 mm, indicating that this galvanometer produces a deflection that can be easily observed.

The Wheatstone bridge is limited to the measurement of resistances ranging from a few ohms to several megohms. The upper limit is set by the reduction in sensitivity to unbalance, caused by high resistance values, because in this case the equivalent Thevenin resistance of Fig. 5-3(c) becomes high, thus reducing the galvanometer current. The lower limit is set by the resistance of the connecting

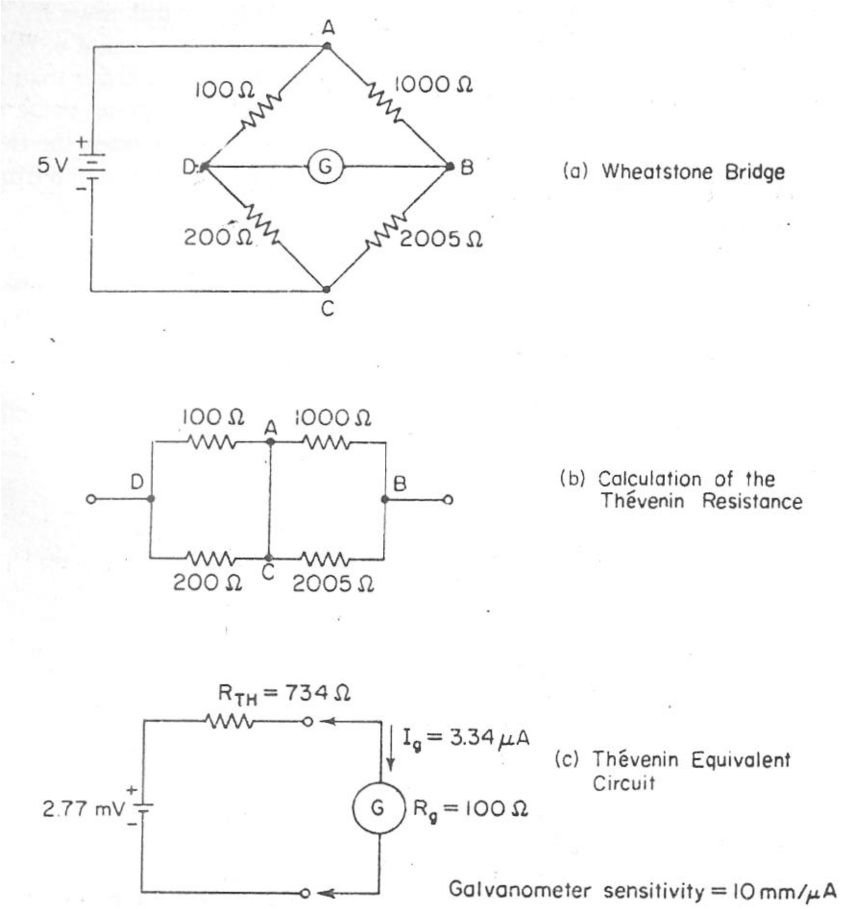


Figure 5-3 Calculation of galvanometer deflection caused by a small unbalance in arm *BC*, using the simplified Thévenin approach.

leads and the contact resistance at the binding posts The resistance of the leads could be calculated or measured, and the final result modified, but contact resistance is very hard to compute or measure. For low-resistance measurements, therefore, the Kelvin bridge is generally the preferred instrument

5-3 KELVIN BRIDGE

5-3.1 Effects of Connecting Leads

The Kelvin bridge is a modification of the Wheatstone bridge and provides greatly increased accuracy in the measurement of low-value resistances, generally below $1\,\Omega$. Consider the bridge circuit shown in Fig. 5-4, where R_y represents the resistance of the connecting lead from R_3 to R_x. Two galvanometer connections are possible, to point in or to point m. When the galvanometer is connected to point m, the resistance R_y of the connecting lead is added to the unknown R_x, resulting in too high an indication for R_x. When connection is made to point n, R_y is added to bridge arm R_3 and the resulting measurement of R_x will be lower than it should be, because now the actual value of R_3 is higher than its nominal value by resistance R_y. If the galvanometer is connected to a point p, in between the two points m and n, in such a way that the ratio of the resistances from *n* top and from *m* to *p* equals the ratio of resistors R_1 and R_2, we can write

$$\frac{R_{np}}{R_{mp}} = \frac{R_1}{R_2} \qquad (5\text{-}10)$$

The balance equation for the bridge yields

$$R_x + R_{np} = \frac{R_1}{R_2}(R_3 + R_{mp}) \qquad (5\text{-}11)$$

Substituting Eq. (5-10) into Eq. (5-11), we obtain

$$R_x + \left(\frac{R_1}{R_1 + R_2}\right) R_y = \frac{R_1}{R_2}\left[R_3 + \left(\frac{R_2}{R_1 + R_2}\right)R_y\right] \tag{5-12}$$

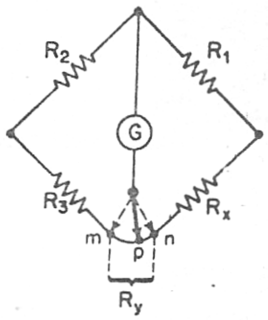


Figure 5-4 Wheatstone bridge circuit, showing resistance R_y of the lead from point *m* to point *n*.

which reduces to

$$R_x = \frac{R_1}{R_2} R_3 \tag{5-13}$$

Equation (5-13) is the usual balance equation developed for the Wheatstone bridge and it indicates that the effect of the resistance of the connecting lead from point in to point ii has been eliminated by connecting the galvanometer to the intermediate position *p*.

This development forms the basis for construction of the Kelvin double bridge, commonly known as the Kelvin bridge.

5-3.2 Kelvin Double Bridge

The term *double* bridge is used because the circuit contains a second set of ratio arms, as shown in the schematic diagram of Fig. 5-5. This second set of arms, labeled a and *b* in the diagram, connects the galvanometer to a point *p* at the appropriate potential between *m* and *n*, and it eliminates the effect of the yoke resistance R_y. An initially established condition is that the resistance ratio of a and *b* is the same as the ratio of R_1 and R_2.

The galvanometer indication will be zero when the potential at *k* equal the potential at *p*, or when $E_{kl} = E_{Imp}$ where

$$E_{kl} = \frac{R_2}{R_1+R_2}E = \frac{R_2}{R_1+R_2}I[R_3+R_x+\frac{(a+b)R_y}{a+b+=R_y}] \qquad (5\text{-}14)$$

and

$$E_{Imp} = I\ \{R_3 + \frac{b}{a++b}[\frac{(a+b)R_y}{a+b+R_y}]\} \qquad (5\text{-}15)$$

We can solve for R1 by equating E_{kl}, and E_{Imp},, in the following manner:

$$\frac{R_2}{R_1+R_2}I[R_3+R_x+\frac{(a+b)R_y}{a+b+R_y}]=[R_3+\frac{b}{a+b}\frac{(a+b)R_y}{a+b+R_y}]$$

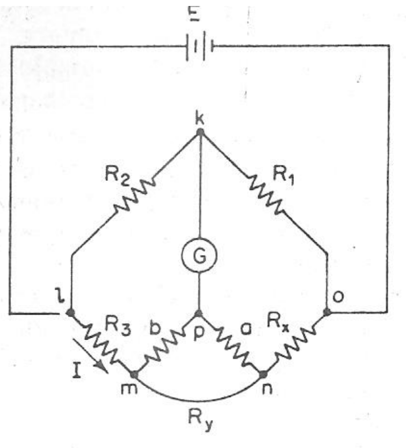


Figure 5-5 Basic Kelvin double bridge circuit.

Or simplifying, we get

$$R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} = \frac{R_1+R_2}{R_2}[R_3 + \frac{bR_y}{a+b+R_y}]$$

and expanding the right-hand member yields

$$R_3 + R_x + \frac{(a+b)\,R_y}{a+b+R_y} = \frac{R_1R_3}{R_2}\,R_3 + \frac{R_1\ +R_2}{R_2}.\frac{bR_y}{a+\mathbf{b}+R_y}$$

Solving for R_x yields

$$R_x = \frac{R_1R_3}{R_2} = \frac{R_1R_3}{R_2}\,R_3 + \frac{R_1\ +R_2}{R_2}.\frac{bR_y}{a+\mathbf{b}+R_y}$$

so that

$$R_x = \frac{R_1R_3}{R_2} + \frac{R_1}{R_2}.\frac{bB_y}{a+b+R_y}\left(\frac{R_1}{R_2} - \frac{a}{b}\right) \qquad (5\text{-}16)$$

Using the initially established condition that *a/b* = R_1/R_2, we see that Eq. (5-16) reduces to the well-known relationship

$$R_x = R_3\ \frac{R_1}{R_2} \qquad (5\text{-}17)$$

Equation (5-17) is the usual working equation for the Kelvin bridge. It indicates that the resistance of the yoke has no effect on the measurement, provided that the two sets of ratio arms have equal resistance ratios.

The Kelvin bridge is used for measuring very low resistances, from approximately $1\,\Omega$ to as low as $0.00001\,\Omega$. Figure 5-6 shows the simplified circuit diagram of a commercial Kelvin bridge capable of measuring resistances from 10 Ω to 0.00001Ω. In this bridge, resistance R_3 of Eq. (5-17) is represented by the variable standard resistor in Fig. 5-6. The ratio arms (R_1 and R_2) can usually be switched in a number of decade steps.

Contact potential drops in the measuring circuit may cause large errors and to reduce this effect the standard resistor consists of nine steps of 0.001Ω each plus a calibrated manganin bar of 0.0011Ω with a sliding contact. The total resistance of the R3 arm therefore amounts to 0.0101Ω and is variable in steps of 0.001Ω plus fractions of 0.0011Ω by the sliding contact. When both contacts are switched to select the suitable value of standard resistor, the voltage drop be- tween the ratio-arm connection points is changed, but the total resistance around the battery circuit is unchanged. This arrangement places any contact resistance in series with the relatively high-resistance values of the ratio arms, and the contact resistance has negligible effect.

The ratio R_1/R_2 should be selected that a relatively large part of the standard resistance is used in the measuring circuit. In this way the value of unknown resistance R_x is determined with the largest possible number of significant figures, and the measurement accuracy is improved.

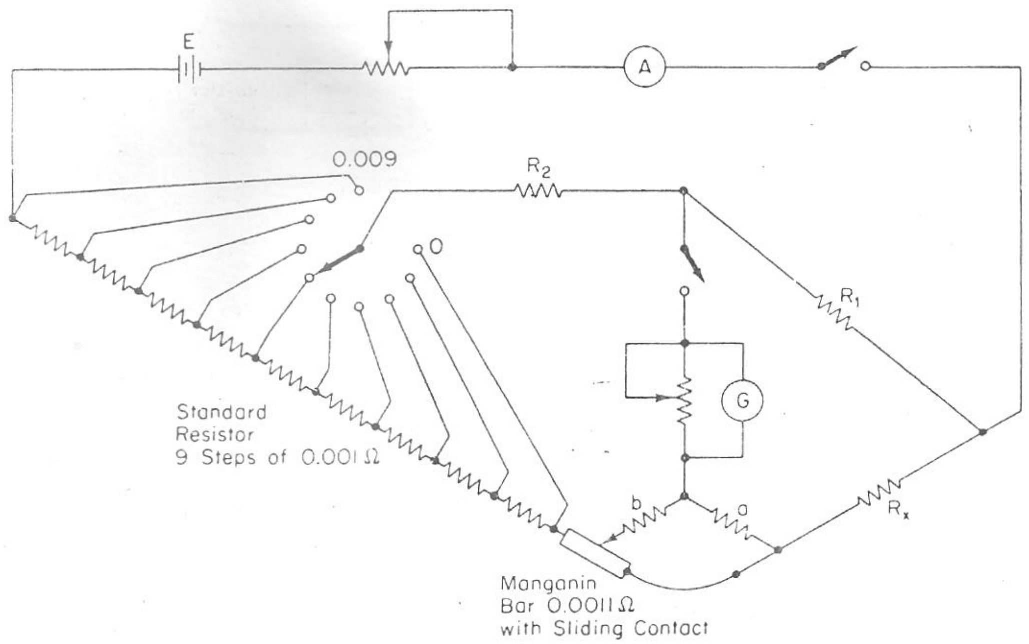


Figure 5-6 Simplified circuit of a Kelvin double bridge used for the measurement of very low resistances.

5- 4 GUARDED WHEATSTONE BRIDGE

5- 4.1 Guard Circuits

The measurement of extremely high resistances, such as the insulation resistance of a cable or the leakage resistance of a capacitor (often on the order of several thousands of megohms), is beyond the capability of the ordinary dc Wheatstone bridge. One of the major problems in high-resistance measurements is the leakage that occurs over and around the component or specimen being measured, or over the binding posts by which the component is attached to the instrument, or within the instrument itself. These leakage currents are undesired because they can enter the measuring circuit and affect the measurement accuracy to a considerable extent. Leakage currents, whether inside the instrument itself or associated with the test specimen and its mounting, are particularly noticeable in high-resistance measurements where high voltages are often necessary to obtain sufficient deflection sensitivity. Also, leakage effects are generally variable from day to day, depending on the humidity of the atmosphere.

The effects of leakage paths on the measurement are usually removed by some form of guard circuit. The principle of a simple guard circuit in the R_x arm of a Wheatstone bridge is explained with the aid of Fig. 5-7. Without a guard circuit, leakage current I_1 along the insulated surface of the binding post adds to current I_x

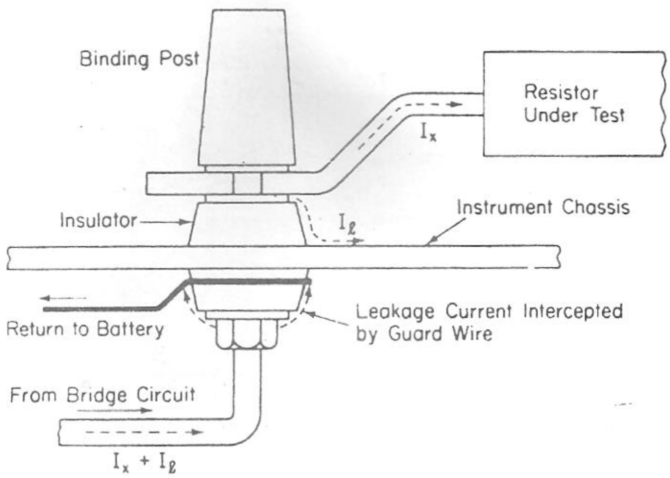


Figure 5-7 Simple guard wire on the R_x terminal of a guarded Wheatstone bridge eliminates surface leakage.

through the component under measurement to produce a total circuit current that can be considerably larger than the actual device current. A guard wire, completely surrounding the surface of the insulated post, intercepts this leakage cur- rent and returns it to the battery. The guard must be carefully placed so that the leakage current always meets some portion of the guard wire and is prevented from entering the bridge circuit.

In the schematic diagram of Fig. 5-8 the guard around the R_x binding post, indicated by a small circle around the terminal, does not touch any part of the bridge circuitry and is connected directly to the battery terminal. The principle of the guard wire on the binding post can be applied to any internal part of the bridge circuit where leakage affects the measurement; we then speak of a guarded Wheatstone bridge.

5- 4.2 Three-Terminal Resistance

To avoid the effects of leakage currents external to the bridge circuitry, the junction of ratio arms R_A and R_B is usually brought out as a separate guard

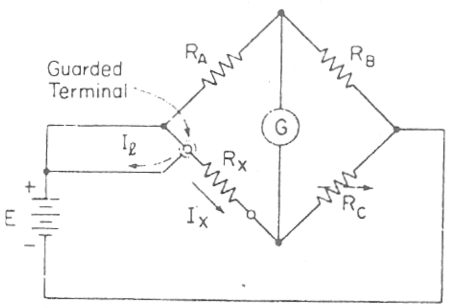


Figure 5-8 Guarded terminal returns leakage current to the battery.

terminal on the front panel of the instrument. This guard terminal can be used to connect a so-called *three-terminal resistance*, as shown in Fig. 5-9. The high is mounted on two insulating posts that are fastened to a metal plate. The two main terminals of the resistor are connected to the R_x terminals of the bridge in the usual manner. The third terminal of the resistor is the common point of resistances R_1 and R_2, which represent the leakage paths from the main terminals along the insulating posts to the metal plate, or guard. The guard is connected to the guard terminal on the front panel of the bridge, as indicated in the schematic of Fig. 5-9. This connection puts R_1 in parallel with ratio arm R_A, but since R_1 is very much larger than R_A, its shunting effect is negligible. Similarly, leakage resistance R_2 is in parallel with the galvanometer, but the resistance of R_2 is so much higher than that of the galvanometer that the only effect is a slight reduction in galvanometer sensitivity. The effects of external leakage paths are therefore removed by using the guard circuit on the three-terminal resistance.

If the guard circuit were not used, leakage resistance R_1 and R_2 would be directly across R_x and the measured value of R_x would be considerably in error.

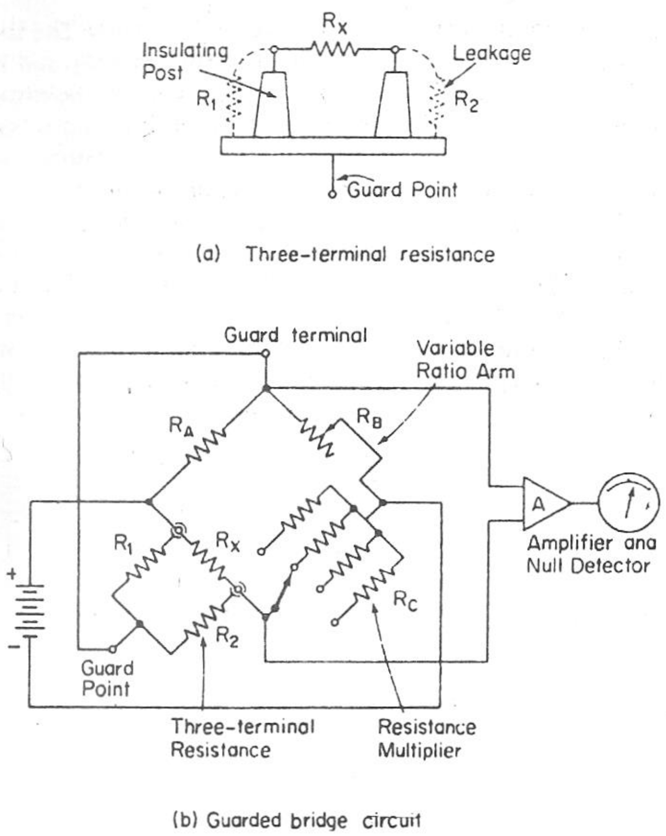


Figure 5-9 Three-terminal resistance, connected to a guarded high-voltage megohm bridge.

Assuming, for example, that the unknown is 100 MΩ and that the leakage resis-tance from each terminal to the guard is also 100 MΩ, resistance R_x would be measured as 67 MΩ, an error of approximately 33 per cent.

5-5 AC BRIDGES AND THEIR APPLICATION

5-5.1 Conditions for Bridge Balance

` The ac bridge is a natural outgrowth of the dc bridge and in its basic form consists of four bridge arms, a source of excitation, and a null detector. The power source supplies an ac voltage to the bridge at the desired frequency. For measurements at low frequencies, the power line may serve as the source of excitation; at higher frequencies, an oscillator generally supplies the excitation voltage. The null detector must respond to ac unbalance currents and in its cheapest (but very effective) form consists of a pair of headphones. In other applications, the null detector may consist of an ac amplifier with an output meter, or an electron ray tube (tuning eye) indicator.

The general form of an ac bridge is shown in Fig. 5-10. The four bridge arms Z_1, Z_2, Z_3, and Z_4 are indicated as unspecified impedances and the detector is represented by headphones. As in the case of the Wheatstone bridge for dc measurements, the balance condition in this ac bridge is reached when the detector response is zero, or indicates a null. Balance adjustment to obtain a null response is made by varying one or more of the bridge arms.

The general equation for bridge balance is obtained by using complex notation for the impedances of the bridge circuit. (Boldface type is used to indicate quantities in complex notation.) These quantities may be impedances or admittances as well as voltages or currents. The condition for bridge balance requires that the potential difference from A to C in Fig. 5-10 be zero. This will be the case when the voltage drop from B to A equals the voltage drop from B to C, in both magnitude and phase. In complex notation we can write

$$E_{BA} = E_{BC} \quad \text{or} \quad I_1 Z_1 = I_2 Z_2 \qquad (5\text{-}18)$$

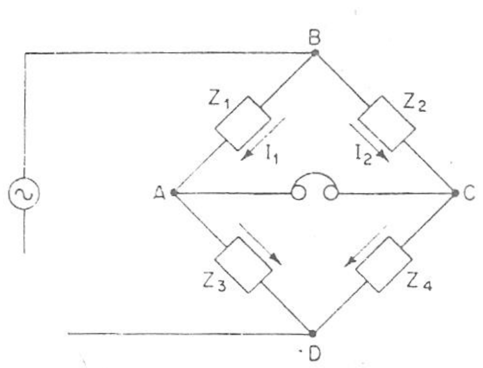


Figure 5-10 General form of the ac bridge.

For zero detector current (the balance condition), the currents are

$$I_1 = \frac{E}{Z_1 + Z_3} \qquad (5\text{-}19)$$

and

$$I_2 = \frac{E}{Z_2 + Z_4} \qquad (5\text{-}20)$$

Substitution of Eqs. (5-19) and (5-20) into Eq. (5-18) yields

$$Z_1 Z_4 + Z_2 Z_3 \qquad (5\text{-}21)$$

or when using admittances instead of impedances.

$$Y_1 Y_4 = Y_2 Y_3 \qquad (5\text{-}22)$$

Equation (5-21) is the most convenient form in most cases and is the general equation for balance of the ac bridge. Equation (5-22) can be used to advantage when dealing with parallel components in bridge arms. Equation (5-21) states that the product of impedances of one pair of opposite arms must equal the product of impedances of the other pair of opposite arms, with the impedances expressed in complex notation. If the impedance is written in the form $Z = Z \underline{/0}$, where Z represents the magnitude and θ the phase angle of the complex impedance, Eq. (5-21) can be written in the form

$$(Z_1 \underline{/\theta_1})(Z_4 \underline{/\theta_4}) = (Z_2 \underline{/\theta_2})(Z_3 \underline{/\theta_3}) \qquad (5\text{-}23)$$

Since in multiplication of complex numbers the magnitudes are multiplied and the phase angles added, Eq. (5-23) can also be written as

$$Z_1 Z_4 \underline{/(\theta_1 + \theta_4)} = Z_2 Z_3 \underline{/(\theta_2 + \theta_3)} \qquad (5\text{-}24)$$

Equation (5-24) shows that two conditions must be met simultaneously when balancing an ac bridge. The first condition is that the magnitudes of the impedances satisfy the relationship

$$Z_1 Z_4 = Z_2 Z_3 \qquad (5\text{-}25)$$

or, in words:

The products of the magnitudes of the opposite arms must be equal.

The second condition requires that the phase angles of the impedances satisfy the relationship

$$\underline{/\theta_1} + \underline{/\theta_4} = \underline{/\theta_2} + \underline{/\theta_3} \qquad (5\text{-}26)$$

Again, in words:

The sum of the phase angles of the opposite arms must be equal.

5-5.2 Application of the Balance Equations

The two balance conditions expressed in Eqs. (5-25) and (5-26) can be applied when the impedances of the bridge arms are given in polar form, with both magnitude and phase angle. In the usual case, however, the component values of the bridge arms are given, and the problem is solved by writing the balance equation in complex notation. The following examples illustrate the procedure.

EXAMPLE 5-3

The impedances of the basic ac bridge of Fig. 5-10 are given as follows:

$\mathbf{Z}_1 = 100\ \Omega\ \angle 80^\circ$ (inductive impedance)

$\mathbf{Z}_2 = 250\ \Omega$ (pure resistance)

$\mathbf{Z}_3 = 400\ \Omega\ \angle 30^\circ$ (inductive impedance)

$\mathbf{Z}_4 =$ unknown

Determine the constants of the unknown arm.

SOLUTION The first condition for bridge balance requires that

$$Z_1 Z_4 = Z_2 Z_3 \qquad (5\text{-}25)$$

Substituting the magnitudes of the known components and solving for Z_4, we obtain

$$Z_4 = \frac{Z_2 Z_3}{Z_1} = \frac{250 \times 400}{100} = 1{,}000\ \Omega$$

The second condition for bridge balance requires that the sums of the phase angles of opposite arms be equal, or

$$\theta_1 + \theta_4 = \theta_2 + \theta_3 \qquad (5\text{-}26)$$

Substituting the known phase angles and solving for θ_4, we obtain

$$\theta_4 = \theta_2 + \theta_3 - \theta_1 = 0 + 30 - 80 = -50^\circ$$

Hence the unknown impedance Z_4 can be written in polar form as

$$\mathbf{Z}_4 = 1{,}000\ \Omega\ \angle -50^\circ$$

indicating that we are dealing with a capacitive element, possibly consisting of a series combination of a resistor and a capacitor.

The problem becomes slightly more complex when the component values of the bridge arms arc specified and the impedances are to be expressed in complex notation. In this case, the inductive or capacitive reactances can only be calculated when the frequency of the excitation voltage is known, as Example 5-4 shows.

