Lecture No. 2

Electrical Measurement and Instrumentation

STATISTICAL ANALYSIS

- A statistical analysis of measurement data allows an **analytical determination of the uncertainty of the final results**.
- The outcome of a certain measurement method may be predicted on the basis of sample data without having detailed information on all the disturbing factors.

STATISTICAL ANALYSIS

- Large number of measurements is usually required.
- Systematic errors should be small compared with residual or random errors, because statistical treatment of data cannot remove a fixed bias contained in all the measurements.

STATISTICAL ANALYSIS

includes

- Arithmetic Mean
- Deviation from arithmetic mean
- Average Deviation
- Standard Deviation

Arithmetic Mean

- The most probable value of a measured variable is the arithmetic mean of the number of readings taken. The arithmetic mean is given by the following expression:

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 \ldots\ldots + x_n}{n} = \frac{\sum x}{n} \qquad (1\text{-}1)$$

Where $\bar{x}$ = arithmetic mean

x_1, x_2, x_n = readings taken

n = number of readings

Deviation from the Mean

- Deviation is the departure of a given reading from the arithmetic mean of the group of readings. If the deviation of the first reading, x_1, is called d_1, and that of the second reading, x_2, is called d_2, and so on, then the deviations from the mean can be expressed as

$$d_1 = x_1 - \bar{x} \qquad d_2 = x_2 - \bar{x} \qquad d_n = x_n - \bar{x} \qquad (1\text{-}2)$$

- Deviation may have a positive or a negative value
- **Algebraic sum of all the deviations must be zero**

Average Deviation

- The average deviation is an indication of the precision of the instruments used in making the measurements.
- Highly precise instruments will yield a low average deviation between readings.
- **Average deviation is the absolute values of the deviations divided by the number of readings**.
- The absolute value of the deviation is the value without respect to sign.

Average Deviation

- Average deviation may be expressed as