EXAMPLES 5-4

The ac bridge of Fig. 5-10 is in balance with the following constants: arm AB, $R = 450\ \Omega$; arm BC, $R = 300\ \Omega$ in series with $C = 0.265\ \mu\text{F}$; arm CD, unknown; arm DA, $R = 200\ \Omega$ in series with $L = 15.9$ mH. The oscillator frequency is 1 kHz. Find the constants of arm CD.

SOLUTION The general equation for bridge balance states that

$$\mathbf{Z}_1\mathbf{Z}_4 = \mathbf{Z}_2\mathbf{Z}_3 \tag{5-21}$$

$$\mathbf{Z}_1 = R = 450\ \Omega$$

$$\mathbf{Z}_2 = R - j/\omega C = (300 - j600)\ \Omega$$

$$\mathbf{Z}_3 = R + j\omega L = (200 + j100)\ \Omega$$

$$\mathbf{Z}_4 = \text{unknown}$$

Substituting the known values in Eq. (5-21) and solving for the unknown yields

$$\mathbf{Z}_4 = \frac{450 \times (200 + j100)}{300 - j600} = +j150\ \Omega$$

This result indicates that $\mathbf{Z}_4$ is a pure inductance with an inductive reactance of 150 Ω at a frequency of 1 kHz. Since the inductive reactance $X_L = 2\pi f L$, we solve for L and obtain $L = 23.9$ mH.

5-6 MAXWELL BRIDGE

The Maxell bridge, whose schematic diagram is shown in Fig. 5-11, measures an unknown inductance in terms of a known capacitance. One of the ratio arms has a resistance and a capacitance in parallel and it may now prove somewhat easier to write the balance equations using the admittance of arm 1 instead of its impedance.

Rearranging the general equation for bridge balance, as expressed in Eq. (5-21), we obtain

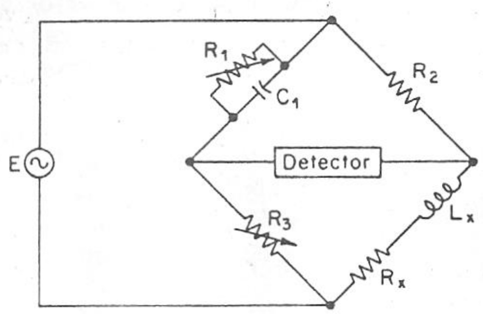


Figure 5-11 Maxwell bridge for inductance measurements.

where Y_1 is the admittance of arm 1. Reference to Fig. 5-11 shows that

Z_2 R_2; $Z_3 = R_3$; and $Y_1 = \frac{1}{R_1} + j\omega C_1$

Substitution of these values in Eq. (5-27) gives

$$Z_x = R_x + j\omega L_x = R_2R_3\left(\frac{1}{R_1} + j\omega C_1\right) \qquad (5\text{-}28)$$

Separation of the real and imaginary terms yields

$$R_x = \frac{R_2 R_3}{R_1} \tag{5-29}$$

and

$$L_x = R_2 R_3 C_1 \tag{5-30}$$

where the resistances are expressed in ohms, inductance in henrys, and capacitance in farads.

The Maxwell bridge is limited to the measurement of medium-*Q* coils ($1 < Q < 10$). This can be shown by considering the second balance condition which states that the sum of the phase angles of one pair of opposite arms must be equal to the sum of the phase angles of the other pair. Since the phase angles of the resistive elements in arm 2 and arm 3 add up to 0^o, the sum of the angles of arm 1 and arm 4 must also add up to 0^o. The phase angle of a high-*Q* coil will he very nearly 90° (positive), which requires that the phase angle of the capacitive arm must also be very nearly 90° (negative). This in turn means that the resistance of R_1 must be very large indeed, which can be very impractical. High-*Q* coils are therefore generally measured on the Hay bridge, presented in Sec. 5-7.

The Maxwell bridge is also unsuited for the measurement of coils with a very low *Q*-value ($Q < 1$) because of balance convergence problems. Very low *Q* values occur in inductive resistors, for example, or in an RF coil if measured at low frequency. As can be seen from the equations for R_x and L_x, adjustment for inductive balance by R_3 upsets the resistive balance by R_1 and gives the effect known as sliding balance. Sliding balance describes the interaction between controls, so that when we balance with R_1 and then with R_3, then go back to R_1, we find a new balance point. The balance point appears to move or slide toward its final point after many adjustments. Interaction does not occur when R_1 and C_1 are used for the balance adjustments, but a variable capacitor is not always suitable.

The usual procedure for balancing the Maxwell bridge is by first adjusting R_3 for inductive balance and then adjusting R_1 for resistive balance. Returning to the R_3 adjustment, we find that the resistive balance is being disturbed and moves to a new value. This process is repeated and gives slow convergence to final balance. For medium-*Q* coils, the resistance effect is not pronounced, and balance is reached after a few adjustments.

5-7 HAY BRIDGE

The Hay bridge of Fig. 5-12 differs from the Maxwell bridge by having resistor R_1 in series with standard capacitor C_1instead of in parallel. It is immediately apparent that for large phase angles, R_1 should have a very low value. The Hay circuit is therefore more convenient for measuring high-*Q* coils.

The balance equations are again derived by substituting the values of the impedances of the bridge arms into the general equation for bridge balance. For the circuit of Fig. 5-12, we find that

$Z_1 = R_1 - \dfrac{j}{\omega C_1}$; $Z_2 = R_2$; $Z_3 = R_3$; $Z_x = R_x + j\omega L_x$

Substituting these values in Eq. (5-21), we get

$$\left(R_1 - \frac{j}{\omega C_1}\right)(R_x + j\omega L_x) = R_2\ R_3 \qquad (5\text{-}31)$$

Which expands to

$$R_1R_x + \frac{L_x}{C_1} - \frac{jR_x}{\omega C_1} + j\omega L_x R_1 = R_2\ R_3$$

Separating the real and imaginary terms, we obtain

$$R_1R_x + \frac{L_x}{C_1} - \frac{jR_x}{\omega C_1} = R_2 R_3 \qquad (5\text{-}32)$$

and

$$\frac{R_x}{\omega C_1} = \omega L_x R_1 \qquad (5\text{-}33)$$

Both Eq. (5-32) and Eq. (5.33) contain L_x and R_x, and we must solve these equations simultaneously. This yields

$$R_x = \frac{\omega^2 C_1^2 R_1 R_2 R_3}{1+\omega^2 C_1^2 R_1^2} \qquad (5\text{-}34)$$

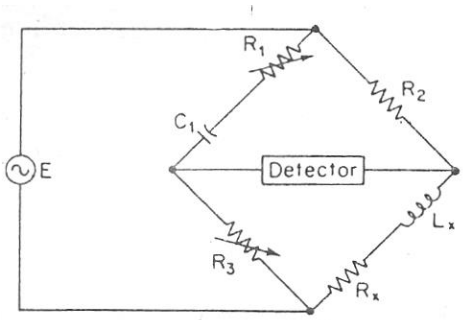


Figure 5-12 Hay bridge for inductance measurements.

$$L_x \ \frac{R_2 R_3 C_1}{1+\omega^2 C_1^2 R_1^2} \qquad (5\text{-}35)$$

These expressions for the unknown inductance and resistance both contain the angular velocity wand it therefore appears that the frequency of the voltage source must be known accurately. That this is not true when a high-*Q* coil is being measured follows from the following considerations: Remembering that the sum of the opposite sets of phase angles must be equal, we find that the inductive phase angle must be equal to the capacitive phase angle, since the resistive angles are zero. Figure 5-13 shows that the tangent of the inductive phase angle equals

$$\text{Tan } \theta_L = \frac{X_L}{R} = \frac{\omega L_x}{R_x} = Q \qquad (5\text{-}36)$$

and that of the capacitive phase angle is

$$\tan = \theta_L = \tan\theta_c \ or \ Q = \frac{1}{\omega C_1 R_1} \qquad (5\text{-}37)$$

When the two phase angles are equal, their tangents are also equal and we can write

$$\tan\ \theta_L\ = \tan\theta_c\ or\ Q = \frac{1}{\omega C_1 R_1} \qquad (5\text{-}38)$$

Returning now to the term (1 ± w2ClRf) which appears in Eqs. (5-34) and (535),we find that, after submitting Eq. (5-38) in the expression for 4, Eq. (535) Reduces to

$$L_x = \frac{R_2 R_3 C_1}{1+(1/Q)^2} \qquad (5\text{-}39)$$

For a value of Q greater than ten, the term $(1/Q)^2$ will be smaller than 1/100 and can be neglected. Equation (5-35) therefore reduces to the expression derived for the Maxwell bridge,

$$L_x = R_2 R_3 C_1$$

The Hay bridge is suite for 10, the measurement (Q) Inductors, especially for those inductors having a Q greater than 10. For Q- values smaller than 100, the term $(1/Q)^2$ becomes important and cannot be neglected. In this case, the Max-well bridge is more suitable.

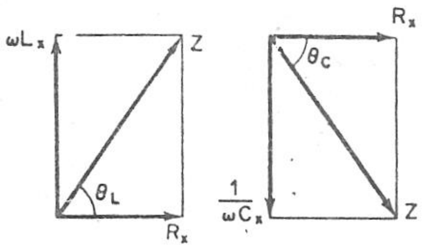


Figure 5-13 Impedance triangles illustrate inductive and capacitive phase angles.

5-8 SCHERING BRIDGE

The Schering bridge, one of the most important ac bridges, is used extensively for the measurement of *capacitors*. Although the Schering bridge is used for capacitance measurements in a general sense, it is particularly useful for measuring insulating properties, i.e., for phase angles very nearly 90°.

The basic circuit arrangement is shown in Fig. 5-14, and inspection of the circuit shows a strong resemblance to the comparison bridge. Notice that arm 1 now contains a parallel combination of a resistor and a capacitor, and the standard arm contains only a capacitor. The standard capacitor is usually high-quality mica capacitor for general measurement work or an air capacitor for insulation measurements. A good-quality mica capacitor has very low losses (no resistance) and therefore a phase angle of approximately 900. An air capacitor, when designed carefully, has a very stable value and a very small electric field; the insulating material to be tested can easily be kept out of any strong fields.

The balance conditions require that the sum of the phase angles of arms 1 and 4 equals the sum of the phase angles of arms 2 and 3. Since the standard-capacitor is in arm 3, the sum of the phase angles of arm 2 and arm 3 will be 0^o + 90° = 90°. In order to obtain the 90°-phase angle needed for balance, the sum of the angles of arm 1 and arm 4 must equal 90°. Since in general measurement work the unknown Will have a phase angle smaller than 90°, it is necessary to give arm 1 a small capacitive angle by connecting capacitor C_1 in parallel with resistor R_1. A small capacitive angle is very easy to obtain, requiring a small capacitor across resistor R_1.

The balance equations are derived in the usual manner, and by substituting the corresponding impedance and admittance values in the general equation, we obtain.

$$\mathbf{Z_x\ Z_2 Z_3 Y_1}$$

or

$$R_x - \frac{j}{\omega C_x} = R_2 \left(\frac{-j}{\omega C_3} \right) \left(\frac{1}{R_1} + j\omega C_1 \right)$$

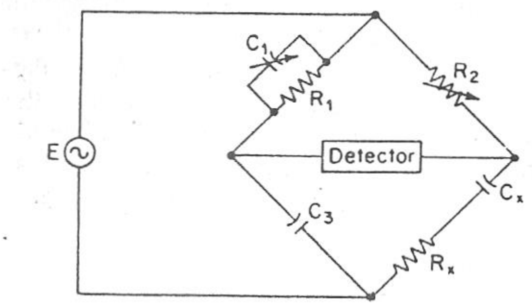


Figure 5-14 Schering bridge for the measurement of capacitance.

and expanding

$$R_x - \frac{j}{\omega C_x} = \frac{R_2 C_1}{C_3} - \frac{jR_2}{\omega C_3 R_1} \qquad (5\text{-}40)$$

Equating the real terms and the imaginary terms, we find that

$$R_x = R_2 \frac{C_1}{C_3} \qquad (5\text{-}41)$$

$$C_x = C_3 \frac{R_1}{R_2} \qquad (5\text{-}42)$$

As can be seen from the circuit diagram of Fig. 5-13. the, two variables chosen for the balance adjustment are capacitor C_1 and resistor R_2. There seems to be nothing unusual about the balance equations or the choice of variable components, but consider for a moment how the quality of a capacitor is defined.

The power factor (PF) of a series *RC* combination is defined as the cosine of the phase angle of the circuit. Therefore the PF of the unknown equals PF = R_x/Z_x. For phase angles very close to 90^o, the reactance is almost equal to the impedance and we can approximate the power factor to

$$PF = \frac{R_x}{X_x} \omega C_x R_x \qquad (5\text{-}43)$$

The dissipation factor of a series RC circuit is defined as the cotangent of the phase angle and therefore, by definition, the dissipation factor

$$D = \frac{R_x}{X_x} = \omega C_x R_x \qquad (5\text{-}44)$$

Incidentally, since the quality of a coil is defined by $Q = X_L/R_L$, we find that the dissipation factor, *D*, is the reciprocal of the quality factor, *Q*, and therefore $D = 1/Q$. The dissipation factor tells us something about the quality of a capacitor; i.e., how close the phase angle of the-capacitor is to the ideal value of 90°. By substituting the value of C_x, in Eq. (5-42) and of R_x in Eq. (5-41) into the expression for the dissipation factor, we obtain

$$D = \omega R_1 C_1 \tag{5-45}$$

If resistor R_1 in the Schering bridge of Fig. 5-14 has a fixed value, the dial of capacitor C_1 may be calibrated directly in dissipation factor D. This is the usual practice in a Schering bridge. Notice that the term u appears in the expression for the dissipation factor [Eq. (5-45)]. This means, of course, that the calibration of the C_1 dial holds for only one particular frequency at which the dial is calibrated. A different frequency can be used, provided that a correction is made by multiplying the C_1 dial reading by the ratio of the two frequencies. Figure 5-15 shows a modern automatic bridge.

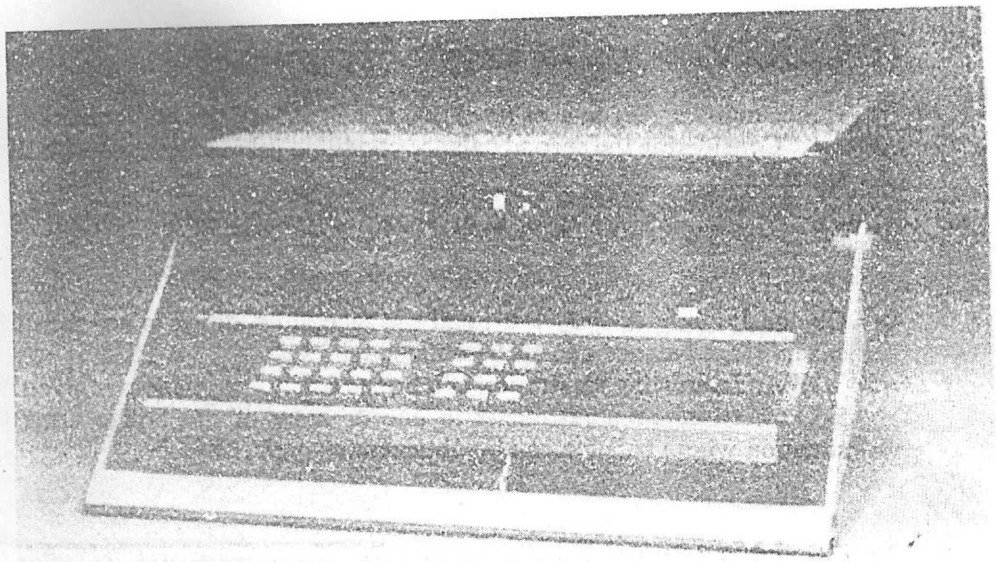


Figure 5-15 Modern bridges measure a variety of parameters and are automatic, such as this unit. (Courtesy of Electro Scientific Industries, Inc.)

5-9 UNBALANCE CONDITIONS

It sometimes happens that an ac bridge cannot be balanced at all simply because one of the stated balance conditions (Sec. 5-5) cannot he met. Consider for example. the circuit of Fig. 5-16, where $\mathbf{Z_1}$ and $\mathbf{Z_4}$ are inductive elements (positive phase angles), $\mathbf{Z_2}$ is a pure capacitance (-90° phase angle), and $\mathbf{Z_3}$ is a variable resistance (zero phase angle). The resistance of R_3 needed to obtain bridge balance can be determined by applying the first balance condition (magnitudes) and

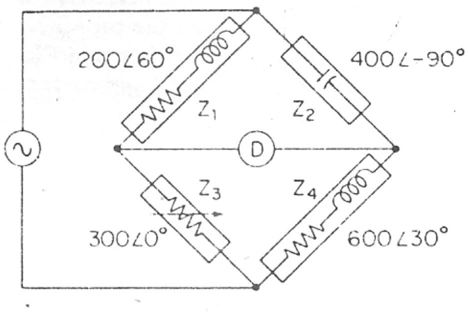


Figure 5-16 Ac bridge that cannot be balanced.

we find that

$$R_3 = \frac{Z_1 Z_4}{Z_2} = \frac{200 \text{ x } 600}{400} = 300\Omega$$

Hence adjusting R_3 to a value of 300 Ω will satisfy the first condition,

Considering the second balance condition (phase angles) yields the following

situation:

$$\theta_1 + \theta_4 = +60^\circ + 30^\circ = 90^\circ$$
$$\theta_2 + \theta_3 = -90^\circ + 0^\circ = 90^\circ$$

Obviously, $\theta_1 + \theta_4 \neq \theta_2 + \theta_3$, and the second condition is not satisified. In this case, bridge balance cannot be obtained.

An interesting illustration of a bridge balancing problem is given in Example 5-5, where minor adjustments to one or more of the bridge arms result in a situation where balance can be obtained.

EXAMPLE 5-5

Consider the circuit of Fig. 5-17(a) and determine whether or not the bridge is in complete balance. If not, show two ways in which it can be made to balance and specify numerical values for any additional components. Assume that bridge arm 4 is the unknown that cannot be modified.

SOLUTION Inspection of the circuit shows that the first balance condition (magnitudes) can easily be met by slightly increasing the resistance of R_3. The second balance condition requires that $\theta_1 + \theta_4 = \theta_2 + \theta_3$ where

$\theta_1 = -90°$ (pure capacitance)

$\theta_2 = \theta_3 = 0°$ (pure resistance)

$\theta_4 < +90°$ (inductive impedance)

Obviously, balance is not possible with the configuration of Fig. 5-17(a) because the sum of θ_1 and θ_4 will be slightly negative while $\theta_2 + \theta_3$ will be exactly 0°. Balance can be restored by modifying the circuit in such a way that the phase angle condition is satisfied. There are basically two methods to accomplish this: The first option is to modify $\mathbf{Z}_1$ so that its phase angle is decreased to less than 90° (equal to θ_4) by placing a resistor in parallel with the capacitor. This modification results in a Maxwell bridge configuration, as shown in Fig. 5-17(b). The resistance of R_1 can be determined by the standard approach of Sec. 5-6, using the admittance of arm 1, and we can write

$$\mathbf{Y}_1 = \frac{\mathbf{Z}_4}{\mathbf{Z}_2\mathbf{Z}_3}$$

where

$$\mathbf{Y}_1 = \frac{1}{R_1} + \frac{j}{1{,}000}$$

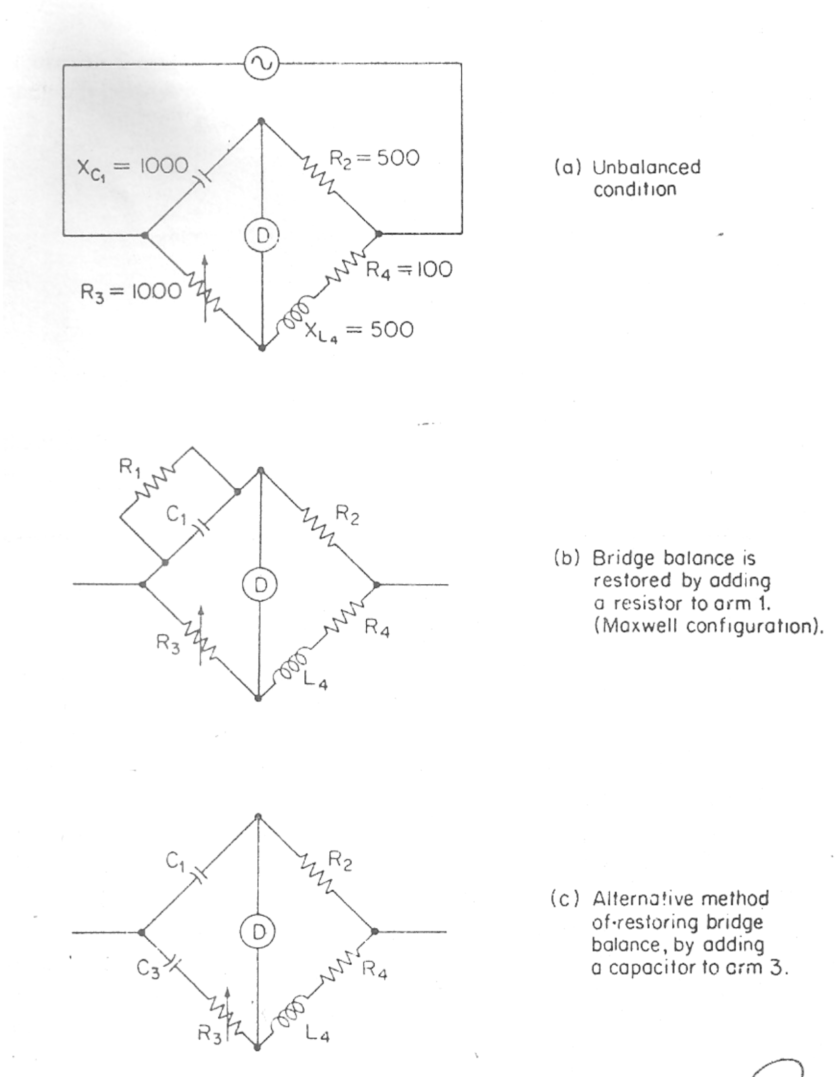


Figure 5-17 Bridge balancing problem.

Substituting the known values and solving for R_1, we obtain

$$\frac{1}{R_1} + \frac{j}{1{,}000} = \frac{100 + j500}{500 \text{ x } 1{,}000}$$

and

It should he noted that the addition of R_1 upsets the first balance condition of the circuit (the magnitude of Z_1 has changed) and variable resistor R_3 should be adjusted to compensate for this effect.

The second option is to modify the phase angle of arm 2 or arm 3 by adding a series capacitor, as shown in Fig. 5-17(c). Again writing the general balance equation, using impedances this time, we obtain

$$Z_3 = \frac{Z_1 Z_4}{Z_2}$$

Substituting the component values and solving for X yields

$$1{,}000 - jXc = \frac{-j1{,}000(100\ j500)}{500}$$

or

$$X_c = 200\,\Omega$$

In this case, also, the magnitude of Z3 has increased so that the first balance condition has changed. A small readjustment of R1 is necessary LO restore balance.

5-10 WIEN BRIDGE

The Wien bridge is presented here not only for its use as an ac bridge to measure frequency, but also for its application in various other useful circuits. We find, for example, a Wien bridge in the harmonic distortion analyzer, where it is used as a notch filter, discriminating against one specific frequency. The Wien bridge also finds application in audio- and HF oscillators as the frequency-determining element. in this chapter, however, the Wien bridge is discussed in its basic form, designed to measure frequency; in other chapters it is shown as an element of different types of instrument.

The Wien bridge has a series RC combination in one arm and a parallel RC combination in the adjoining arm (see Fig. 5-18). The impedance of arm 1 is $Z_1 = R_1 - j/\omega C_1$. The admittance of arm 3 is $\mathbf{Y_3} = 1/R_3 + j/\omega C_3$. Using the basic equation for bridge balance and substituting the appropriate values, we obtain

$$R_2 \left(R_1 - \frac{j}{\omega C_1} \right) R_4 \left(\frac{1}{R_3} + j\omega C_3 \right) \qquad (5\text{-}46)$$

Expanding this expression, we get

$$R_2 = \frac{R_1 R_4}{R_3} + j\omega C_3 R_1 R_4 - \frac{jR_4}{\omega C_1 R_3} + \frac{R_4 C_3}{C_1} \qquad (5\text{-}47)$$

Equating the real terms, we obtain

$$R_2 = \frac{R_1 R_4}{R_3} + \frac{R_4 C_3}{C_1} \qquad (5\text{-}48)$$

which reduces to

$$\frac{R_2}{R_4} = \frac{R_1}{R_3} \frac{C_3}{C_1} \qquad (5\text{-}49)$$

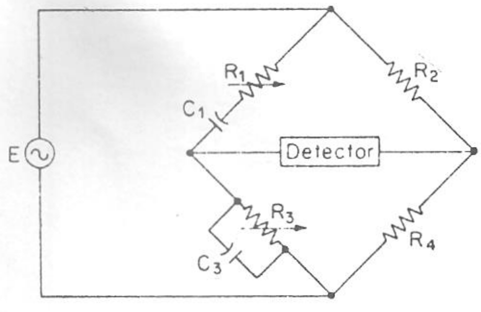


Figure 5-18 Frequency measurement with the Wien bridge.

Equating the imaginary terms, we obtain

$$\omega C_3 R_1 R_4 = \frac{R_4}{\omega C_1 R_3} \qquad (5\text{-}50)$$

where $\omega = 2\pi f$, and solving for f, we get

$$f = \frac{1}{2\pi\sqrt{C_1 C_3 R_1 R_3}} \qquad (5\text{-}51)$$

Notice that, the two conditions for bridge balance now result in an expression determining the required resistance ratio, R_2/R_4, and another expression determining the frequency of the applied voltage, in other words, if we satisfy Eq. (5-49), and also excite the bridge with a frequency described by Eq. (5-51), the bridge will be in balance.

In most Wien bridge circuits, the components are chosen such that $R_1 = R_3$ and $C_1 = C_3$. This reduces Eq. (5-49) to $R_2/R_4 = 2$ and Eq. (5-51) to

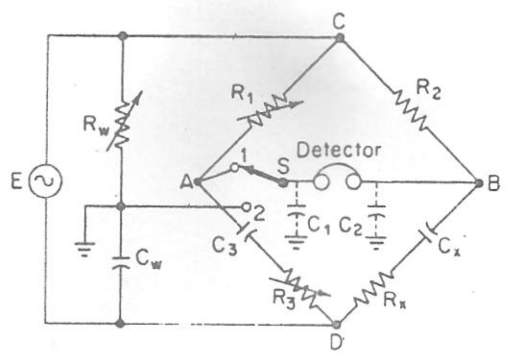
$$f = \frac{1}{2\pi RC} \qquad (5\text{-}52)$$

which is the general expression for the frequency of the Wien bridge. In a practical bridge, capacitors C_1 and C_3 are fixed capacitors, and resistors R_1 and R_3 are variable resistors controlled by a common shaft. Provided now that $R_2 = 2R_4$, the bridge may be used as a frequency-determining device balanced by a single control. This control may be calibrated directly in terms of frequency.

Because of its frequency sensitivity, the Wien bridge may be difficult to balance (unless the waveform of the applied voltage is purely sinusoidal). Since the bridge is not balanced for any harmonics present in the applied voltage, these harmonics will sometimes produce an output voltage masking the true balance point.

5.11 WAGNER GROUND CONNECTION

The- discussion so far has assumed that the four bridge arms consist of simple lumped impedances which do not interact in any way. In practice, however, stray capacitances exist between the various bridge elements and ground, and also



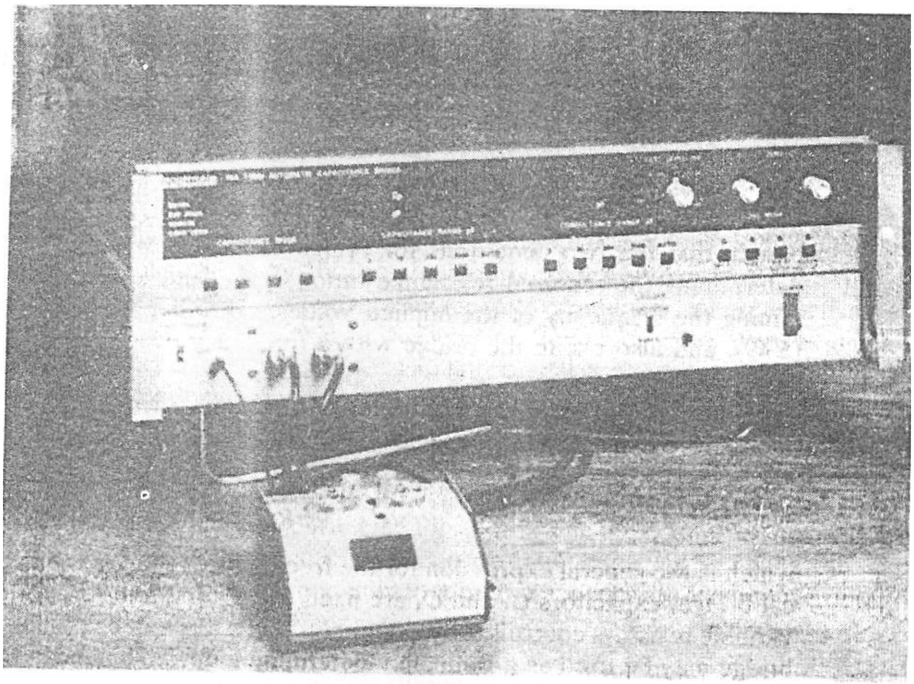


Figure 5-19 (a) The Wagner ground connection eliminates the effect of stray capacitances across the detector; (b) completely automatic capacitance bridge that can be computer interfaced. (Photograph courtesy of Boonton Electronics Corporation.)

between the bridge arms themselves. These stray capacitances shunt the bridge arms and cause measurement errors, particularly at the higher frequencies or when small capacitors or large inductors are measured. One way to control stray capacitances is by shielding the arms and connecting the shields to ground. This does not eliminate the capacitances but at least makes them constant in value, and they can therefore b compensated.

One of the most widely used methods for eliminating some of the effects of capacitance in a bridge circuit is the Wagner ground connection. This circuit eliminates the troublesome capacitance which exists between the detector terminals and ground. Figure 5-19(a) shows the circuit of a capacitance bridge. where C_1 and C_2 represent these stray capacitances. The oscillator is removed from its usual ground connection and bridged by a series combination of resistor R_w and capacitor C_w. The junction of R_w and C_w is grounded and is called the Wagner ground connection. The procedure for initial adjustment of the bridge is as follows: The detector is connected to point 1, and R_1 is adjusted for null or minimum sound in the headphones. The switch is then thrown to position 2, which connects the detector to the Wagner ground point. Resistor R_w, is now' adjusted for minimum sound. When the switch is thrown to position 1 again. some unbalance will probably be shown. Resistors R_1 and R_3 are then adjusted for minimum detector response, and the switch is again thrown to position 2. A few adjustments of R_w and R_1 (and R_3) may be necessary before a null is reached on both switch positions. When null is finally obtained, points 1 and 2 are at the same potential, and this is ground potential. Stray capacitances C_1 and C_2 are then effectively shorted out and have no effect on normal bridge balance. There are also capacitances from points C and D to ground, but the addition of the Wagner ground point eliminates them from the detector circuit, since current through these capacitances will enter through the Wagner ground connection.

The capacitances across the bridge arms arc not eliminated by the Wagner ground connection and they will still affect the accuracy of the measurement. The idea of the Wagner ground can also be applied to other bridges, as long as care is taken that the grounding arms duplicate the impedance of one pair of bridge arms across which they are connected. Since the addition of the Wagner ground connection does not affect the balance conditions, the procedure for measurement remains unaltered.

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PROBLEMS

5-1. The standard resistor arm of the bridge shown in Fig. P5-l has a range from 0 to $100\,\Omega$ with a resolution of $0.001\,\Omega$. The galvanometer has an internal resistance of $100\,\Omega$ and can be read to $0.5\,\mu A$. When the unknown resistance is 500, what is the resolution of the bridge in both ohms and per cent of the unknown?

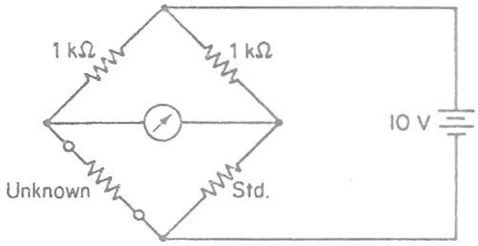


Figure P5-1

5-2. The ratio arms of the Kelvin bridge of Fig. 5-5 are 100 Ω each. The galvanometer has an internal resistance of 500 Ω and a current sensitivity of 200 mm/μA. The unknown resistance R_x 0.1002 Ω and the standard resistance is set at 0.1000 Ω. A dc current of 10 A is passed through the standard and the unknown from a 2.2-V battery in series with a rheostat. The resistance of the yoke may be neglected. Calculate (a) the deflection of the galvanometer, and (b) the resistance unbalance required to produce a galvanometer deflection of 1 mm.

5-3. The ratio arms of a Kelvin bridge are 1,00011 each. The galvanometer has an internal resistance of 100 Ω and a current sensitivity of 500 mm/μA. A dc current of 10 A is passed through the standard arm and the unknown from a 2.2-V battery in series with a rheostat. The standard resistance is set at 0.1000 Ω and the galvanometer deflection is 30 mm. Neglecting the resistance of the yoke, determine the value of the unknown.

5-4. A balanced ac bridge has the following constants: arm AB, R = 2,000 Ω in parallel with C = 0.047 μF; arm BC, R = 1,000 Ω in series with C = 0.47

μF; arm CD, unknown; arm DA, C = 0.5 μF. The frequency of the oscillator is 1,000 Hz. Find the constants of arm CD.

5-5. A bridge is balanced at 1,000 Hz and has the following constants: AB, 0.2 μF pure capacitance; BC, 500Ω pure resistance; CD, unknown; DA, R = 300Ω in parallel with C = 0.1 μF. Find the R and C or L constants of arm CD, considered as a series circuit.

5-6. A 1,000-Hz bridge has the following constants: arm AB, R = 1,000Ω in parallel with C = 0.5μF; BC, R = 1,000Ω in series with C= 0.5μF; CD, L = 30 mH in series with R = 200Ω. Find the constants of arm DA to balance the bridge. Express the result as a pure R in series with a pure C or L and also a pure R in parallel with a pure C or L.

5-7. An ac bridge has in arm AB a pure capacitance of 0.2 μF; in arm BC, a pure resistance of 500 Ω: in arm CD, a series combination of R = 50Ω and L = 0.1 H. Arm DA consists of a capacitor C = 0.4 μF in series with a variable resistor R_s. w = 5,000 rad/s. (a) Find the value of R_s to obtain bridge balance. (b) Can complete balance be attained by the adjustment of R_s? If not, specify the position and value of an adjustable resistance to complete the balance.

5-8. An ac bridge has the following constants: arm AB, R = 1,000Ω in parallel with C = 0.159 μF; BC, R = 1,000Ω; CD, R = 500Ω; DA, C = 0.636 μF in series with an unknown resistance. Find the frequency for which this bridge is in balance and deter mine the value of the resistance in arm DA to produce this balance.

Electronic Instruments for Measuring Basic Parameters

6-1 INTRODUCTION

The measuring instruments discussed in the previous chapters used the movement of an electromagnetic meter to measure voltage, current, resistance, power, etc. Although the bridges and multimeters used electrical components for these measurements, the instruments described used no amplifiers to increase the sensitivity of the measurements. The heart of these instruments was the d'Arsonval meter, which typically cannot be constructed with a full-scale sensitivity of less than about 50 μA. Any measurement system using the d'Arsonval meter, without amplifiers, must obtain at least 50 μA from the circuit under test for a full-scale deflection. For the measurement of currents of less than 50 MA full scale, an amplifier must be employed. The resistance of a (very) sensitive meter, such as a 50-μA meter for a volt-ohm-milliammeter, is on the order of a few hundred ohms and represents a small but finite amount of power. As an example, 50 μA through a 200-Ω meter represents microwatt (μW). This represents the power required for the meter for full-scale deflection and does not represent the power dissipated in the series resistor, and thus the total power required by the example meter would be greater than k MW and would depend on the voltage range. This doesn't sound like much power, but many electronic circuits cannot tolerate this much power being drained from them: Consider, also, the voltage across a 200-Ω, 50-

μA meter at full scale, which, by Ohm's law, is 10 my. The most sensitive voltmeter that could be constructed from the 50-μA meter, without an amplifier, would be 10 mV full scale. The schematic of this sensitive meter would have no external resistor but only the internal resistance of the meter itself.

As shown above, an amplifier is required to increase the current sensitivity below 50 μA, the voltage below 10 my, and the power required below $\frac{1}{2}$ μW. For the case of ac measurements, the amplifier is even more necessary for sensitive measurements.

In addition to instruments for making measurements of small currents and voltages, included in this chapter are electronic instruments for measuring other parameters, such as resistance, inductance, and capacitance. -

6-2 AMPLIFIED DC METER

A simple amplified voltmeter is shown in Fig. 6-1. This meter decreases the amount of power drawn from a circuit under test by increasing the input impedance using an amplifier with unity gain. A source follower drives an emitter follower. This combination is capable of a thousand-fold or more increase in impedance, while maintaining a voltage gain of very nearly one. The input impedance of this meter is 10 MΩ, which would require 0.025 μW of power for a 0.5-V deflection, as compared to 25 MW for an unamplified meter, an increase in sensitivity of 100 times.

Because the emitter follower must have some bias current present, the emitter voltage does not go to zero volts with zero input voltage. Thus the meter must be returned not to ground, but to a voltage that can be set to be equal to the quiescent point of the emitter-follower output. This tends to vary somewhat with temperature, and in many practical meters this is made adjustable from the front

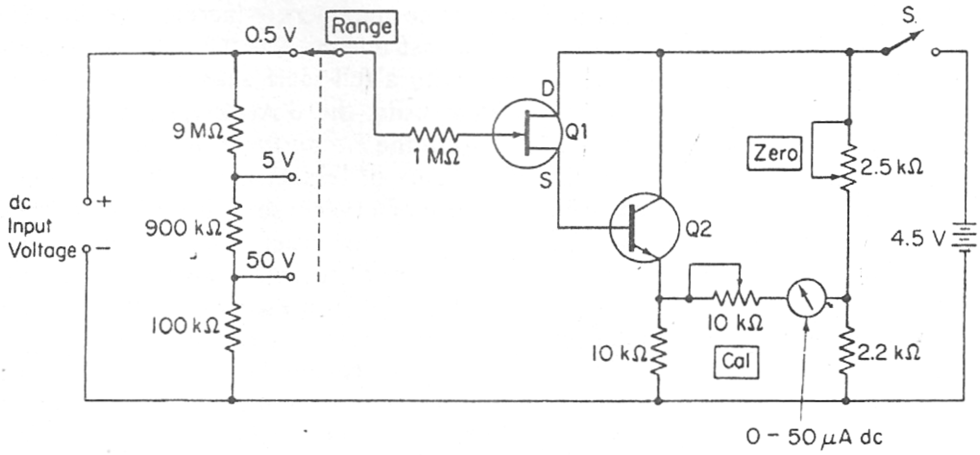


Figure 6-1 Basic dc voltmeter circuit with FET input.

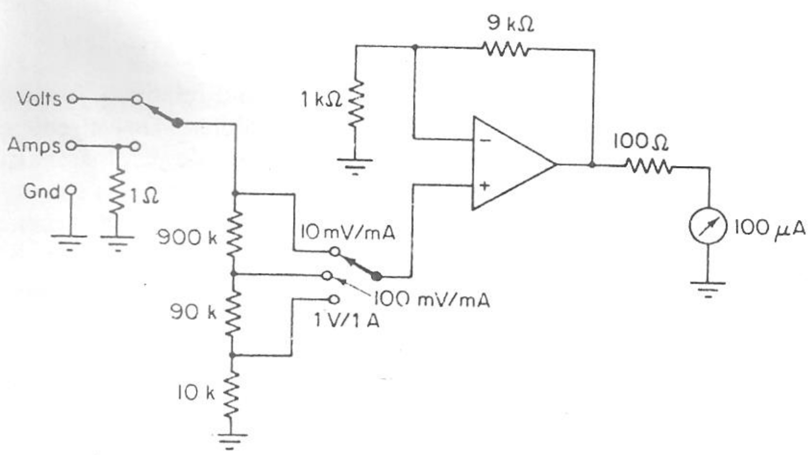


Figure 6-2 Amplified voltage and current meter.

panel of the meter. Because the setting of the Zero control affects the total resistance in series with the meter, a Cal (calibrate) control is also supplied. This control is not necessary for amplified meters using a differential amplifier because there is no interaction between the zero adjustment and the calibration of the meter.

A block diagram of a meter capable of measuring small voltages and currents is shown in Fig. 6-2. The input voltage is amplified and applied to a meter. If the amplifier has a gain of 10, the sensitivity of the measurement is increased by a like amount. A dc-coupled amplifier, that is, an amplifier with no coupling capacitors and having a well-controlled dc gain, is used to provide the necessary amplification. An amplifier capable of a fixed dc gain of 10 is not difficult to construct and to keep stable. A simple op-amp plus the required feedback components will do a suitable job for this application.

De gains of much more than 10 are required to use a standard d'Arsonval meter movement to measure very small currents and voltages such as microvolt and nanoamperes. To amplify nanoamperes to drive a milliampere meter requires a gain of 10^6. In theory, this requires an op-amp and two resistors, and a simple circuit. However, when gains this large are desired, all the defects of an operational amplifier become significant. Offset current, offset voltage, and bias currents become so troublesome that it is practically impossible to achieve acceptable performance with a standard op-amp. Many of these defects can be reduced or eliminated by the use of trim adjustments accessible from the front panel in a similar fashion as the Cal and Zero functions discussed above. However, temperature- and time-induced drifts would soon render the amplifier unusable, and the adjustment would have to be repeated. Direct-coupled amplifiers that have been optimized for low-temperature drift and low offset and bias currents are called instrumentation amplifiers and are manufactured by semiconductor suppliers.

6-2.1 Chopper-Stabilized

One technique for amplifying direct currents and relatively low-frequency alternating currents is the chopper-stabilized. This circuit eliminates the effects of dc offset currents of other dc parameters by using an ac coupled amplifier for the necessary gain. The technique, as shown in Fig. 6-3, is to convert the input signal to an ac signal and, after high-gain amplification, reconstruct the dc from the amplified ac sigmal.

The input signal is converted to an ac signal by chopping, which simply involves switching the input of an amplifier between the input and ground with an electronic switch or an electromechanical chopper, which is similar to a relay. The output of the chopper is an ac signal with a peak value equal to the input dc voltage. Because the chopped input has a negative peak of zero and a positive peak of the input voltage, the resulting ac waveform has a dc component of approximately one-half of the input dc voltage. The actual dc component of the chopped waveform is not important, as this will be fed to an ac-coupled amplifier where the dc component will be lost.

The amplified signal is chopped in a similar fashion as the input and in synchronism with the input chopper. The synchronized chopping restores the dc value of the input signal amplified by the ac gain of the amplifier. Because the amplifier did not provide any dc gain, the effects of dc offset voltages and currents are eliminated.

Enormous gains can be achieved in this fashion, and the chopper-stabilized amplifier can provide gains of more than 106 with excellent dc stability. All this does not come without problems. First, when dealing with very small currents and voltages, unusual problems can occur. One significant problem is with the chopper. This device must be specially made to avoid generating voltages from

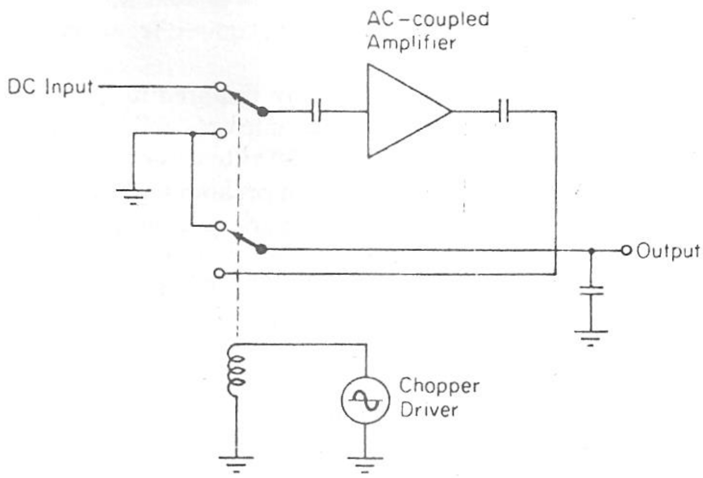


Figure 6-3 Ac-coupled amplifier can be used to amplify dc signals if the input and output are chopped using the circuit shown.

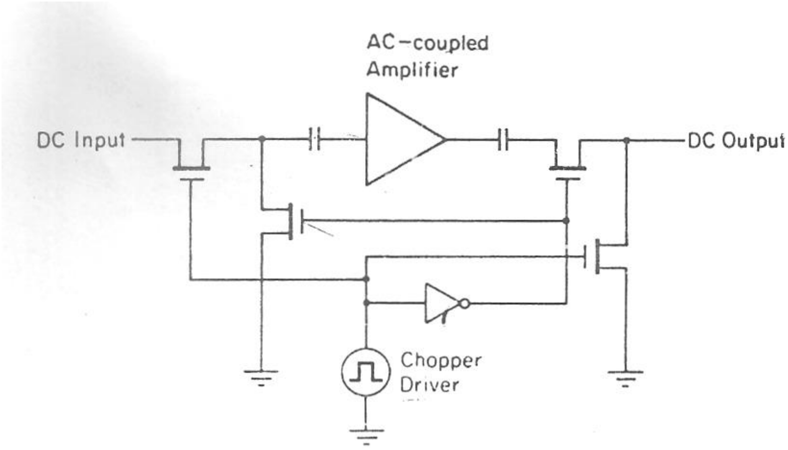


Figure 6-4 All-electric chopper circuit using field effect transistors.

thermocouple effects. When two dissimilar metals are joined, depending on the temperature, small voltages can be generated. The chopper is specially made to reduce these thermally generated voltages.

The electromechanical chopper, being a mechanical device, has a relatively short life span when compared to other electronic devices. Various types of all electronic choppers have been devised to replace the venerable mechanical chopper. The most important characteristic of the chopper is that it must not inject any current into the circuit being chopped, especially for the input chopper. Bipolar transistors, light-activated devices, and field effect transistors have been used for choppers with the. MOS field effect transistor being the most successful. Because the MOS transistor has no junction as a source of leakage current, very little current is transmitted from the chopping signal to the input. Figure 6-4 shows a series-shunt chopper using two MOS field effect transistors. The chopping signal is fed to the inverter, which drives the two chopper FETs, one on each half of the chopping cycle.

The input impedance of the chopper-stabilized amplifier is very high for direct current. Looking into the chopper-stabilized amplifier, the series chopper switches the input to the ac-coupled amplifier every half-cycle; however, because the amplifier is ac coupled, it appears as an infinite resistance to direct current. The series chopper switch is always open before the shunt switch is closed, and thus there is no path to ground.

6-3 AC VOLTMETER USING RECTIFIERS

Electronic ac voltmeters are basically identical to dc voltmeters except that the ac input voltage must be rectified before it can be applied to the dc meter circuit. In some instances, rectification takes place before amplification, in which case a simple diode rectifier circuit precedes the, amplifier and meter, as in Fig. 6-5(a).

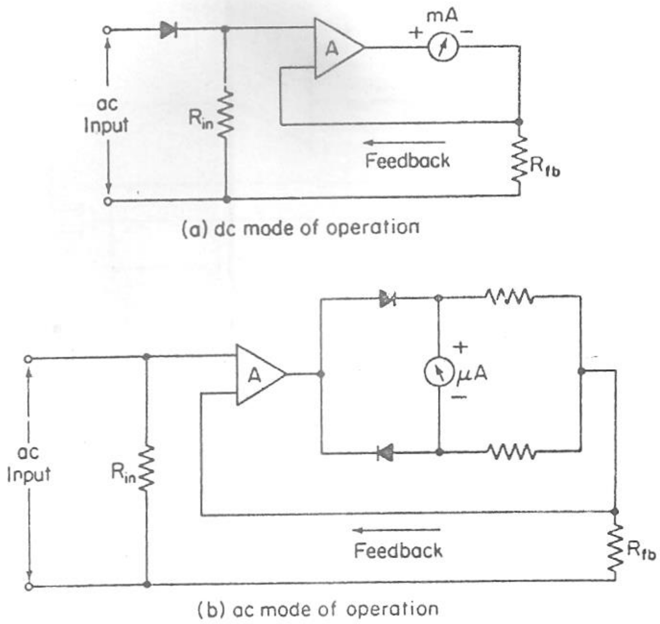


Figure 6-5 Basic ac voltmeter circuits: (a) the ac input signal is first rectified and then applied to a dc amplifier and meter movement; (b) the ac input signal is first amplified and then applied to a full-wave rectifier in the meter circuit.

This approach ideally requires a de amplifier with zero drift characteristics and unity voltage gain, and a dc meter movement with adequate sensitivity.

In another approach the ac signal is rectified after amplification, as in Fig. 6-5(b) where full-wave rectification takes place in the meter circuit connected to the output terminals of the ac amplifier. This approach generally requires an ac amplifier with high open-loop gain and large amounts of negative feedback to overcome the nonlinearity of the rectifier diodes.

Ac voltmeters are usually of the average-responding type, with the meter scale calibrated in terms of the rms value of a sine wave. Since so many wave forms in electronics are sinusoidal, this is an entirely satisfactory solution and certainly much less expensive than a true rms-responding voltmeter. No sinusoidal waveforms, however, will cause this type of meter to read high or low, depending on the form factor of the waveform.

A few basic rectifier circuits are shown in Fig. 6-6. The series-connected diode of Fig. 6-6(a) provides half-wave rectification, and the average value of the half-wave voltage is developed across the resistor and applied to the input terminals of the dc amplifier. Full-wave rectification can be obtained by the bridge circuit of Fig. 6-6(b), where the average value of the sine wave is applied to the amplifier and meter circuit. In some cases, there may be a requirement to mea- sure the peak value of a waveform instead of the average value; the circuit of Fig. 6-6(c) may then be used. In this circuit the rectifier diode charges the small

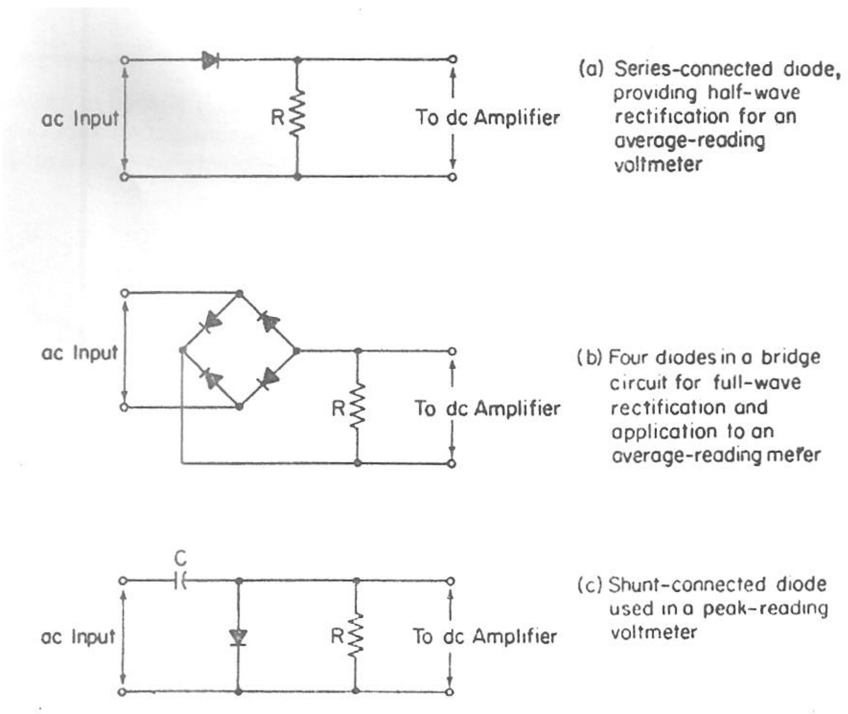


Figure 6-6 Rectifier circuits used in ac voltmeters.

capacitor to the peak of the applied input voltage and the meter will therefore indicate the peak voltage. In most cases, the meter scale is calibrated in terms of both the rms and peak values of the sinusoidal input waveform.

The rms value of a voltage wave that has equal positive and negative excursions is related to the average value by the form factor. The form factor, as the ratio of the rms value of the average value of this waveform, for a sinusoid can be expressed as

$$K = \frac{\sqrt{(1/T)\int_o^t e^2\,dt}}{(2/T)\int_0^{T/2} e\,dt} = \frac{\sqrt{(\omega/2\pi)\int_0^{2\pi/\omega} E_m \sin^2 \omega t\,dt}}{(\omega/2\pi)\int_0^{2\pi/\omega} E_m \sin^2 \omega t\,dt}$$

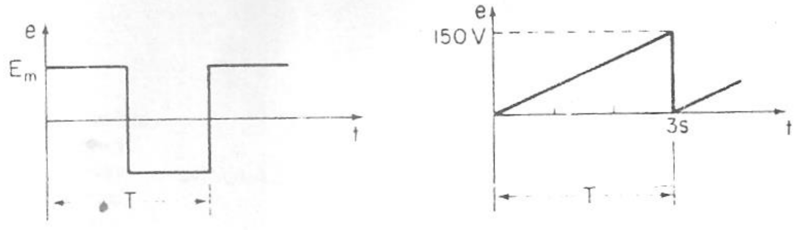
$$= \frac{\sqrt{(E_m^2/4\pi)[\omega t - \sin\omega t\cos\omega t]_0^{2\pi/\omega}}}{(E_m/\pi)[-\cos\omega t]_0^{\pi/\omega}} = \frac{E_m\,0.707}{E_m\,0.636} = 1.11 \qquad (6\text{-}1)$$

Therefore when an average-responding voltmeter has scale markings corresponding to the rms value of the applied sinusoidal input waveform, those markings are actually corrected by a factor of 1.11 from the true (avenge) value of applied voltage.

Nonsinusoidal waveforms, when applied to this voltmeter, will cause the meter to read either high or low, depending on the form factor of the waveform. An illustration of the effect of nonsinusoidal waveforms on ac voltmeter is given in Examples 6-1 and 6-2.

EXAMPLE 6-1

The symmetrical square-wave voltage of Fig. 6-7(a) is applied to an average-responding ac voltmeter with a scale calibrated in terms of the rms value of a sine wave. Calculate (a) the form factor of the square-wave voltage; (b) the error in the meter indication.



(a) Symmetrical square wave (b) Sawtooth waveform

Figure 6-7 Waveforms used in Examples 6-1 and 6-2.

SOLUTION (a) The rms value of the square-wave voltage is

$$E_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T e^2\, dt} = E_m$$

and the average value is

$$E_{\text{av}} = \frac{2}{T}\int_0^{T/2} e\, dt = E_m$$

so that the form factor equals, by definition,

$$k = \frac{E_{\text{rms}}}{E_{\text{av}}} = 1$$

(b) The meter scale is calibrated in terms of the rms value of a sine-wave voltage, where $E_{\text{rms}} = k \times E_{\text{av}} = 1.11E_{\text{av}}$. For the square-wave voltage, $E_{\text{rms}} = E_{\text{av}}$, since $k = 1$. Therefore the meter indication for the square-wave voltage is *high* by a factor $k_{\text{sine wave}}/k_{\text{square wave}} = 1.11$. The percentage error equals

$$\frac{1.11 - 1}{1} \times 100\% = 11\%$$

EXAMPLE 6-2

Repeat Example 6-1 if the voltage applied to the meter consists of a sawtooth waveform with a peak value of 150 V and a period of 3 s as shown in Fig. 6-7(b).

SOLUTION (a) The analytical expression for the sawtooth waveform between the limits of $t = 0$ and $t = T = 3$ s is $e = 50t$ V. Therefore

$$E_{rms} = \sqrt{\frac{1}{T}\int_0^T e^2\,dt} = \sqrt{\frac{1}{3}\int_0^3 (50t)^2\,dt} = 50\sqrt{3}\text{ V}$$

$$E_{av} = \frac{1}{T}\int_0^T e\,dt = \frac{1}{3}\int_0^3 50t\,dt = 75\text{ V}$$

$$\text{Form factor, } k = \frac{50\sqrt{3}}{75} = 1.155$$

(b) The ratio of the two form factors is

$$\frac{k_{\text{sine wave}}}{k_{\text{sawtooth}}} = \frac{1.11}{1.155} = 0.961$$

The meter indication is *low* by a factor of 0.961. The percentage error equals

$$\frac{0.961 - 1}{1} \times 100\% = -3.9\%$$

6-4 TRUE RMS-RESPONDING VOLTMETER

Complex waveforms are most accurately measured with an rms-responding voltmeter. This instrument produces a meter indication by sensing waveform heating power, which is proportional to the square of the rms value of the voltage. This heating power can be measured by feeding an amplified version of the input waveform to the heater element of a thermocouple whose output voltage is then proportional to E^2_{rms}

One difficulty with this technique is that the thermocouple is often nonlinear in its behavior. This difficulty is overcome in some instruments by placing two thermocouples in the same thermal environment, as shown in the block diagram of the true rms-responding voltmeter of Fig. 6-8. The effect of the nonlinear behav

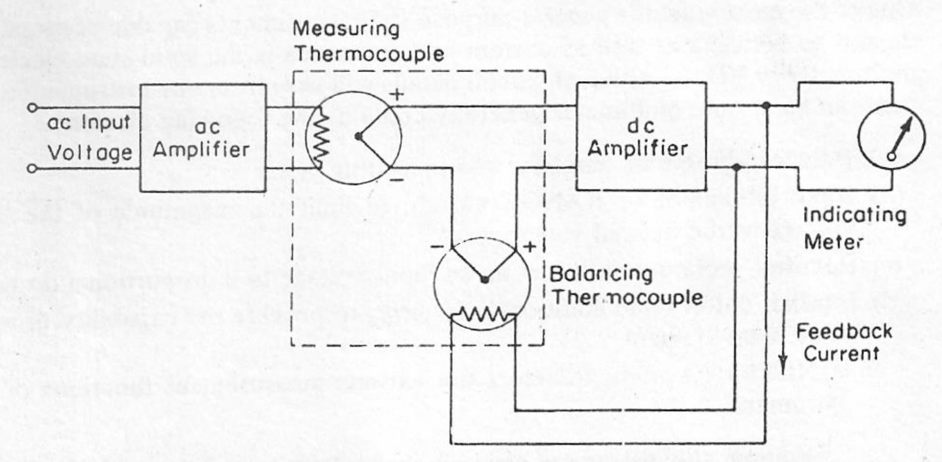


Figure 6-8 Block diagram of a true rms-reading voltmeter. The measuring and balancing thermocouples are located in the same thermal environment.

ior of the couple in the input circuit (the measuring thermocouple) is canceled by similar nonlinear effects of the couple in the feedback circuit (the balancing thermocouple). The two couple elements form part of a bridge in the input circuit of a dc amplifier. The unknown ac input voltage is amplified and applied to the heating element of the measuring thermocouple. The application of heat produces an output voltage that upsets the balance of the bridge. The unbalance voltage amplified by the dc amplifier and fed back to the heating element of the balancing thermocouple. Bridge balance will be reestablished when the feedback current delivers sufficient heat to the balancing thermocouple, so that the voltage outputs of both couples are the same. At this point the dc current in the heating clement of the feedback couple is equal to the ac current in the input couple. This dc current is therefore directly proportional to the effective, or rms, value of the input voltage and is indicated on the meter movement in the output circuit of the dc amplifier. The true rms value is measured independently of the waveform of the ac signal, provided that the peak excursions of the waveform do not exceed the dynamic range of the ac amplifier, A Lypical laboratory-type rms-responding voltmeter provides accurate rms readings of complex waveforms having a crest factor (ratio of peak value to rms value) of 1011. At 10 per cent of full-scale meter deflection, where there is less chance of amplifier saturation, waveforms with crest factors as high as 100/1 could be accommodated. Voltages throughout a range of 100 μV to 300 V within a frequency range of 10 Hz to 10 MHz may be measured with most good instruments.

6-5 ELECTRONIC MULTIMETER

6.5.1 Basic Circuit

One of the most versatile general-purpose shop instruments capable of measuring dc and ac voltages as well as current and resistance is the solid. state electronic multi meter or VOM. Although circuit details will vary from one instruments to the next, au electronic multimeter generally contains the following elements:

(a) Balanced-bridge tic amplifier and indicating meter

(b) Xnput attenuator or RANGE switch, to limit the magnitude of the input voltage to the desired value

(c) Rectifier section. to convert an ac input voltage to a proportional dc value

(d) Internal battery and additional circuitry, to provide the capability of resistance measurement

(e) FUNCTION switch, to select the various measurement functions of the instrument

Inaddition, the instrument generally has a built-in power supply for ac line operation and, in most cases, one or more batteries for operation as a portable test instrument.

Figure 6-9 shows the schematic diagram of a balanced-bridge dc amplifier using field effect transistors or FETs. This circuit also applies to a bridge amplifier with ordinary bipolar transistors or BJTs. The circuit shown here consists of FETs which sould be reasonably well matched for current gain to ensure thermal stability of the circuit. The two FETs form the upper arms of a bridge circuit. Source resistors R_1 and R_2, together with ZERO adjust resistor R_3, form the lower bridge arms. The meter movement is connected between the source terminals of the FETs, representing two opposite corners of the bridge.

Without an input signal, the gate terminals of the FETs are at ground potential and the transistors operate under identical quiescent conditions. In this case, the bridge is balanced and the meter indication is zero. In practice, however, small differences in the operating characteristics of the transistors, and slight tolerance differences in the various resistors, cause a certain amount of unbalance in the drain currents, and the meter shows a small deflection from zero. To return the meter to zero, the circuit is balanced by ZERO adjust control R_3 for a true null indication.

When a positive voltage is applied to the gate of input transistor Q_1, its drain current increases which causes the voltage at the source terminal to rise. The resulting unbalance between the Q_1 and Q_2 source voltages is indicated by the meter movement, whose scale is calibrated to agree with the magnitude of the applied input voltage.

The maximum voltage that can be applied to the gate of Q_1 is determined by the operating range of FET and is usually on the order of a few volts. The range of input voltages can easily be extended by an input attenuator or RANGE switch, as shown in Fig. 6-10. The unknown dc input voltage is applied through a large resistor in the probe body to a resistive voltage divider. Thus, with the RANGE switch in the 3-V position as shown, the voltage at the gate of the input FET is

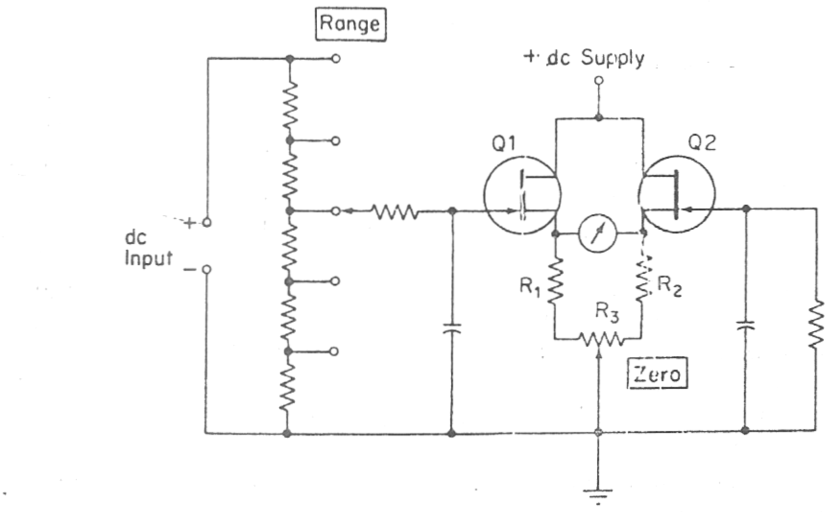


Figure 6-9 Balanced-bridge dc amplifier with input attenuator and indicating meter.

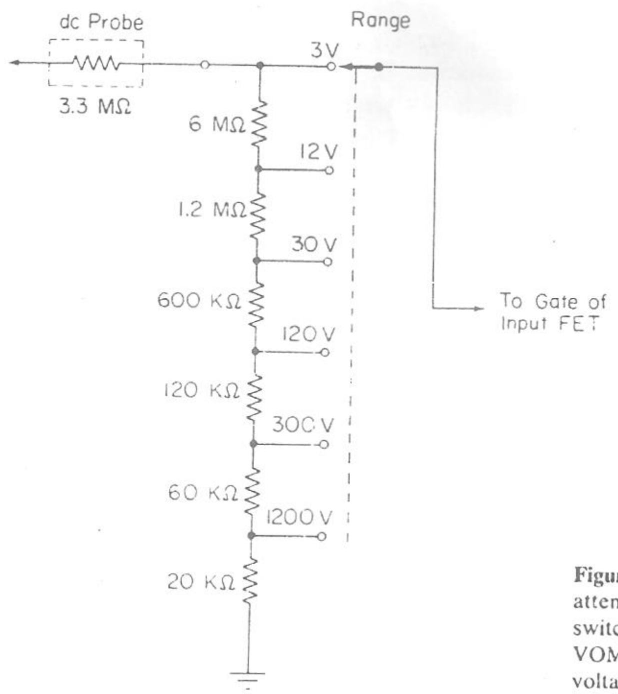


Figure 6-10 Typical input voltage attenuator for a VOM. The RANGE switch on the front of the panel of the VOM allows selection of the desired voltage range.

developed across 8 MΩ of the total resistance of 11.3 MΩ and the circuit is so arranged that the meter deflects full scale with 3 V applied to the tip of the probe. With the RANGE switch in the 12-V position, the gate voltage is developed across 2 MΩ of the total divider resistance of 11.3 MΩ and an input voltage of 12 V is required to cause the same full-scale meter deflection.

6-5.2 Resistance Ranges

When the function switch of the multimeter is placed in the OHMS position, the unknown resistor is connected in series with an internal battery, and the meter simply measures the voltage drop across the unknown. A typical circuit is shown in Fig. 6-11, where a separate divider network, used only for resistance measurements, provides for a number of different resistance ranges. When unknown resistor R_x is connected to the OHMS terminals of the multimeter, the 1.5-V battery supplies current through one of the range resistors and the unknown resistor to ground. Voltage drop V_x and R_x is applied to the input of the bridge amplifier and causes a deflection on the meter. Since the voltage drop across R_x is directly proportional to its resistance, the meter scale can be calibrated in terms or resistance.

Note chat the resistance scale of the multimeter reads increasing resistance from left to right, opposite to the way resistance scales read on conventional multimeters (Sec. 4-9). This can be expected because the electronic multimeter

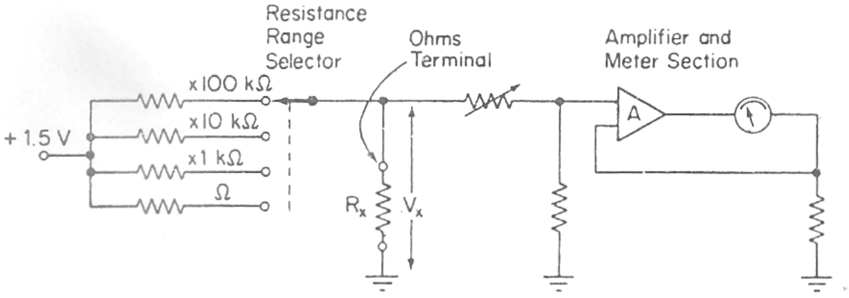


Figure 6-11 Resistance range selector circuit of a VOM.

reads a larger resistance as a higher voltage, whereas the ordinary multimeter indicates a higher resistance as a smaller current.

6-5.3 Commercial Multimeter

The simplified metering circuit of a commercial solid-state VOM is given in Fig. 6-12. The dc voltage from the input voltage divider (Fig. 6-9) is applied to the

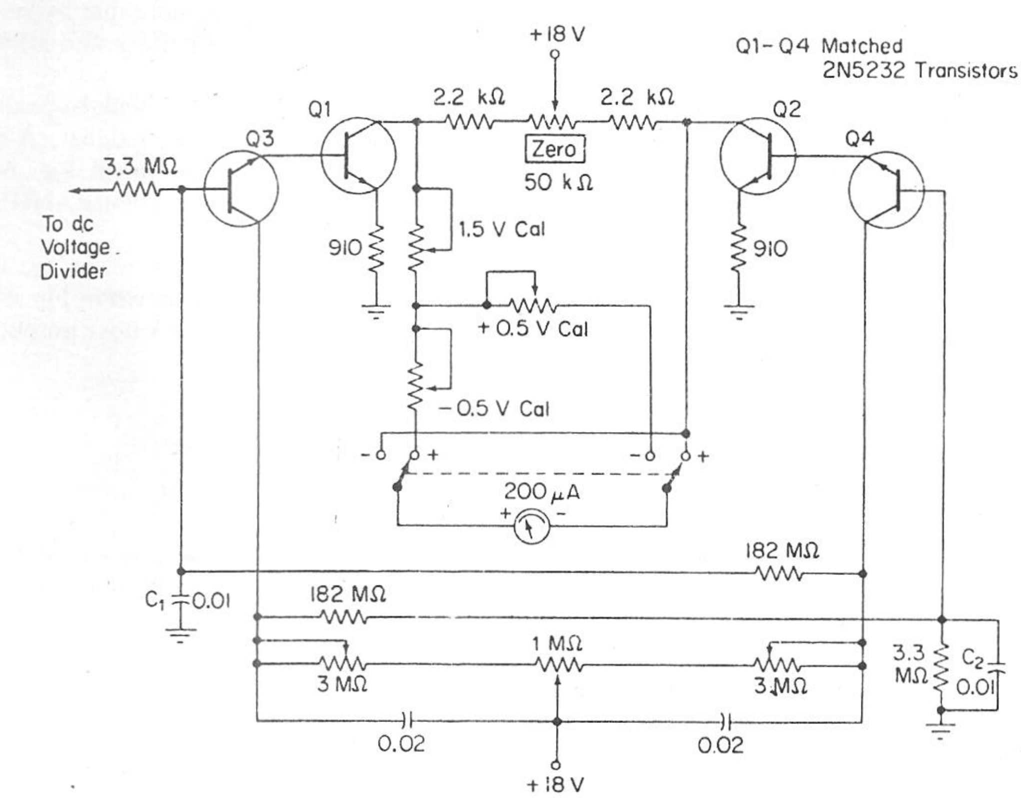


Figure 6-12 Typical metering circuit of a solid-state VOM.

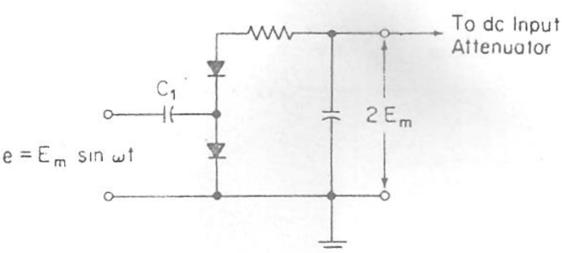


Figure 6-13 Full-wave peak-to-peak rectifier, also known as a voltage doubler.

bases of bridge preamplifier transistors Q_3 and Q_4. These emitter followers provide nearly infinite input impedence and therefore present a minimum load to the high-resistance input voltage divider. Preamplifier transistors Q_3 and Q_4, drive the bases of bridge amplifier transistors Q_1 and Q_2, respectively. The input impedances of Q_1 and Q_2 are very high because of their unbypassed emitter resistors, which prevent loading of the Q_3 and Q_4. emitters. The output voltage of the bridge amplifier is indicated on the 200-μA meter, connected between the collectors of Q_1 and Q_2. The front-panel ZERO control balances the meter amplifier output with zero input signal. Internal adjustments allow for meter calibration with two accurate test voltages of 0.5 V and 1.5 V, respectively. Also note that bypass capacitors C_1 and C_2 prevent ac signals from reaching the amplifier and affecting the meter reading.

Ac voltages being measured are applied to a full-wave peak-to-peak rectifier that charges a capacitor to the peak-to-peak value of the ac signal. A circuit of this type is also known as a voltage doubler and is shown in Fig. 6-13. The rectified ac voltage is then fed to the amplifier through the regular RANGE voltage divider.

When resistance is being measured, 1.5 V dc is applied to the unknown resistor through one of the resistance range resistors, as shown in Fig. 6-11. The known and the unknown resistances form a voltage divider whose output is fed to the amplifier and read on the meter in terms of resistance.

6-6 CONSIDERATIONS IN CHOOSING AN ANALOG

VOLTMETER

The most appropriate instrument for a particular voltage measurement depends on the performance required in a given situation. Some important considerations in choosing a voltmeter are summarized below.

6-6. 1 Input Impedance

To avoid loading effects, the input resistance or impedance of the voltmeter should be at least a-n order of magnitude higher than the impedance of the circuit under measurement. For example, when a voltmeter with a 10-MΩ input resistance is used to measure the voltage across a 100-kΩ reads a larger resistance as a higher voltage, whereas the ordinary multimeter indicates a higher resistance as a smaller current. resistor, the circuit is hardly disturbed and the loading effect of the meter on the circuit is negligible. The same meter placed across a 10-MΩ resistor, however, seriously loads the circuit and causes an error in -measurement of approximately 50 per cent.

The input impedance of the voltmeter is a function of the inevitable shunt capacitance across the input terminals. The loading effect of the meter is particularly noticeable at the higher frequencies, when the input shunt capacitance greatly reduces the input impedance.

In some applications, a passive voltage-divider probe can be used to reduce the input capacitance at the point of measurement at the sacrifice of perhaps 20 dB of sensitivity. With such a probe, measurements can be easily made at random points without disturbing the circuit under test.

6-6.2 Voltage Ranges

The voltage ranges on the meter scale may be in the 1-3-10 sequence with 10dB of separation, or in the 1.5-5-15 sequence, or in a single scale calibrated in decibels. In any case, the scale divisions should be compatible with the accuracy of the instrument. For example, a linear meter with a 1 per cent full-scale accuracy should have 100 divisions on the 1.0-V scale so that 1 per cent can be easily resolved. An instrument with an accuracy of 1 per cent or less should also have a mirror-backed scale to reduce parallax and improve accuracy.

6-6.3 Decibels

Use of the decibel scale can be very effective in measurements that cover a wide range of voltages. A measurement of this kind is found, for example, in the frequency response curve of an amplifier or a filter, where the output voltage is measured as a function of the frequency of the applied input voltage Almost all voltmeters with dB scales are calibrated in dBm, referenced to some particular impedance. The 0-dBm reference for a 600-Ω system is 0.7746 V; for a 50-Ω system it is 0.2236 V. In many applications only a 0-dB reference is needed. In this case, 0 dBv (relative to 1 V) can be used for any impedance system.

6-6.4 Sensitivity Versus Bandwidth

Noise is a function of bandwidth. A voltmeter with a broad bandwidth will pick up and generate more noise than one operating over a narrow range of frequencies. In general, an instrument with a bandwidth of 10 Hz to 10 MHz has a sensitivity of 1 mV. A voltmeter whose bandwidth extends only to 5 MHz could have a sensitivity of 100 μV.

6-6.5 Battery Operation

For field work, a voltmeter powered by an internal battery is essential. If an area contains troublesome groundloops, a battery-powered instrument is preferred over a mains-powered voltmeter to remove the groundpaths.

6-6.6 AC Current Measurements

Current measurements can be made by a sensitive ac voltmeter and a series resistance. In the usual case, however, an ac current probe is used which enables the operator to measure an ac current without disturbing the circuit under test. The current probe simply clips around the wire carrying the unknown current and in effect makes the wire the one-turn primary of a transformer formed by a ferrite core and a many-turn secondary within the current-probe body. The signal induced in the secondary winding is amplified and the output voltage of the amplifier is applied to a suitable ac voltmeter for measurement. Normally, the amplifier is designed so that 1 mA in the wire being measured produces I mV at the amplifier output. The current is then read directly on the voltmeter, using the same scale as for voltage measurements.

In summarizing the preceding considerations, the following general guidelines can be stated:

(a) For measurements involving dc applications, select the meter with the broadest capability meeting the circuit's requirements.

(b) For ac measurements involving sine waves with only modest amounts of distortion (<10 per cent), the average-responding voltmeter provides the best accuracy and most sensitivity per dollar investment.

(c) For high-frequency measurements (>10 MHz), the peak-responding voltmeter with a diode-probe input is the most economical choice. Peak-responding circuits are acceptable if the inaccuracies caused by distortion in the input waveform can be tolerated.

(d) For measurements where it is important to determine the effective power of waveforms which depart from the true sinusoidal form, the rms-responding voltmeter is the appropriate choice.

6-7 DIGITAL VOLTMETERS

6-7.1 General Characteristics

The digital voltmeter (DVM) displays measurements of dc or ac voltages as discrete numerals instead of a pointer deflection on a continuous scale as in analog devices. Numerical readout is advantageous in many applications because it reduces human reading and interpolation errors, eliminates parallax error, increases reading speed, and often provides outputs in digital form suitable for further processing or recording.

The DVM is a versatile and accurate instrument that can be used in many laboratory measurement applications. Since the development and perfection of integrated circuit (IC) modules, the size, power requirements, and cost of the

>>>>>>>>>>>>>>>

DVM have been drastically reduced so that DVMs can actively compete with conventional analog instruments, both in portability and price.

The DVM's outstanding qualities can best be illustrated by quoting• some typical operating and performance characteristics. The following specifications do not all apply to one particular instrument, but they do represent valid information on the present state of the art:

a. Input range: from ± 1.000000 V to ± 1,000.000 V, with automatic range selection and overload indication

b. Absolute accuracy: as high as ± 0.005 percent of the reading

c. Stability: short-term. 0.002 percent of the reading for a 24-hr period; long term, 0.008 percent of the reading for a 6-month period.

d. Resolution: 1 part in $l0^{6}$ (1 μV can be read on the 1-V input range)

e. Input characteristics: input resistance typically 10 MΩ; input capacitance typically 40 pF

f. Calibration: internal calibration standard allows calibration independent of the measuring circuit; derived from stabilized reference source

g. Output signals: print command allows output to printer; BCD (binary-coded-decimal) output for digital processing or recording.

Optional features may include additional circuitry to measure current, resistance, and voltage ratios. Other physical variables may be measured by using suitable transducers.

Digital voltmeters can be classified according to the following broad categories:

a. Ramp-type DVM
b. Integrating DVM
c. Continuous-balance DVM
d. Successive-approximation DVM

6-7.2 Ramp-Type DVM

The operating principle of the ramp-type DVM is based on the measurement of the time it takes for a linear ramp voltage to rise from 0 V to the level of the input voltage, or to decrease from the level of the input voltage to zero. This time interval is measured with an electronic time-interval counter, and the count is displayed as a number of digits on electronic indicating tubes.

Conversion from a voltage to a time interval is illustrated by the waveform diagram of Fig. 6-14. At the start of the measurement cycle, a ramp voltage is initiated; this voltage can be positive-going or negative-going. The negative-going ramp, shown in Fig. 6-1,4, is continuously compared with the unknown input voltage. At the instant that the ramp voltage equals the unknown voltage, a

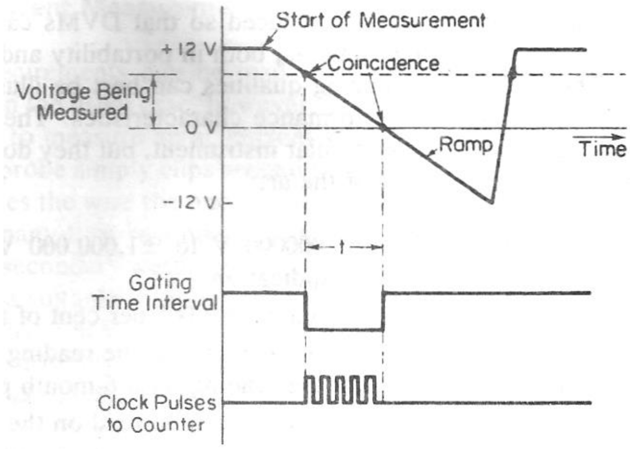


Figure 6-14 Voltage-to-time conversion using gated clock pulses.

coincidence circuit, or comparator, generates a pulse which opens a gate. This gate is shown in the block diagram of Fig. 6-15. The ramp voltage continues to decrease with time until it finally reaches 0 V (or ground potential) and a second comparator generates an output pulse which closes the gate.

An oscillator generates clock pulses which are allowed to pass through the gate to a number of decade counting units (DCUs) which totalize the number of pulses passed through the gate. The decimal number, displayed by the indicator tubes associated with the DCUs, is a measure of the magnitude of the input voltage.

The sample-rate multivibrator determines the rate at which the measurement cycles are initiated. The oscillation of this multivibrator can usually be adjusted by a front-panel control, marked rate, from a few cycles per second to as high as

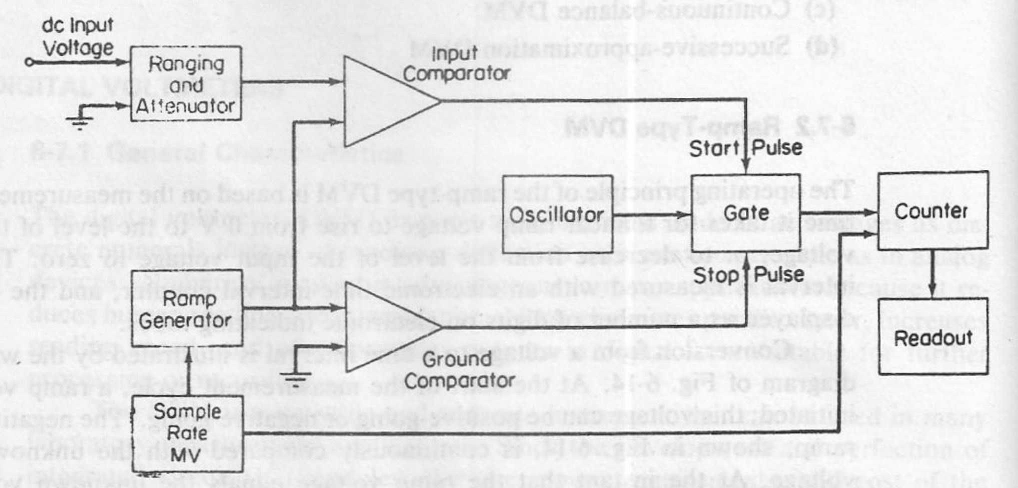


Figure 6-15 Block diagram of a ramp-type digital voltmeter.

1,000 or more. The sample-rate circuit provides an initiating pulse for the ramp generator to start its next ramp voltage. At the same time, a reset pulse is generated which returns all the DCUs to their 0 state, removing the display momentarily from the indicator tubes.

6-7.3 Staircase-Ramp DVM

The staircase-ramp DVM is given in block diagram form in Fig. 6-16. It ; a variation of the ramp-type DVM but is somewhat simpler in overall design. resulting in a moderately priced general-purpose instrument that can he used in the laboratory, on production test-stands, in repair shops, and at inspection stations.

This DVM makes voltage measurements by comparing the Flit voltage to an internally generated staircase-ramp voltage. The instrument shown in Fig. 6-16 contains a 10-MΩ input attenuator, providing five input ranges from 100 mV to 1,000 V full scale. The dc amplifier, with a fixed gain of 100, deliver, 10 V to the comparator at any of the full-scale voltage settings of the input divider. The comparator senses coincidence between the amplified input voltage and the staircase-ramp voltage which is generated as the measurement proceeds through its cycle.

When the measurement cycle is first initiated, the clock (a 4.5-kHz relaxation oscillator) provides pulses to three DCUs in cascade. The units counter provides a carry pulse to the tens decade at every tenth input pulse. The tens decade counts the carry pulses from the units decade and provides its own carry pulse after it has counted ten carry pulses. This carry pulse is fed to the hundreds decade which provides a carry pulse to an overrange circuit. The ovcrrange circuit causes a front panel indicator to light up, warning the operator that the input capacity of the instrument has been exceeded. The operator should then switch to the next higher setting on the input attenuator.

Each decade counter unit is connected to a digital-to-analog (DIM converter. The outputs of the D/A converters are connected in parallel and provide an output current proportional to the current count of the DCUs. The staircase amplifier converts the DIA current into a staircase voltage which is applied to the comparator. When the comparator senses coincidence of the input voltage and the staircase voltage, it provides a trigger pulse to stop the oscillator. The current content of the counter is then proportional to the magnitude of the input voltage.

The sample rate is controlled by a simple relaxation oscillator. This oscillator triggers and resets the transfer amplifier at a rate of two samples per second. The transfer amplifier provides a pulse that transfers the information stored In the decade counters to the front panel display unit. The trailing edge of this pulse triggers the reset amplifier which sets the three decade counters to zero and initiates a new measurement cycle by starting the master oscillator or clock.

The display circuits store each reading until a new reading is completed, eliminating any blinking or counting during the computation.

The ramp type of AID converter requires a precision ramp to achieve accuracy. Maintaining the quality of the ramp requires a precision, stable capacitor and resistor in the integrator. In addition, the offset voltages and currents of the operational amplifier used in the integrator are critical in the accurate ramp generator. One method of reducing the dependence of the accuracy of the conversion on the resistor, capacitor, and operational amplifier is to use a technique called the dual-slope converter.

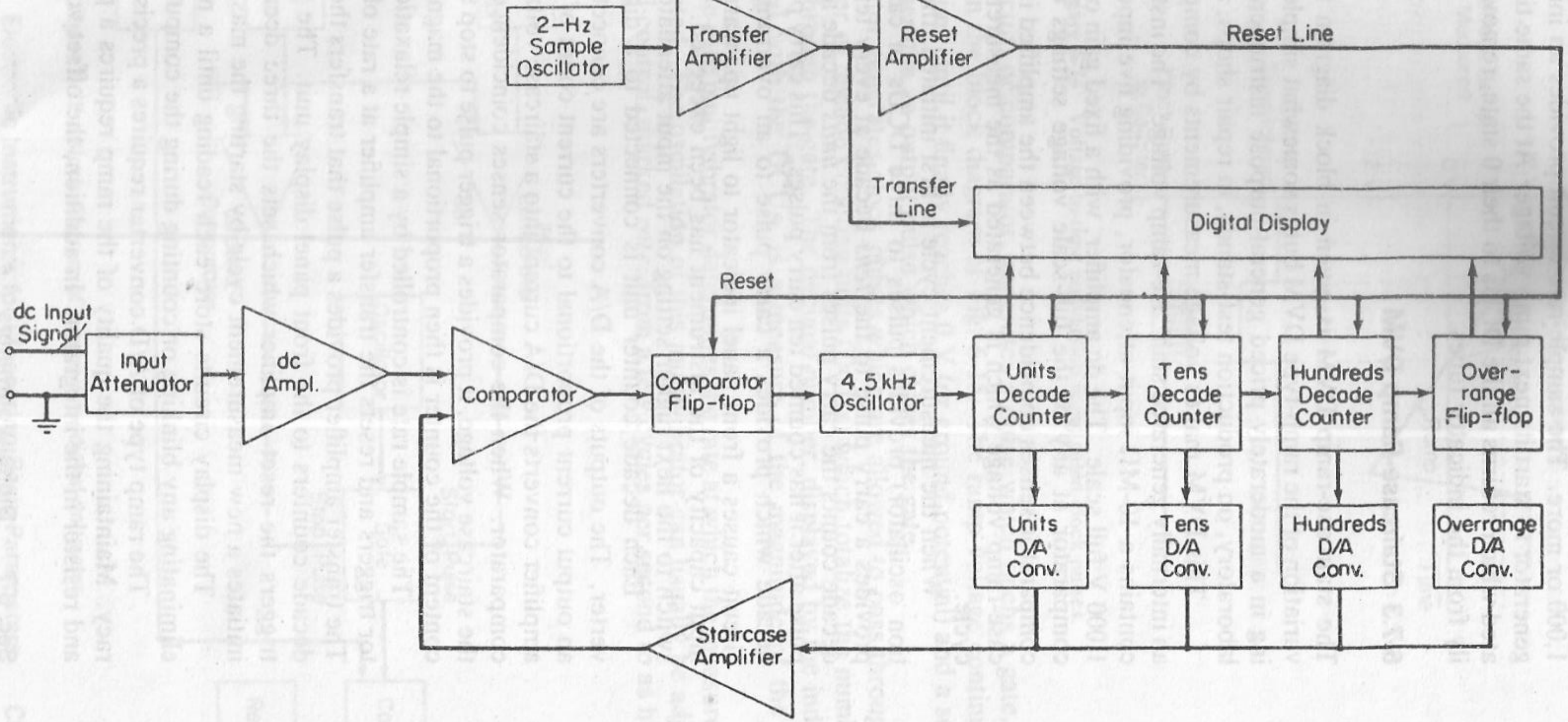


Figure 6-16 Block diagram of a staircase-ramp digital voltmeter.

In the dual-slope technique, an integrator is used to integrate an accurate voltage reference for a fixed period of time. The same integrator is then used to integrate with the reverse slope, the input voltage, and the time required to return to the starting voltage is measured.

It does not matter which of the two integrations occurs first, and for ease of understanding. the case where the unknown is used to integrate first and then the reference will be considered.

The output of an integrator shown in Fig. 6-17(a) is

$$V_{out} = -\frac{V_x t}{RC} \qquad (6\text{-}2)$$

where V_x = steady input voltage relative to ground

V_{out} = the output voltage from the integrator

R, C = integrator time-constant components

t = elapsed time from when the integration began

Equation (6-2) also assumes that the integrator capacitor started with no charge and thus the output of the integrator started at zero volts.

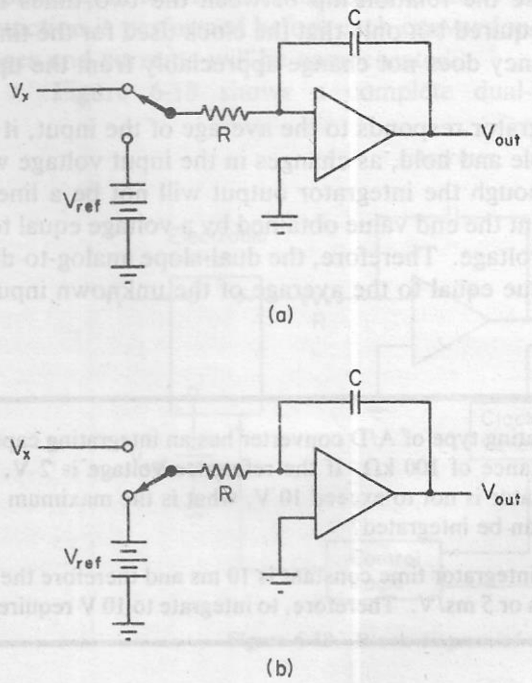


Figure 6-17 Functional schematic of the integrator of an integrating DVM.

If the integration were allowed to continue for a fixed period of time T_1, the output voltage would be

$$V_1 = -\frac{V_x}{RC}T_1 \qquad (6\text{-}3)$$

Notice that the integrator output has gone in the opposite polarity as the input. That is, a positive input voltage produces a negative integrator output.

If a reference voltage, 141, were substituted for the input voltage V_x, as shown in Fig. 6-17(b), the integrator would begin to ramp toward zero at a rate of V_{ref}/RC assuming that the reference voltage was of the opposite polarity as the unknown input voltage. The integrator for this situation does not start at zero but at an output voltage of V_1 and the output voltage can be represented as

$$V_{out} = V_1 + \frac{V_{ref}}{RC} t \qquad (6\text{-}4)$$

Notice that the second term in Eq. (6-4) has a negative sign due to its polarity. Setting the output voltage of the integrator to zero and solving for V_x yields

$$V_x = \frac{T_x}{T_1} V_{ref} \qquad (6\text{-}5)$$

where T_x is the time required to ramp down from the output level of V_1 to zero volts.

Notice that the relationship between the reference voltage and the input voltage does not include R or C of the integrator but only the relationship between the two times. Because the relationship between the two times is a ratio, an accurate clock is not required but only that the clock used for the timing be stable enough that the frequency does not change appreciably from the up ramp to the down ramp.

Because the integrator responds to the average of the input, it is not necessary to provide a sample and hold, as changes in the input voltage will not cause significant errors. Although the integrator output will not be a linear ramp. the integration will represent the end value obtained by a voltage equal to the average of the unknown input voltage. Therefore, the dual-slope analog-to-digital conversion will produce a value equal to the average of the unknown input.

EXAMPLE 6-3

A dual-slope integrating type of A/D converter has an integrating capacitor of 0.1 μF and a resistance of 100 kΩ. If the reference voltage is 2 V, and the output of the integrator is not to exceed 10 V, what is the maximum time the reference voltage can be integrated?

SOLUTION The integrator time constant is 10 ms and therefore the integrator output is 200 V/s or 5 ms/V. Therefore, to integrate to 10 V requires 50 ms.

The dual-slope type of A/D conversion is a very popular method for digital voltmeter applications. When compared to other types of analog-to-digital conversion techniques, the dual-slope method is slow but is quite adequate for a digital voltmeter used for laboratory measurements. For data acquisition applications, where a number of measurements are required, faster techniques are recommended. Many refinements have been made to the technique and many large- scale-integration (LSI) chips are available to simplify the construction of DVMs.

When a dual-slope AID converter is used for a DVM the counters may be decade rather than binary and the segment and digit drivers may be contained in the chip. (Multiplexed counter displays are discussed in Chapter 10.) When the converter is to interface to a microprocessor, and many high-performance DVMs use microprocessors for data manipulation, the counters employed are binary.

One significant enhancement made to the dual-slope converter is automatic zero correction. As with any analog system, amplifier offset voltages, offset currents, and bias currents can cause errors. In addition, in the dual-slope AID converter, the leakage current of the capacitor can cause errors in the integration and consequentially, an error. These effects, in the dual-slope

AID converter, will manifest themselves as a reading of the DVM when no input voltage is present. Figure 6-17 shows a method of counteracting these effects. The input to the converter is grounded and a capacitor, the auto zero capacitor, is connected via an electronic switch to the output of the integrator. The feedback of the circuitry is such that the voltage at the integrator output is zero. This effectively places an equivalent offset voltage on the automatic zero capacitor so that there is no integration. When the conversion is made, this offset voltage is present to counteract the effects of the input circuitry offset voltages. This automatic zero function is performed before each conversion, so that changes in the offset voltages and currents will be compensated.

Figure 6-18 shows a complete dual-slope AID converter. Electronic

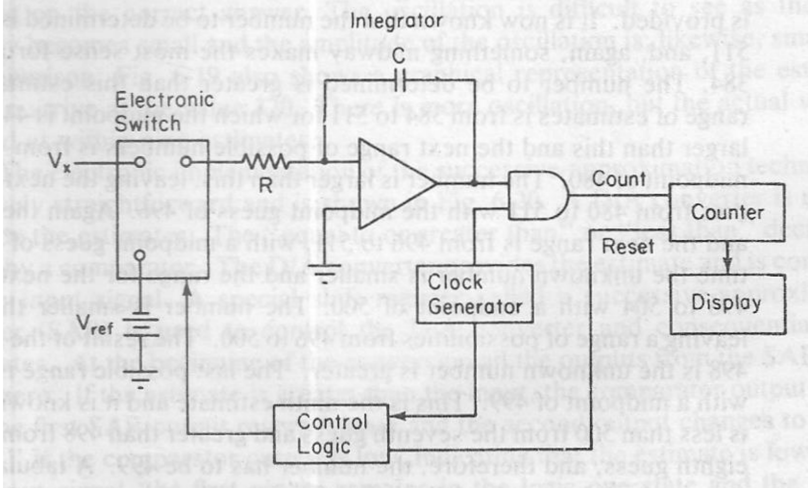


Figure 6-18 Block diagram of a dual-slope DVM.

switches, usually PET switches, are used to switch the input of the integrator alternately between the reference voltage and the unknown. Another pair of switches apply the integrator output to the automatic zero capacitor and ground the input for the automatic zero function.

All of the switch timing and the counting of the clock pulses to determine the unknown voltage are under control of the control logic. The output is made available to the external electronics after the conversion is complete.

If, in this example, the reference voltage were 1.000 V and the integrator were allowed to integrate the reference for 1,000 counts, the display would read 1V full scale with a resolution of 1 mV.

The actual frequency of the clock is not critical, as previously explained, but has an effect on the speed of the conversion. As an example, a l0-kHz clock would allow a maximum conversion time of 0.2 s for the example described above.

6-7.4 Successive-Approximation Conversion

A very effective and relatively inexpensive method of analog-to-digital conversion is the method of successive approximation. This is an electronic implementation of a technique called binary regression.

Assume that one is to determine the value of a number and is allowed to make estimates. Each estimate would be evaluated and it would be known if the estimate was (1) equal 10 or less than or (2) greater than the number to be determined. The maximum and minimum value of the possible number is also known.

Consider, as an example, that a number to he determined is between 0 and 511. The best first guess would be some number midway between the extremes and, ideally, 256. To further the example. assume that the number to be determined is 499. The number is greater than the estimate of 256 and this information is provided. It is now known that the number to be determined is between 256 and 511, and, again, something midway makes the most sense for a guess, which is 384. The number to be determined is greater than this estimate, and the next range of estimates is from 384 to 511 for which the midpoint is 448. The number is larger than this and the next range of possible numbers is from 448 to 511, with a midpoint of480. The number is larger than this, leaving the next range of possibilities from 480 to 511 with the midpoint guess of 496. Again the number is larger and the next range is from 496 to 511, with a midpoint guess of 504. For the first time the unknown number is smaller and the range for (he next estimate is from 496 to 504 with a midpoint of 500. The number is smaller than this estimate, leaving a range of possibilities from 496 to 500. The result of the midpoint guess of 498 is the unknown number is greater. The last possible range is from 498 to 500. With a midpoint of 499. This is the ninth estimate and it is known that the number is less than 500 from the seventh guess and greater than 498 from the results of the eighth guess, and therefore, the number has to be 499. A tabular synopsis of the guesses and the results follows.

Estimate	Result
256	Equal to or less than
256 + 128 = 384	Equal to or less than
384 + 64 = 448	Equal to or less than
448 + 32 = 480	Equal to or less than
480 + 16 = 496	Equal to or less than
496 + 8 = 504	Greater than
496 + 4 = 500	Greater than
496 + 2 = 498	Equal to or less than
498 + 1 = 499	Correct

There are some interesting observations to be made from the tabulation. First, there were eight estimates set forth when the actual answer was known, After the eighth estimate the actual value was known to lie between 598 and 500, which is knowing the answer to an 8-bit accuracy plus or minus one bit.

Can all numbers between 0 and 512 be determined in eight guesses or less using this method? To determine the answer to this question, consider the following. The first estimate cannot be in error by more than 256. The second estimate cannot be in error by more than 128, the third by more than 64, and so on. A total of nine estimates are required to produce the final estimate, which is in error by no more than I, which is the minimum possible error. Numbers from 0 to 511 can be represented by 9 binary bits. It is clear that this analysis can be extended to any number of binary bits, and the number of estimates required is exactly equal to the number of bits required of the analog-to-digital conversion.

A graphical representation of the estimates of the successive approximation conversion illustrates the converging nature of this technique. Figure 6-19 shows a graphical representation of the example of 499. As can be seen, the estimates approach the value from below, oscillating around the desired answer before settling on the correct answer. The oscillation is difficult to see as the error quickly becomes small and the amplitude of the oscillation is, likewise, small. As a comparison, Fig. 6-19 also shows a graphical representation of the estimates used to arrive at the value 320. There is more oscillation, but the actual value is arrived at within nine estimates.

The electronic implementation of the successive-approximation technique is relatively straightforward and is shown in Fig. 6-20. A D/A converter is used to provide the estimates. The “equal to or greater than” or “less than” decision is made by a comparator. The DIA converter provides the estimate and is compared to the input signal. A special shift register called a successive-approximation register (SAR) is used to control the D/A converter and consequentially the estimates. At the beginning of the conversion all the outputs from the SAR are at logic zero. If the estimate is greater than the input, the comparator output is high and the first SAR output reverses state and the second output changes to a logic "one." If the comparator output is low, indicating that the estimate is lower than the input signal, the first output remains in the logic one state and the second

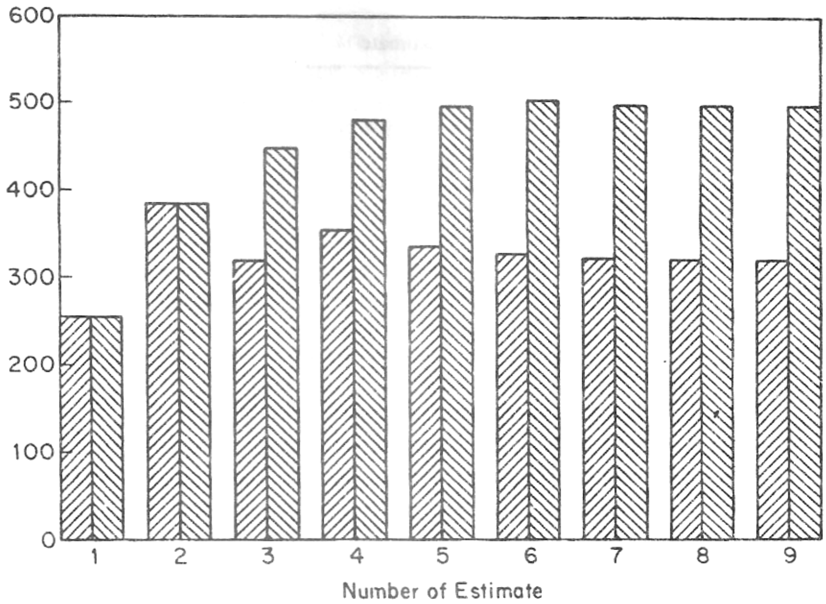


Figure 6-19 Graphical display of the values of estimates from a successive approximation type DVM.

output assumes the logic one state. This continues to all the states until the conversion is complete.

This sequence of events performs, electronically, the same estimating procedure that was outlined previously. An estimate is made on the edge of the SAR clock. For an N-bit conversion after N clocks, the actual value of the input is known. The least significant bit is the state of the comparator. In some systems

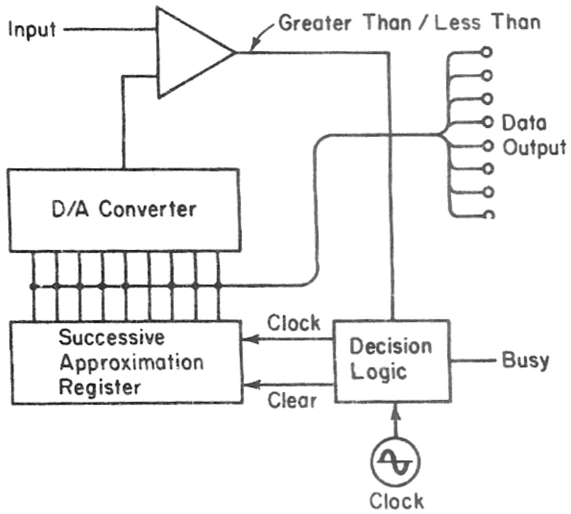


Figure 6-20 Block diagram of a successive-approximation DVM.

an additional clock is used to store the last bit in the SAR and thus $N + 1$ clocks are required for a conversion.

6-7.5 Quantizing Error

An electrical parameter, whether it is voltage, current, power, or something else, can assume any value within the possible range of that parameter. When the quantity is converted to a digital form, there are only a finite number of values that the quantity may assume. As an example, if a digital number consists of four bits, which has 16 different combinations, there are 16 levels that the analog quantity can be described.

Consider a voltage range of zero to 15 V to be digitized to a 4-bit number. There is a binary number for each volt of that range. What if the actual analog value is between the quantizing levels such as 2.25 V? The digitizing can produce a value equal to 2 V, which is represented as 0010, or 3 V, represented as 0011. The solution is simple. Round off the number to 2.0 and accept the digitized value of 0010. There is an error, however. The difference between the actual value of the analog quantity and the digitized value is 0.25. The number of bits of the quantized number could be increased by two, to 6 bits, and the number could be represented as 0010.01, which is exact with no error. What if the actual analog value were 2.27. Now the closest possible values of the binary number would be 2.25 or 2.50. Clearly, the closest is 2.25, which expresses the analog quantity with an error of only 0.02.

It would be clear, regardless of how many bits are used to express an analog quantity, that there is always a possibility of error when the quantities to be digitized fall between the exact digital values. The maximum error is equal to plus or minus one-half of the value of the least significant bit, which is called the quantizing error.

Older analog meters that used a meter scale as an indicator device required some sort of ranging circuitry so that the meter could be used over a large range of input parameters. As an example, if the full scale of a meter is 1,000 V, it would be nearly impossible to see the effects of a 1-V input. Therefore, a switched attenuator was used in the meter to provide a 1,000-, 100-, 10- and 1-V full-scale reading so that the desired meter deflections could be easily readable.

In the case of a digital meter, if a four-digit meter had a full-scale reading of 999.9 V, a 1-V reading would appear as 001.0. This represents two significant digits for the 1W reading. The meter, however, is a four-digit meter, and 99 percent of the meter capability is not used when reading I V. This is based on the fact that a four-digit meter can resolve 1 part in 10,000, while the two significant digits reflected by the 001.0-V display would represent 1 part in 100 or only 1 percent of 1 part in 10,000,. A switchable attenuator in the digital meter would achieve a similar effect. If an attenuator were used so that the full-scale readings of the digital meter were 999.9, 99.99, 9.999, and 0.9999 V, the 1-V reading would be 1.000, which is four significant digits and does not waste any of the capability of the meter.

Modern digital meters are capable of electronically switching the input attenuator, which makes the meter fully automatic. The electronics must determine if the present reading is less than the next-lower range of higher than full scale.

If the present reading is less than the full scale of the next-lower range, the attenuation is reduced. The attenuation continues to be reduced until the reading is between the next lower range and the full scale of the present range.

An opposite scenario takes place when the present reading is more than full scale. In this case attenuation is increased until the present reading is less than full scale.

As an example, assume that the 1-V of the previous example was measured using a digital voltmeter that was presently on the 999.9-V range. The attenuator is in decade steps, which is 999.9, 99.99, 9.999, and 0.9999 V full scale. Since the reading is 001.0, this is less than the full scale of the next-lower range and the attenuation is decreased. This results in a reading of 01.00 V, which is still less than the full scale of the next-lower range and the attenuation is reduced automatically. The next reading is 1.000, which is greater than the next-lower-range full- scale reading of 0.9999 and the attenuation is reduced no further.

Because the variation en input voltage levels can be rather great, the input attenuator is often switched with relays rather than electronic switches. In addition, there are times when the input voltage is much greater than the full-scale reading and the input amplifiers and attenuators must be capable of withstanding a significant overload for a short period of time before the proper attenuation is found. This technique is called auto ranging.

A similar technique is used to counteract the effects of various offset voltages within a digital meter. The input of the meter is electronically switched to ground while the input is disconnected. The meter will now read the results of the offset voltages, leakage currents, or other effects.

This offset reading is compensated by either subtracting the offset from the display or by feeding an analog offset in the opposite polarity. The offset check performed at a regular rate to ensure that the change in offset is accounted for

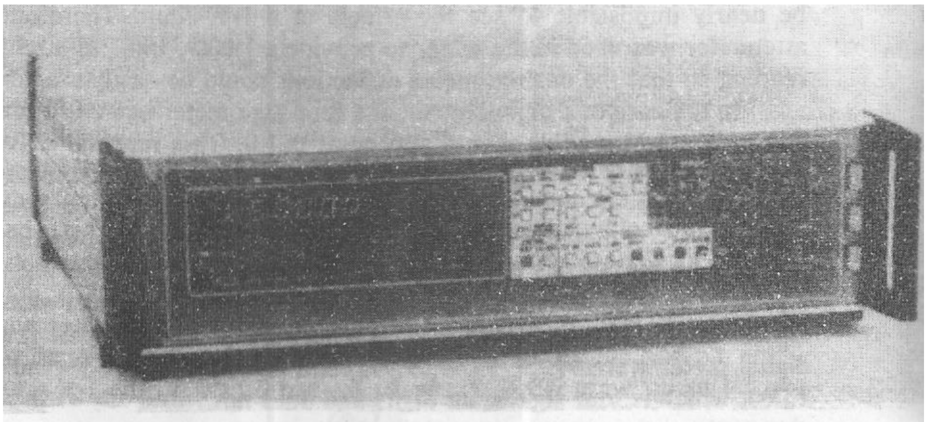


Figure 6-21 *Example of a high performance digital mutumeter.*
(Courtesy of John Fluke Mfg. Co., Inc.)

This technique is called auto zero and is necessary for instruments of high accuracy. Fig. 6-21 shows a high-performance auto-ranging multimeter with a true rms capability for ac measurements.

6-8 COMPONENT MEASURING INSTRUMENTS

Bridges for measuring component values of resistance, inductance, and capacitance were discussed in Chapter 5. Bridges are potentially very accurate and reliable for component measurements using measuring frequencies to the low megahertz region. They have some disadvantages in that they involve a variable inductor, resistor, or capacitor, depending on the type of bridge, and this usually involves an operator. This adjustment makes it difficult to automate or computerize the measurement since an actual mechanical movement is required. For manual measurements, this tends to slow down the measurements, but for computer interface, this tends to make the task nearly impossible.

6-8.1 All-Electronic Component Measurements

Chapter 5 discussed the Wheatstone bridge for resistance measurements, and the simple ohmmeter was discussed in Chapter 4. This is an example of a bridge and an all-electronic instrument for measuring resistance. (In the case of the moving- coil meter, the actual meter movement is mechanical, but this could be replaced with a digital readout, making the resistance measurement all-electronic.)

There are several methods of performing an all-electronic inductance or capacitance measurement where the measurement is not performed by a comparison, as is the case with a bridge. Fig. 6-22 shows one possible method of measuring the value of a capacitor, where a voltage is applied to the capacitor and the current through the capacitor can be measured. The relationship between the current through a capacitor and the voltage applied to the capacitor is

$$I_C = \frac{V}{X_C} = V(2\pi f C) \qquad (6\text{-}6)$$

where V = applied voltage

f = applied frequency

C = capacitance

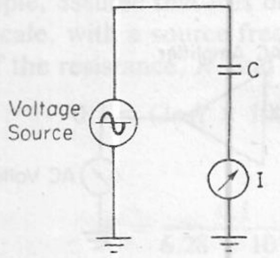


Figure 6-22 This circuit can measure the value of a capacitor by measuring the current through the capacitor with a known applied voltage.

The meter is simply calibrated in capacitance because of the linear relationship between the capacitance and the current. Although in theory this is a useful circuit, it is not practical because of the typical values of capacitors encountered in the electronics industry. Capacitors of a few picofarads are not unusual, and these capacitors typically could have working voltages of less than 25 V. RF current measuring devices, essentially thermocouple instruments, are not available for currents of less than a few hundred milliamperes, and thus the current expected must be greater than a few hundred milliamperes. If, as an example, a capacitor of 10 pF were to produce a current of 100 mA, with an applied voltage of 10 V rms, which would be safe for a 25-V capacitor, the frequency would have to be higher than 1,600 MHz. At this frequency. most capacitors have ceased to behave as capacitors and lead inductance, dissipation resistance, and other parasitic impedances will dominate the measurement. In addition, the accuracy of the measurement is dependent on the frequency of the generator, which would be difficult to control at 1,600 MHz. Therefore, smaller currents must be used for capacitance measurements.

An alternative method is shown in Fig. 6-23. In this example the current through the capacitor is sampled across a known resistance and the resultant voltage is amplified and measured. The amplifier provides the necessary gain so that the current through the capacitor can be quite small and within practicality. The voltage across the resistor can be expressed as

$$V = \frac{RV_{in}}{\sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}} \tag{6-7}$$

where R = resistance

V_{in} = generator voltage

V = voltage across the resistor

C = capacitance of the unknown capacitor

f = frequency of the generator.

If V_{in}, f and R are kept constant, the voltage V is a function of the unknown capacitance. The scale would have to be calibrated in a nonlinear fashion because of the relationship of Eq. (6-7). An applied frequency of a few megahertz can provide a practical system using this technique. The actual movement of the meter depends on not only the constants mentioned above, but on the gain of the

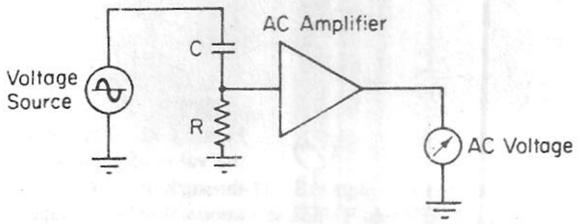


Figure 6-23 Measuring the current through a capacitor using a sampling resistor and an amplifier.

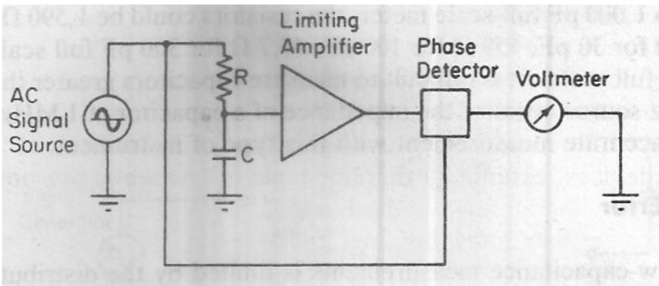


Figure 6-24 Capacitance measuring meter using the phase shift characteristics of *RC* circuit.

amplifier. It can be difficult to maintain a constant gain in an amplifier at several megahertz, especially for the large dynamic range encountered while measuring capacitance using this system. An alternative approach is shown in Fig. 6-24. In this example the phase angle between the applied voltage and the voltage across the capacitor is measured. An amplifier is used in this scheme except that the gain of the amplifier is not a factor in the measurement. Typically, a limiting amplifier such as that found in an FM receiver would be used. The phase angle can be expressed as

$$\theta = \tan^{-1}\frac{R}{X_C} = \tan^{-1}(2\pi fRC) \qquad (6\text{-}8)$$

The angle, θ, will be read by the meter in this circuit and the meter can be calibrated in capacitance since this angle is a function of the unknown capacitance. This would result in a nonlinear but useful display.

Using the Taylor expansion, the expression for the angle 0 can be rewritten:

$$\theta = \tan^{-1}(2\pi fRC) = (2\pi fRC) - \frac{1}{3}(2\pi fRC)^3 + \frac{1}{5}(2\pi fRC)^2 \;\ldots.. \qquad (6\text{-}9)$$

As can be seen from the Taylor expansion, the value of the arctangent will approach the angle, in radians, if the value of (2πfRC) is small. To gain an idea of how small the arctangent must be so that just one term of the Taylor expansion may be used, that is, the first term, consider an arctangent of less than 0.1. The value of the Taylor expansion using the first term only is, of course, 0.1. The actual value of the arctangent is 0.0996687, which is only 0.3% less than the actual angle, in radians. If the meter in this technique were calibrated directly in capacitance and the phase angle were restricted to less than 0.1 rad, the error due to this approximation would not exceed 0.3%. Therefore, 0 = (2πfRC) for less than 0.1.

The capacitance meter based on the circuit of Fig. 6-24 could be configured to cover several ranges by changing the value of fl, such that the full-scale reading is 0.1 rad. As an example, assume that it is desired that the lowest range cover from 0 to 100 pF full scale, with a source frequency of 1

MHz. Therefore, at I MHz the phase shift of the resistance, R, and 100 pF must be 0.1 rad or

$$0.1 = (2\pi R \times 100 \text{ pF}) \quad (6\text{-}10)$$

Solving for R gives

$$R = \frac{0.1}{6.28 \times 10^{-10}} \quad (6\text{-}11)$$

To cover from 10 to 1,000 pF full-scale meter, the resistors could be 1,5908 for 10 pP full scale, 477 Ω for 30 pF, 159Ω for 100 pF, 417 0 for 300 pP full scale, and 15.9 Ω for 1,000 pF full scale. It is difficult to measure capacitors greater than 100 pF using the 1-MHz source because the impedance of a capacitor at 1 MHz is too low to achieve an accurate measurement with this type of instrument.

6-8.2 Sources of Error

The accuracy of low-capacitance measurements is limited by the distributed Capacitance of the measuring circuits. Figure 6-25 shows the basic measuring circuit with the parasitic capacitances added. The series resistance, R, has some series inductance and the input of the amplifier will have a certain amount of input capacitance. Primarily, the amplifier input capacitance will have the greatest effect on the accuracy of the measurement. It would be difficult to design an amplifier with an input capacitance low enough to allow measurements of capacitors below 10pF without some form of compensation.

Figure 6-26 shows a modified measuring circuit allowing the effects of the input capacitance of the amplifier to be nailed out. In this example, the resistor has been placed at the amplifier input, and the signal source is applied to a transformer to create an out-of-phase component. The effects of the input capacitance are nulled out by injecting some of the out-of-phase signal through a variable capacitor. Except for the trimming circuits, the operation of this capacitance measuring system is similar to the previous example.

Another source of error is the harmonic distortion of the signal source. The phase shift of the RC circuit, which is the heart of the capacitance measuring system, will satisfy the equations presented only if the signal source is a pure sine function without any harmonic distortion. For an accuracy of 0.3 percent, which was the theoretical limit for the linear approximation using the Taylor expansion. the harmonic content of the signal source must he better than 50dB down from the nominal level. A crystal oscillator is capable of supplying a signal purity of this magnitude only if the output is carefully coupled from the oscillator. In addition to the coupling point, the signal should be passed through a low-pass Jilter.

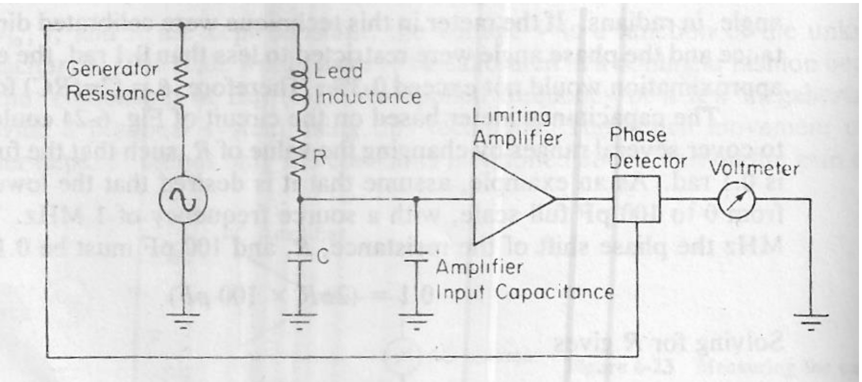


Figure 6-25 Capacitance measuring meter. showing the parasitic inductance, resistance. and capacitance.

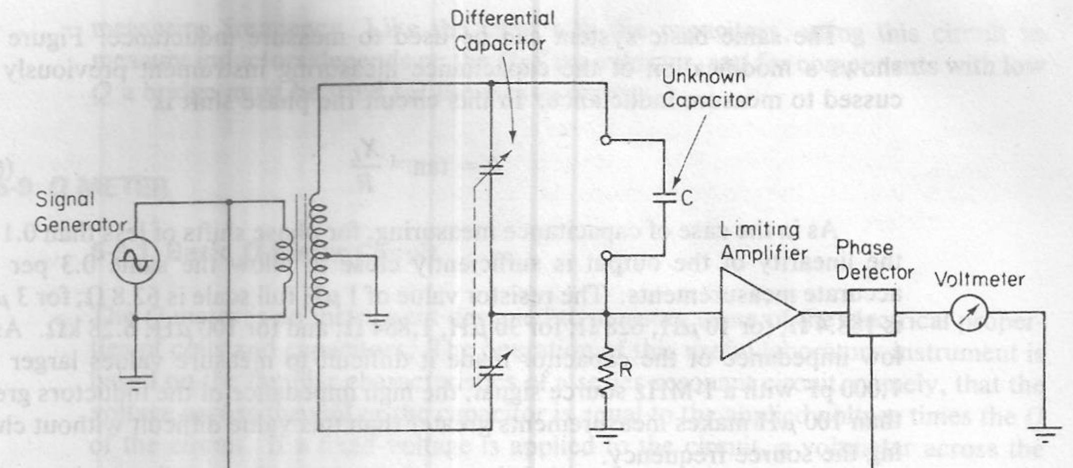


Figure 6-26 Nulling circuit for canceling the effects of the input stray capacitance of the amplifier.

By far the largest source of error is the equivalent series or parallel resistance. The series resistance called equivalent series resistance, or ESR, adds to the resistance of the circuit, but the phase measurement is made relative not to the capacitance but to the point where the ESR and the circuit resistance join, as shown in Fig. 6-27. this causes an error because the phase shift is not being measured accurately. Likewise, an equivalent parallel resistance, which is due to leakage resistance, will cause an erroneous reading because it changes the equivalent resistance as seen by the capacitor and hence changes the phase shift. This capacitance measuring method is not suitable for measuring capacitors with high dissipation factors or high ESR. Corrections can be made if the actual dissipation factor or ESR is known, but capacitance and dissipation factor can both be measured in a capacitance bridge. Generally, the quality of capacitors in the region of capacitance measured by this instrument is very good with insignificant ESR and dissipation factors, and the errors caused by these resistances are negligible.

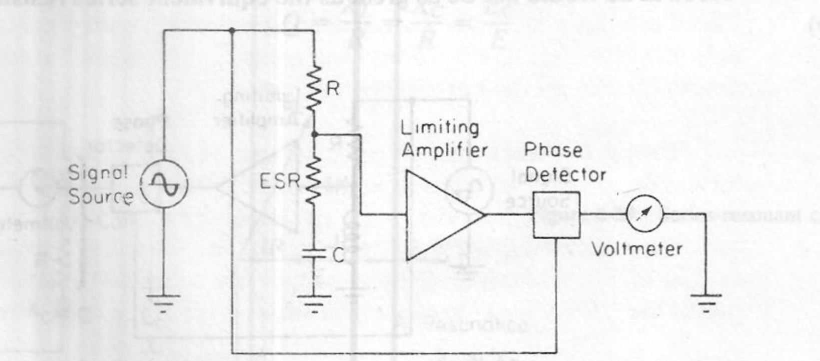


Figure 6-27 Effects of equivalent series resistance on capacitor measurement.

The same basic system can be used to measure inductance. Figure 6-28 shows a modification of the capacitance measuring instrument previously discussed to measure inductance. In this circuit the phase shift is

$$\theta = \tan^{-1} \frac{X_L}{R} \qquad (6\text{-}12)$$

As in the ease of capacitance measuring, for phase shifts of less than 0.1 rad, the linearity of the output is sufficiently close to allow the same 0.3 percent accurate measurements. The resistor value of 1 μH full scale is 62.8 Ω; for 3 μH it is 188.4Ω; for 10 μH, 628Ω; for 30 μH, 1.884Ω; and for 100 ΩH, 6.28 kΩ. As the low impedance of the capacitor made it difficult to measure values larger than 1,000 pF with a 1-MHz source signal, the high impedance of the inductors greater than 100 μH makes measurements greater than this value difficult without changing the source frequency.

One of the more important applications of the all-electronic inductance or capacitance measuring system is its ability to be interfaced with a computer. This would require that the output of the phase detector be digitized and made available to the computer. Although there are digital phase detectors that supply a digital representation of the phase angle to within 1 part in 10,000 for a source frequency as high as 1 MHz, these devices are not practical. For the typical computer application, the output of the phase detector would be digitized using an analog-to-digital converter.

There are sources of error in this system, just as in the capacitor measuring system described previously, and, as in the capacitor measurements, they are primarily due to resistance. The equivalent series resistance of an inductor is expressed indirectly as the inductor Q. Mathematically, $Q = X_L/R$,, where X_L is the capacitive reactance and R is the equivalent series resistance. From the equation it can be seen that, for smaller values of R, the Q or quality of an inductor goes up. It must be pointed out that the value of R is not that value of resistance that would be obtained if the inductor were measured with a do bridge or ohmmeter. The value of R is due to losses in the inductor's core material and the variation of resistance due to the skin effect. Therefore, the ohmic resistance measured at dc would not be as great as the equivalent series resistance at 1 MHz, the

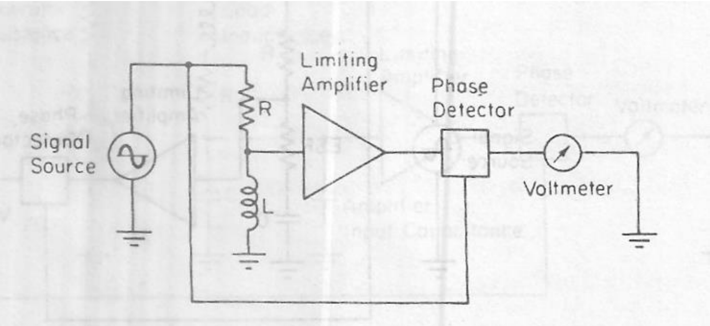


Figure 6-28 Modification of the capacitance measuring circuit described previously, allowing the measurement of inductance.

measuring frequency. Like the case with the capacitors, using this circuit to measure inductors depends on the *Q* of the inductor, and for components with low *Q* a bridge must be used for the measurement.

6-9 Q METER

6-9.1 Basic a-Meter Circuit

The *Q* meter is an instrument designed to measure some of the electrical properties of coils and capacitors. The operation of this useful laboratory instrument is based on the familiar characteristics of a series-resonant circuit, namely, that the voltage across the coil or the capacitor is equal to the applied voltage times the *Q* of the circuit. If a fixed voltage is applied to the circuit, a voltmeter across the capacitor can be calibrated to read *Q* directly.

The voltage and current relationships of a series-resonant circuit are shown in Fig. 6-29. At resonance, the following conditions are valid:

$X_c = X_L$

$E_c = IX_c = IX_L$

$E = IR$

where E = applied voltage

I = circuit current

E_c= voltage across the capacitor

X_c= capacitive reactance

X_L= inductive reactance

R = coil resistance

The magnification of the circuit, by definition is *Q*, where

$$Q = \frac{X_L}{R} = \frac{X_C}{R} = \frac{E_C}{E} \qquad (6\text{- }13)$$

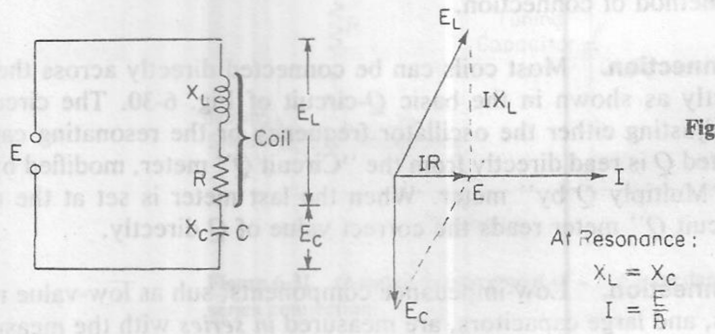


Figure 6-29 Series-resonant circuit.

Therefore if *E* is maintained at a constant and known level, a voltmeter connected across the capacitor can be calibrated directly in terms of the circuit *Q*.

A practical *Q*-meter circuit is shown in Fig. 6-30. The wide-range oscillator with a frequency range from 50 kHz to 50 MHz delivers current to a low-value shunt resistance RSH. The value of this shunt is very low, typically on the order of 0.02 0. It introduces almost no resistance into the oscillatory circuit and it therefore represents a voltage source of magnitude E with a very small (in most cases negligible) internal resistance. The voltage E across the shunt, corresponding to E in Fig. 6-29, is measured with a thermocouple meter, marked “Multiply *Q* by.” The voltage across the variable capacitor, corresponding to E_c in Fig. 629, is measured with an electronic voltmeter whose scale is calibrated directly in *Q* values.

To make a measurement, the unknown coil is connected to the test terminals of the instrument, and the circuit is tuned to resonance either by setting the oscillator to a given frequency and varying the internal resonating capacitor or by presetting the capacitor to a desired value and adjusting the frequency of the oscillator. The *Q* reading on the output meter must be multiplied by the index setting of the “Multiply *Q* by” meter to obtain the actual *Q* value.

The indicated *Q* (which is the resonant reading on the “Circuit *Q*” meter) is called the circuit *Q* because the losses of the resonating capacitor, voltmeter, and insertion resistor are all included in the measuring circuit. The effective *Q* of the measured coil will be somewhat greater than the indicated *Q*. This difference can generally be neglected, except in certain cases where the resistance of the coil is relatively small in comparison with the value of the insertion resistor. (This problem is discussed in Example 6-7.)

The inductance of the coil can be calculated from the known values of frequency (f) and resonating capacitance (C), since

$$X_L = X_c \text{ and } L = \frac{1}{(2\pi f)^2 C} \text{ henry} \qquad (6\text{-}14)$$

6-9.2 Measurement Methods

There are three methods for connecting unknown components to the test terminals of *Q* meter: direct, series, and parallel. The type of component and its size determine the method of connection.

Direct connection. Most coils can be connected directly across the terminals, exactly as shown in the basic Q-circuit of Fig. 6-30. The circuit resonated by adjusting either the oscillator frequency or the resonating capacitor. The indicated Q is read directly from the "Circuit Q" meter, modified by setting of the "Multiply Q by" meter. When the last meter is set at the mark, the "Circuit Q" meter reads the correct value of Q directly.

Series connection. Low-impedance components,-such as low-value resistors, small coils, and large capacitors, are measured in series with the measuring

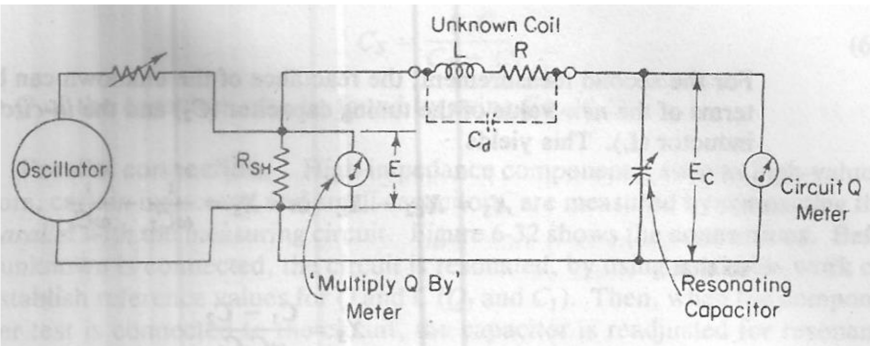


Figure 6-30 Basic Q-meter circuit.

circuit. Figure 6-31 shows the connections. The component lobe measured, here indicated by [Z], is placed in series with a stable work coil across the test terminals. (The work coil is usually supplied with the instrument.) Two measurements are made: In the first measurement the unknown is short-circuited by a small shorting strap and the circuit is resonated, establishing a reference condition. The values of the tuning capacitor (C_1) and the indicated Q (Q_1) are noted. In the second measurement the shorting strap is removed and the circuit is returned, giving a new value for the tuning capacitor (C_2) and a change in the Q value from Q_1 to Q_2.

For the reference condition,

$$X_{C1} = X_L \text{ or } \frac{1}{\omega C_1} = \omega L \qquad (6\text{-}15)$$

and neglecting the resistance of the measuring circuit,

$$Q_1 = \frac{\omega L}{R} = \frac{1}{\omega C_1 R} \qquad (6\text{-}16)$$

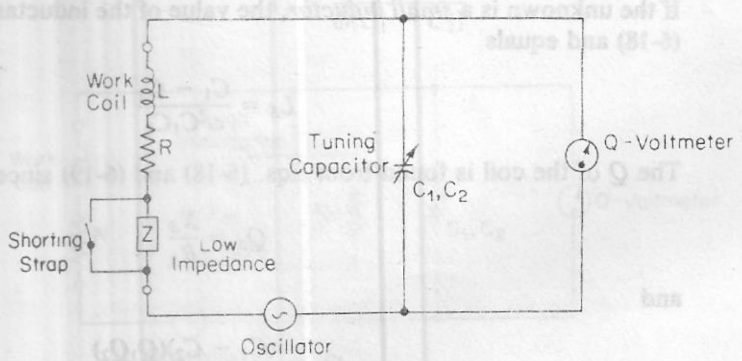


Figure 6-31 *Q*-meter measurement of a low-impedance component in the series connection.

For the second measurement, the reactance of the unknown can be expressed in terms of the new value of the tuning capacitor (C_2) and the in-circuit value of the inductor (L). This yields

$$X_s = X_{c2} - X_L \text{ or } X_s = \frac{1}{\omega C_2} - \frac{1}{\omega C_1} \qquad (6\text{-}17)$$

so that

$$X_s = \frac{C_1 - C_2}{\omega C_1 C_2} \qquad (6\text{-}18)$$

X_s is inductive if $C_1 > C_2$ and capacitive if $C_1 < C_2$. The resistive component of the unknown impedance can be found in terms of reactance Xs and the indicated values of circuit *Q*, since

$$R_1 = \frac{X_1}{Q_1} \qquad and \qquad R_2 = \frac{X_2}{Q_2}$$

Also,

$$R_s = R_2 - R_1 = \frac{1}{\omega C_2 Q_2} - \frac{1}{\omega C_1 Q_1}$$

so that

$$R_s = \frac{C_1 Q_1 - C_2 Q_2}{\omega C_1 C_2 Q_1 Q_2} \tag{6-19}$$

If the unknown is purely resistive, the setting of the tuning capacitor would not have changed in the measuring process, and $C_1 = C_2$. The equation for resistance reduces to

$$R_s = \frac{Q_1 - Q_2}{\omega C_1 Q_1 Q_2} = \frac{\Delta Q}{\omega C_1 Q_1 Q_2} \tag{6-12}$$

If the unknown is a small inductor, the value of the inductance is found from (6-18) and equals

$$L_s = \frac{C_1 - C_2}{\omega^2 C_1 C_2} \tag{6-21}$$

The *Q* of the coil is found from Eqs. (6-18) and (6-19) since, by definition,

$$Q_s = \frac{X_s}{R_s}$$

and

$$Q_s = \frac{(C_1 - C_2)(Q_1 Q_2)}{C_1 Q_1 - C_2 Q_2} \qquad (6\text{-}22)$$

If the unknown is a large capacitor, its value is determined from Eq. (6-18). and

$$Cs = \frac{C_1 C}{C_2 - C_1} \qquad (6\text{-}23)$$

The *Q* of the capacitor may be found by using Eq. (6-22).

Parallel Connection. High-impedance components, such as high-value resistors, certain inductors, and small capacitors, are measured by connecting them in parallel with the measuring circuit. Figure 6-32 shows the connections. Before the unknown is connected, the circuit is resonated, by using a suitable work coil, to establish reference values for Q and C (Q_1 and C_1). Then, when the component under test is connected to the circuit, the capacitor is readjusted for resonance, and a new value for the tuning capacitance (C_2) is obtained and a change in the value of circuit Q (ΔQ) from Q_1 to Q_2.

In a parallel circuit, computation of the unknown impedance is best approached in terms of its parallel components X_p and R_p, as indicated in Fig. 6-32. At the initial resonance condition, when the unknown is not yet connected into the circuit, the working coil (L) is tuned by the capacitor (C_1). Therefore

$$\omega L = \frac{1}{\omega C_1} \tag{6-24}$$

and

$$Q_1 = \frac{\omega L}{R} = \frac{1}{\omega C_1 R} \tag{6-25}$$

When the unknown impedance is not connected into the circuit and the capacitor is tuned for resonance, the reactance of the working coil (X_L) equals the parallel reactances of the tuning capacitor (X_{C_2}) and the unknown (X_p). Therefore

$$X_L = \frac{(X_{C_2})(X_P)}{X_{C_2} + X_P}$$

which reduces to

$$X_P = \frac{1}{\omega(C_1 - C_2)} \tag{6-26}$$

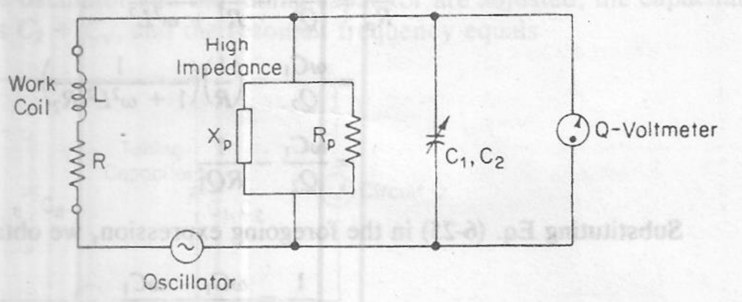


Figure 6-32 *Q*-meter measurement of a high-impedance component in the parallel connection.

If the unknown is inductive, $X_P = \omega L_P$, and Eq. (6-26) yields the value of the unknown impedance:

$$L_P = \frac{1}{\omega^2 (C_1 - C_2)} \qquad (6\text{-}27)$$

If the unknown is capacitive, $X_p = 1/\omega C_p$ and Eq. (6-26) yields the value of the unknown capacitor:

$$C_p = C_1 - C_2 \qquad (6\text{-}28)$$

In a parallel resonant circuit the total resistance at resonance is equal to the product of the circuit Q and the reactance of the coil. Therefore

$$R_T = Q_2X_L$$

or by substitution of Eq. (6-24),

$$R_T = Q_2X_{c1} = \frac{Q_2}{\omega C_1} \qquad (6\text{-}29)$$

The resistance (R_p) of the unknown impedance is most easily found by computing the conductance's in the circuit of Fig. 6-32. Let

G_T = total conductance of the resonant circuit

G_P = conductance of the unknown impedance

G_L = conductance of the working coil

Then

$$G_T = G_P + G_L \text{ or } G_P = G_T - G_L \qquad (6\text{-}30)$$

From Eq. (6-29),

$$G_T = \frac{1}{R_T} = \frac{\omega C_1}{Q_2}$$

Therefore

$$\frac{1}{R_p} = \frac{\omega C_1}{Q_2} - \frac{R}{R^2 + \omega^2 L^2}$$
$$= \frac{\omega C_1}{Q_2} - \left(\frac{1}{R}\right)\left(\frac{1}{1 + \omega^2 L^2/R_2}\right)$$
$$= \frac{\omega C_1}{Q_2} - \frac{1}{RQ_1^2}$$

Substituting Eq. (6-25) in the foregoing expression, we obtain

$$\frac{1}{R_p} = \frac{\omega C_1}{Q_2} - \frac{\omega C_1}{Q_1}$$

and after simplifying, we obtain

$$R_p = \frac{Q_1 Q_2}{\omega C_1 (Q_1 - Q_2)} = \frac{Q_1 Q_2}{\omega C_1 \Delta Q} \qquad (6\text{-}31)$$

The Q of the unknown is then found by using Eqs. (6-26) and (6-3 1) so that

$$Q_p = \frac{R_p}{X_p} = \frac{(C_1 - C_2)(Q_1 Q_2)}{C_1 (Q_1 - Q_2)} = \frac{(C_1 - C_2)(Q_1 Q_2)}{C_1 \Delta Q} \qquad (6\text{-}32)$$

6-9.3 Sources of Error

Probably the most important factor affecting measurement accuracy, and the most often overlooked, is the distributed capacitance or self-capacitance of the measuring circuit. The presence of distributed capacitance in a coil modifies the actual or effective Q and the inductance of the coil. At the frequency at which the self-capacitance and the inductance of the coil are resonant, the circuit exhibits a purely resistive impedance. This characteristic may be used for measuring the distributed capacitance.

One simple method of finding the distributed capacitance (C_d) of a coil involves making two measurements at different frequencies. The coil under test is connected directly to the test terminals of the Q meter, as shown in the circuit of Fig. 6-33. The tuning capacitor is set to a high value, preferably to its maximum position, and the circuit is resonated by adjusting the oscillator frequency. Resonance is indicated by maximum deflection on the "Circuit Q" meter. The values of the tuning capacitor (C_1) and the oscillator frequency (f_1) are noted. The frequency is then increased to twice its original value ($f_2 = 2f_1$) and the circuit is return by adjusting the resonating capacitor (C_2).

The resonant frequency of an LC circuit is given by the well-known equation

$$f = \frac{1}{2\pi\sqrt{LC}} \qquad (6\text{-}33)$$

At the initial resonance condition, the capacitance of the circuit equals $C_1 + C_d$, and the resonant frequency equals

$$f_1 = \frac{1}{2\pi\sqrt{L(C_1 + C_d)}} \qquad (6\text{-}34)$$

After the oscillator and the tuning capacitor are adjusted, the capacitance of the circuit is $C_2 + C_d$, and the resonant frequency equals

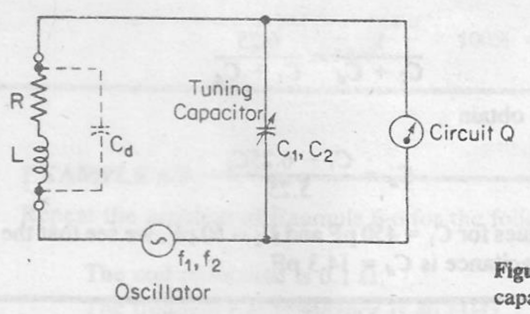


Figure 6-33 Determination of the distributed capacitance of an inductor.

$$f_2 = \frac{1}{2\pi\sqrt{L(C_2 + C_d)}}$$

Since $f_2 = 2f_1$, Eqs. (6-34) and (6-35) are related so that

$$\frac{1}{2\pi\sqrt{L(C_2 + C_d)}} = \frac{2}{2\pi\sqrt{L(C_2 + C_d)}}$$

and

Solving for the distributed capacitance yields

$$C_d = \frac{C_1 - 4C_2}{3} \qquad (6\text{-}36)$$

EXAMPLE 6-4

The self-capacitance of a coil is to be measured by using the procedure just outlined. The first measurement is at $f_1 = 2$ MHz and $C_1 = 460$ pF. The second measurement, at $f_2 = 4$ MHz, yields a new value of tuning capacitor, $C_2 = 100$ pF. Find the distributed capacitance, C_d.

SOLUTION Using Eq. (6-36), we obtain

$$C_d = \frac{C_1 - 4C_2}{3} = \frac{460 - 400}{3} = 20 \text{ pF}$$

EXAMPLE 6-5

Compute the value of self-capacitance of a coil when the following measurements are made: At frequency $f_1 = 2$ MHz, the tuning capacitor is set at 450 pF. When the frequency is increased to 5 MHz, the tuning capacitor is tuned at 60 pF.

SOLUTION Since $f_2 = 2.5 f_1$, Eqs. (6-34) and (6-35) are related as follows:

$$\frac{1}{2\pi\sqrt{L(C_2 + C_d)}} = \frac{2.5}{2\pi\sqrt{L(C_2 + C_d)}}$$

This reduces to

$$\frac{1}{C_2 + C_d} = \frac{6.25}{C_1 + C_d}$$

Solving for C_d, we obtain

$$C_d = \frac{C_1 - 6.25C_2}{5.25}$$

Substituting the values for $C_1 = 450$ pF and $C_2 = 60$ pF, we see that the value of the distributed capacitance is $C_d = 14.3$ pF.

The effective Q of a coil with distributed capacitance is less than the true Q by a factor that depends on the value of the self-capacitance and the resonating capacitor. It can be shown that

$$\text{true Q} = Q_e = \left(\frac{C + C_d}{C}\right) \qquad (6\text{-}37)$$

where Q_e = effective Q of the coil

C = resonating capacitance

C_d = distributed capacitance

The effective Q can usually be considered the indicated Q.

For many measurements, the residual or insertion resistance (R_{sH}) of the Q-meter circuit of Fig. 6-26 is sufficiently small to be considered negligible. Under certain circumstances, it can contribute an error to the measurement of Q. The effect of the insertion resistor on the measurement depends on the magnitude of the unknown impedance and, of course, on the size of the insertion resistor. For instance, the 0.02Ω of insertion resistance maybe neglected in comparison with a coil resistance of 10 Ω, but it assumes importance when compared to a coil resistance of 0.1 Ω. The effect of the 0.02-Ω insertion resistance is illustrated by Examples 6-6 and 6-7.

EXAMPLE 6-6

A coil with a resistance of 10 Ω is connected in the "direct-measurement" mode. Resonance occurs when the oscillator frequency is 1.0 MHz and the resonating capacitor is set at 65 pF. Calculate the percentage error introduced in the calculated value of Q by the 0.02-Ω insertion resistance.

SOLUTION The *effective* Q of the coil equals

$$Q_e = \frac{1}{\omega CR} = \frac{1}{(2\pi)(10^6)(65 \times 10^{-12})(10)} = 244.9$$

The *indicated* Q of the coil equals

$$Q_i = \frac{1}{\omega C(R + 0.02)} = 244.4$$

The percentage error is then

$$\frac{244.9 - 244.4}{244.9} \times 100\% = 0.2\%$$

EXAMPLE 6-7

Repeat the problem of Example 6-6 for the following conditions:

The coil resistance is 0.1 Ω.
The frequency at resonance is 40 MHz.
The tuning capacitor is set at 135 pF.

SOLUTION The *effective* Q of the coil is

$$Q_e = \frac{1}{\omega CR} = \frac{1}{2\pi \times 40 \times 10^6 \times 135 \times 10^{-12} \times 0.1} = 295$$

The *indicated* Q of the coil is

$$Q_i = \frac{1}{\omega C(R + 0.02)} = 246$$

The percentage error equals

$$\frac{295 - 246}{295} = 100\% = 17\%$$

Other sources of error include the residual inductance of the instrument, which is usually in the order of 0.015 μH and affects the measurement of only very small inductors (<0.5 μH). The conductance of the Q voltmeter has a slight shunting effect on the tuning capacitor at the higher frequencies, but this effect can usually be neglected.

6-10 VECTOR IMPEDANCE METER

Impedance measurements are concerned with both the magnitude (Z) and the phase angle (θ) of a component. At frequencies below 100 MHz, measurement of voltage and current is usually sufficient to determine the magnitude of the impedance. The phase difference between the voltage waveform and the current waveform indicates whether the component is inductive or capacitive. If the phase angle can be determined, for example, by using a CRO displaying a Lissajous pattern, the reactance can be calculated. If a component must be fully specified, its properties should be determined at several different frequencies, and many measurements may be required. Especially at the higher frequencies, these measurements become rather elaborate and time consuming, and many steps may be required to obtain the desired information.

The development of such instruments as the vector impedance meter makes impedance measurements over a wide frequency range possible. Sweep-frequency plots of impedance and phase angle versus frequency, providing complete coverage within the frequency band of interest, can also be mad.

The vector impedance meter, shown in Fig. 6-34, makes simultaneous measurements of impedance and phase angle over a frequency range of from 400 kHz to 110 MHz. The unknown component is simply connected across the input terminals of the instrument, the desired frequency is selected by turning the front panel controls, and the two front panel readouts indicate the magnitude of the impedance and the phase angle.

The operation of the vector impedance meter is best understood by referring

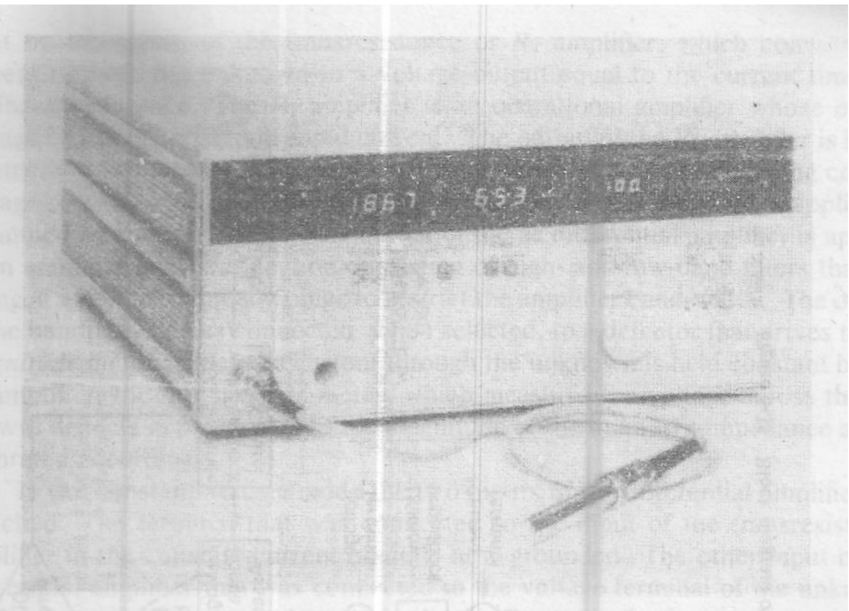


Figure 6-34 Vector impedance meter. (Courtesy of Hewlett-Packard Company.)

to the block diagram of Fig. 6-35 of a representative instrument, Two measurements take place: (1) The magnitude of the impedance is determined by measuring the current through the unknown component when a known voltage is applied across it, or by measuring the voltage across the component when a known current is passed through it; (2) the phase angle is found by determining the phase difference between the voltage across the component and the current through the component.

The block diagram of Fig. 6-35 shows that the instrument contains a signal source (Wien bridge oscillator) with two front panel controls to select the frequency range and to continuously adjust the selected frequency. The oscillator output is fed to an AGC amplifier which allows accurate gain adjustment by means of its feedback voltage. This gain adjustment is an internal control actuated by the setting of the impedance range switch, to which the AGC amplifier output is connected. The impedance range switch is a precision attenuator network controlling the oscillator output voltage and at the same time determining the manner in which the unknown component will be connected into the circuitry that follows the range switch.

The impedance range switch permits operation of the instrument in two modes: the constant-current mode and the constant-voltage mode. The three lower ranges (x1, x 10, and x 100) operate in the constant-current ‘mode and the four higher ranges (x 1k, x 10k, x 100k, and x 1M) operate in the constant-voltage’ mode.

In the constant-current mode the unknown component is connected across the input of the ac differential amplifier. The current supplied to the unknown depends on the setting of the impedance range switch. This current is held constant by the action of the trans-resistance or R_T amplifier, which converts the current through the unknown to a voltage output equal to the current times its feedback resistance. The R_T amplifier is an operational amplifier whose output voltage is proportional to its input current. The output of the R_1 amplifier is fed to a detector circuit and compared to a de reference voltage. The resulting control voltage regulates the gain of the AGC amplifier and hence the voltage applied to the impedance range switch. The output of the ac differential amplifier is applied to an amplifier and filter section consisting of high- and low-band filters that are changed with the frequency range to restrict the amplifier band-width. The output of the band pass filter is connected, when selected, to a detector that drives the Z-magnitude meter.

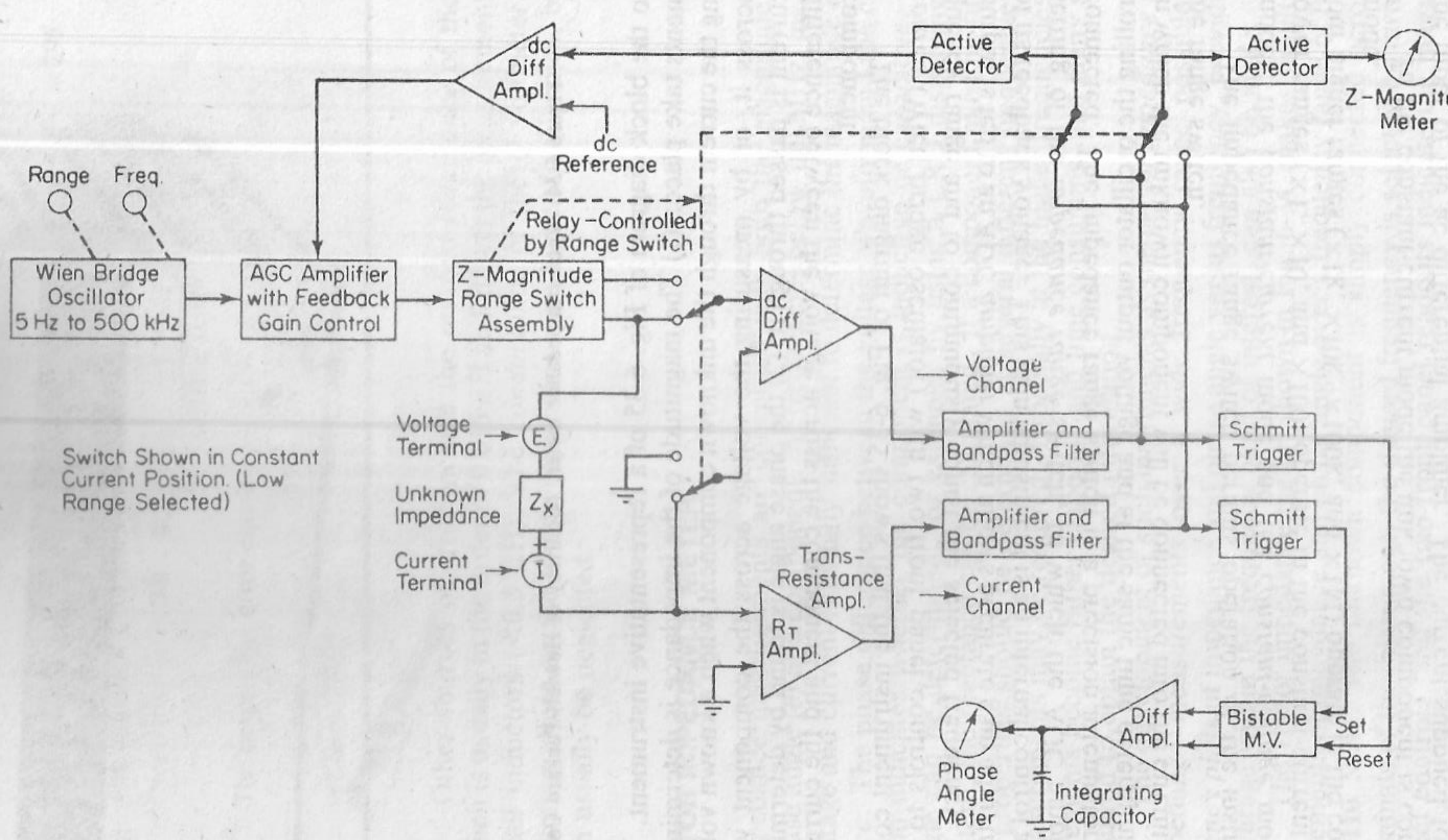


Figure 6-35 Block diagram of the vector impedance meter. (Courtesy of Hewlett-Packard Company.)

Since the current through the unknown is held constant by the R_T amplifier, the Z-magnitude meter, which measures the voltage across the unknown, deflects in proportion to the magnitude of the unknown impedance and is calibrated accordingly.

In the constant-voltage mode the two inputs to the differential amplifier are switched. The terminal that was connected to the input of the trans-resistance amplifier in the constant-current mode is now grounded. The other input of the differential amplifier that was connected to the voltage terminal of the unknown component is now connected to a point on the Z-magnitude range switch which is held at a constant potential. The voltage terminal of the unknown is connected to this same point of constant potential, or depending on the setting of the Z-magnitude range switch, to a decimal fraction of this voltage. In any case, the voltage across the unknown is held at a constant level. The current through the unknown is applied to the trans-resistance amplifier which again produces an output voltage proportional to its input current.

The roles of the ac differential amplifier and the trans-resistance amplifier are now reversed. The voltage output of the R_T amplifier is applied to the detector and then to the Z-magnitude meter. The output voltage of the differential amplifier controls the gain of the AGC amplifier in the same manner that the R_T amplifier did in the constant-current mode.

Phase-angle measurements are carried out simultaneously. The outputs of both the voltage channel and the current channel are amplified and each output is connected to a Schmitt trigger circuit. The Schmitt trigger circuits produce a positive-going spike every time the input sine wave goes through a zero crossing. These positive spikes are applied to a binary phase detector circuit. The phase detector consists of a bistable multivibrator, a differential amplifier, and an integrating capacitor. The positive-going pulse from the constant-current channel sets the multivibrator, and the pulse from the constant-voltage channel resets the multivibrator. The "set" time of the MV is therefore determined by the zero-crossings of the voltage and current waveforms. The "set" and "reset" outputs of the MV are applied to the differential amplifier, which applies the difference voltage to an integrating capacitor. The capacitor voltage is directly proportional to the zero-crossing time interval and is applied to the phase-angle meter which then indicates the phase difference, in degrees, between the voltage and current waveforms.

Calibration of the vector impedance meter is usually performed by connecting standard components to the input terminals. These components may be standard resistors or capacitors. An electronic counter is needed to accurately determine the period of the applied test frequency. When the value of the component under test and the frequency of the test signal are both known accurately, the impedance or reactance can be calculated and compared to the indication on the Z-magnitude meter. With a standard resistor connected to the input terminals, the phase-angle meter should read 0°.

6-11 VECTOR VOLTMETER

A vector voltmeter measures the amplitude of a signal at two points in a circuit and simultaneously measures the phase difference between the voltage waveforms at these two points. This instrument can be used in a wide variety of applications. especially in situations where other methods are very difficult or time consuming. The vector voltmeter is useful in VHF applications and can be used successfully in such measurements as:

a. Amplifier gain and phase shift

b. Complex insertion loss

c. Filter transfer functions

d. Two-port network parameters

The vector voltmeter basically converts two RF signals of the same fundamental frequency (from 1 MHz to GHz) to two IF signals with 20-KHz fundamental frequencies. These IF signals have the same amplitudes, waveforms, and phase relationships as the original RF signals. Consequently, the fundamental components of the IF signals have the same amplitude and phase relationships as the fundamental components of the RF signals. These fundamental components are filtered from the IF signals and are measured by a voltmeter and a phase meter.

The block diagram of Fig. 6-36 shows that the instrument consists of five major sections as follows: two RF-to-IF converters, an automatic phase control section, a phase meter circuit, and a voltmeter circuit. The RF-to-IF converters and the phase control section produce two 20-kllz sine waves with the same amplitudes and the same phase relationship as the fundamental components of the RF signals applied to channels A and B. The phase meter section continuously monitors these two 20-kHz sine waves and indicates the phase angle between them. The voltmeter section can be switched to channel A or channel B to provide a meter display of the amplitude.

Each RF-to-IF converter consists of a sampler and a tuned amplifier. The sampler produces a 20-kHz replica of the RF input waveform, and the tuned amplifier extracts the 20-kHz fundamental component from this waveform replica. Sampling is a time-stretching process, with which a high-frequency repetitive signal is duplicated at a much lower frequency. The process is illustrated in

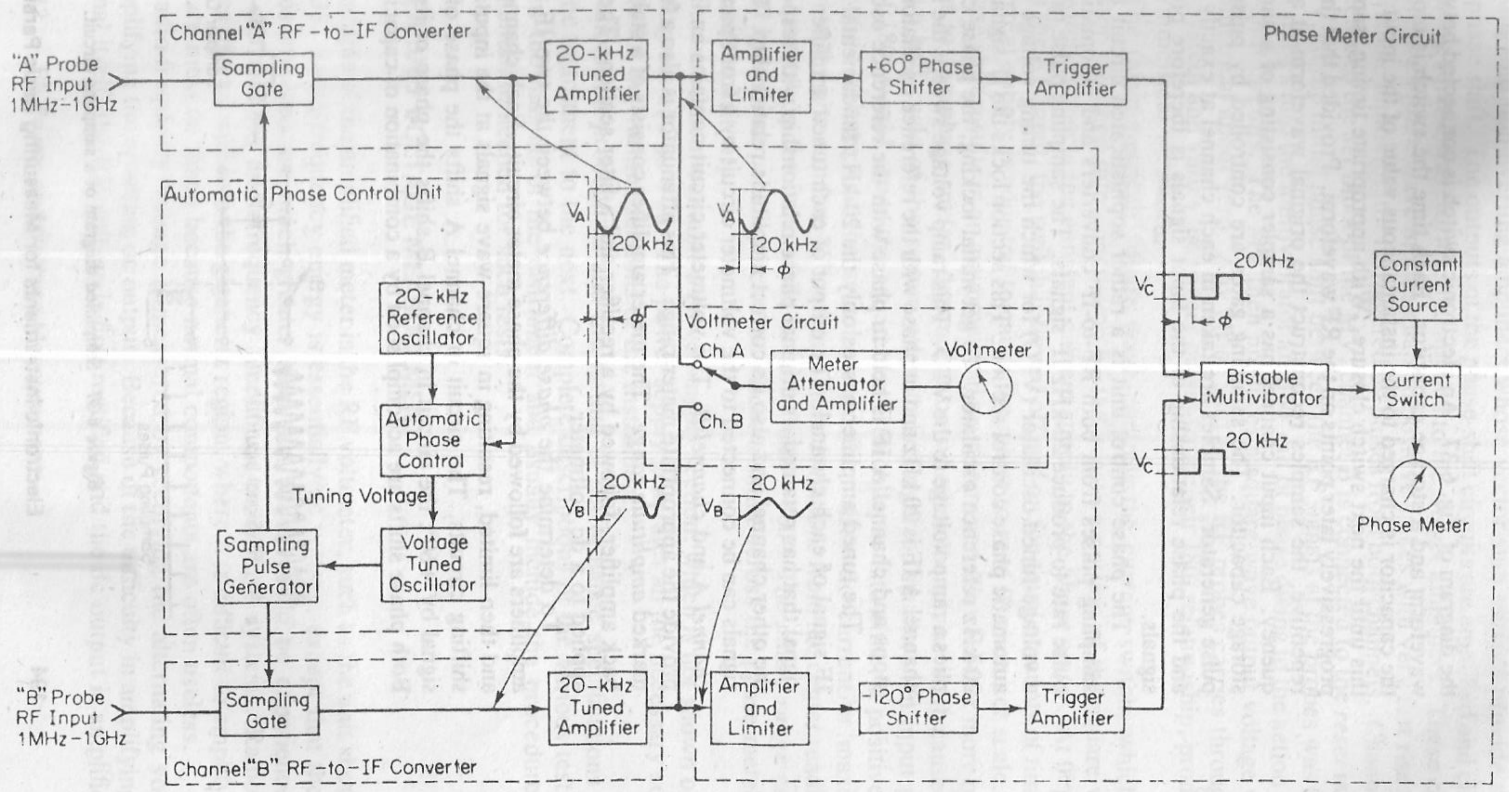


Figure 6-36 Block diagram of the vecto: voltmeter (Courtesy of Hewlett-Packard Company.)

the diagram of Fig. 6-37. An electronic switch is connected between the RF input waveform and a storage capacitor. Each time the switch is momentarily closed, the capacitor is charged to the instantaneous value of the input voltage and holds this until the next switch closure. With appropriate timing, samples are taken at progressively later points on the RF waveform. Provided that the RF waveform is repetitive, the samples reconstruct the original waveform at a much lower frequency. Each input channel has a sampler consisting of a sampling gate and a storage capacitor. The sampling gates are controlled by pulses from the same pulse generator. Samples are taken in each channel at exactly the same instant, and the phase relationship of the input signals is therefore preserved in the IF signals.

The phase control unit is a rather sophisticated circuit that generates the sampling pulses from both RF-to-IF converters and automatically controls the pulse rate to produce 20-kHz IF signals. The sampling pulse rate is controlled by a voltage-tuned oscillator (VTO) for which the tuning voltage is supplied by the automatic phase control section. This section locks the IF signal of channel A to a 20-kHz reference oscillator. To get initial locking, the phase control section applies a ramp voltage to the VTO. This ramp voltage sweeps the sampling rate until channel A IF is 20 kHz and in phase with the reference oscillator. Then the sweep stops and channel A IF is held in phase with the reference oscillator.

The tuned amplifier passes only the 20-kHz fundamental component of the IF signal of each channel. The output of each tuned amplifier then consists of a signal that has retained its original phase relationship with respect to the signal in the other channel and also its correct amplitude relationship. The two filtered IF signals can be connected to the voltmeter circuit by a front panel switch, marked channel A and channel B. The voltmeter circuit contains an input attenuator to provide the appropriate meter range. This attenuator is also a front panel control, marked amplitude range. The meter amplifier consists of a stable fixed-gain feedback amplifier, followed by a rectifier and a filter section. The rectified signal is applied to a dc voltmeter.

To determine the phase difference between the two IF signals, the tuned amplifiers are followed by the phase meter circuit. Each channel is first amplified and then limited, resulting in square-wave signals at the inputs to the IF phase- shifting circuits. The circuit in channel A shifts the phase of the square-wave signal by +600^{o}; the circuit in channel B shifts the phase of its signal by -120^{o}. Both phase shifts are accomplished by a combination of capacitive networks and

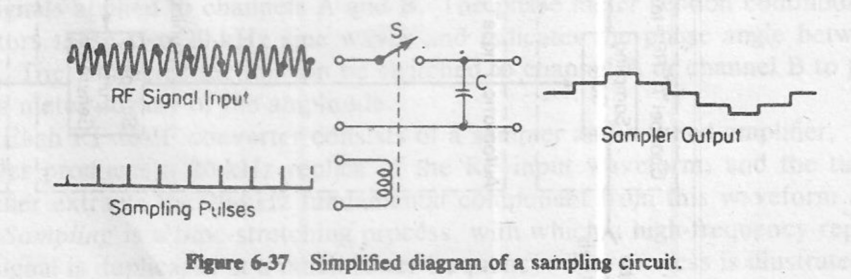


Figure 6-37 Simplified diagram of a sampling circuit.

inverting and non-inverting amplifiers whose vector-sum outputs provide the desired phase shift. The outputs of the phase-shift circuits are amplified and clipped. producing square waveforms, and applied to the trigger amplifiers.

These circuits convert the square-wave input signals to positive spikes with very fast rise times. The bistable multivibrator is triggered by pulses from both channels. Channel A is connected to the set input of the MV; channel B is connected to the reset input of the MV. If the initial phase shift between the RF signals at the probes was 0^o, the trigger pulses into the multivibrator are 180° out of phase owing to the action of the phase-shift circuits. The MV then produces a square-wave output voltage which is symmetrical about zero. Any phase shift at the RF probes carries through the entire system and varies the trigger pulses from their 1800 relationship, producing an asymmetrical waveform.

The (asymmetrical) square wave controls the current switch, which is a transistor switched into conduction by the negative portion of the square wave. The switch connects the constant current supply to the phase meter. At 0° phase shift at the RF input, the switch is turned off and on for equal amounts of time and the current supply is adjusted to cause the meter to read 0° or center scale. Any RF phase shift results in an asymmetrical waveform and allows either more or less current to the phase meter, depending on whether the phase shift caused the negative half-cycle of the square wave to be larger or smaller. An input phase shift of 180° would cause the square wave to collapse into either a positive or a negative dc voltage and the switch would then allow no current or maximum current to the phase meter. These maximum deviations from the center reading of 0° are marked on the meter face as + 180° and -180°. The phase range can be selected by a front panel switch that places a shunt across the phase meter and changes its sensitivity.

The instrument contains a power supply section, which is not shown on the block diagram of Fig. 6-36. The power supply generates all the necessary supply voltages for the various sections of the instrument.

Calibration procedures and the testing of performance specifications vary from one instrument to the next. Complete descriptions of the various tests are given in the manual of the instrument and usually include the procedure and instrumentation needed for such tests.

6-12 RF POWER AND VOLTAGE MEASUREMENT

One example of the amplified meter is the RF voltmeter, such as the unit shown in Fig. 6-38. Radio-frequency energy is essentially ac voltage, except that the frequencies involved are much greater than that which would be experienced in power distribution, audio frequency amplifiers, or control systems. Radio frequencies extend well into the gigahertz region, where it is difficult to amplify and great care must be taken because normal components are often useless.

Radio-frequency voltage is measured by rectifying the alternating voltage and amplifying the resulting dc output. Because of the difficulty in amplifying the RF signal itself, the RF voltage is first rectified and the dc output is amplified.

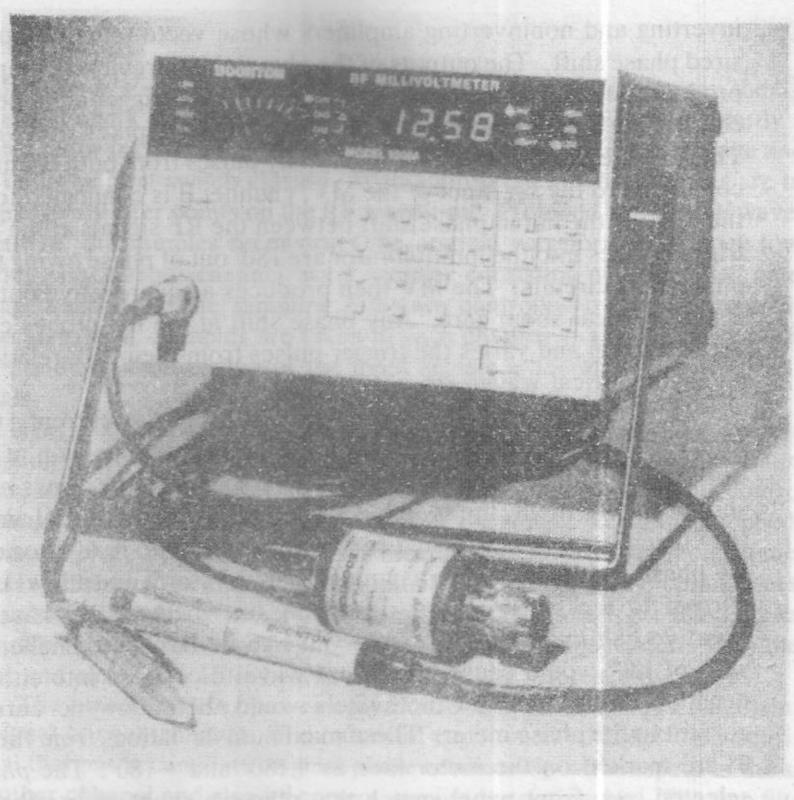


Figure 6-38 RF millivoltmeter for measuring RF power and voltage. (Courtesy of Boonton Electronics Corporation.)

The diodes used to rectify the RF waveform are not like the rectifiers used in a conventional ac meter, discussed in Chapter 4. The diodes used to rectify the RF signal are either Schottky barrier or point contact diodes. Conventional junction diodes with small geometries can be used for lower frequencies, but most detector diodes are not PN junction diodes. There are two significant problems with diodes used for RF rectification. First, most diodes have excessive capacitance for high-frequency RF rectification, and, second, most diodes have excessive reverse recovery time.

When diodes are operated at low forward-biased potentials, the rectified output does not equal the peak of the input. This means that for rather low amplitude RF voltages the resulting dc output is even lower, and a chopper-stabilized amplifier or other amplifier stabilized for dc drifts is required. Figure 6-39 shows a block diagram of a sensitive RF millivoltmeter. The actual RF rectifier or detector is usually mounted on a probe so that measurements can be made with the least amount-of interconnecting RF cable, as even the losses of coaxial cable can cause significant errors at very high frequencies. The detected output is in the very low millivolt region, and often even lower, and is amplified via a chopper-stabilized amplifier, digitized and displayed on a digital readout.

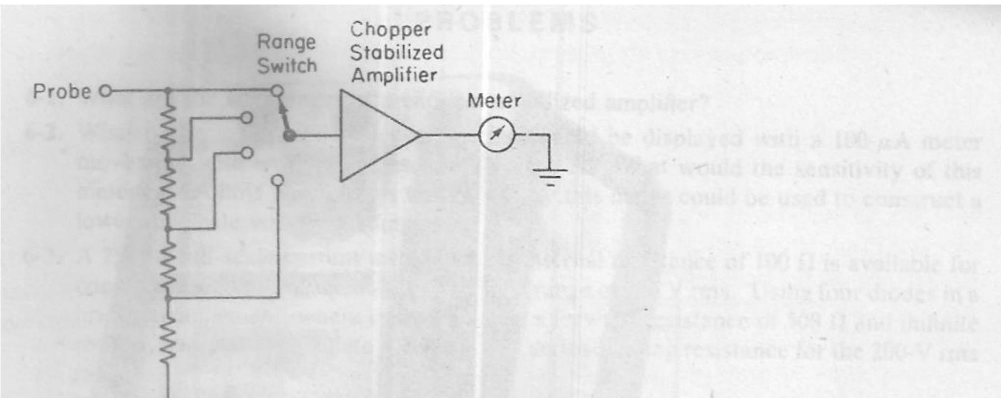


Figure 6-39 Block diagram of the RF millivoltmeter.

The type of measurement made by the RF millivoitmeter depends on the type of probe used. Voltage measurements are made with a probe similar to the one shown in Fig. 6-40(a). Voltage measurements are made with a relatively high impedance, but some capacitance is inescapable. This probe would be used within circuits where the impedances vary and the circuit cannot be isolated and terminated externally.

Many high-frequency circuits can be disconnected and terminated, usually in 50 Ω. externally, and the probe of Fig. 6-40(b) is used. This probe is more a power measuring probe rather than a voltage probe and can be used to measure powers to the nanowatt region. This power measurement is not a true rms measurement, and care must be taken in interpreting measurements, especially when the signal being measured has modulation applied.

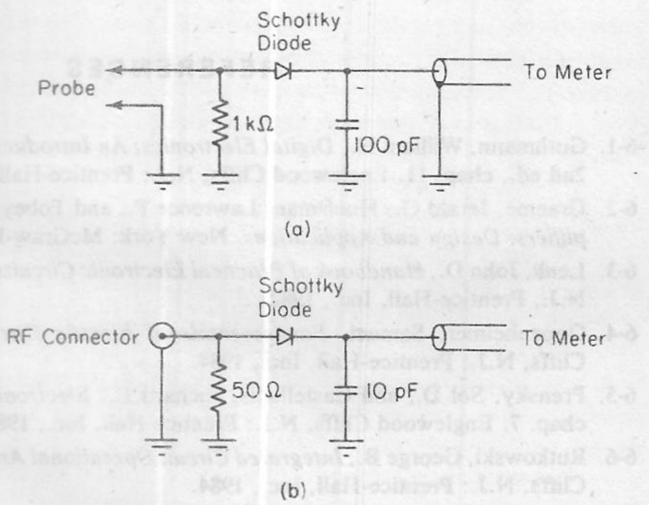


Figure 6-40 Two different RF probes for use with an RF millivoltmeter.

Figure 6-41 The Biddle Megger is a familiar test set for testing (Courtesy of Biddle Instruments.)

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PROBLEMS

6-1. What are the advantages of a chopper-stabilized amplifier?

6-2. What is the lowest voltage that could be displayed with a 100-μA meter movement with an resistance of 150 Ω? What would the sensitivity of this meter be in ohms per volt? Is there any way this meter could be used to construct a Lower full-scale voltage reading?

6-3. A 25-mA full-scale current meter with an internal resistance of 100 Ω is available for constructing an ac voltmeter with a voltage range of 200 V rms, Using four diodes in a bridge arrangement, where each diode has a forward resistance of 500 Ω and infinite reverse resistance, calculate the necessary series-limiting resistance for the 200-V rms range.

6-4. For measuring small values of capacitance, a 60-MHz signal source is to be used in a capacitance meter. What value of series resistance is required if the phase shift is to kept below 5.7 degrees for full-scale capacitance readings of 1, 10, and 100 pF?

6-5. What would a true-rms reading meter indicate if a pulse waveform of 5 V peak and a 25 percent duty cycle were applied? What would the meter indicate if a 5-V dc input were applied (assume the meter has dc capability)?

6-6. To check the distributed capacitance of a coil, the coil is resonated at 10 MHz with 120 pF and then is resonated at 15MHz with 40 pF. What is the inductance of the coil and what is the equivalent distributed capacitance?

6-7. A coil with a resistance of 3 Ω is connected to the terminals of the Q-meter of Fig. 6-34. Resonance occurs at an oscillator frequency of 5 MHz and resonating capacitance of 100 pF Calculate the percentage of error introduced by the insertion resistance, $R_{SH} = 0.1\Omega$.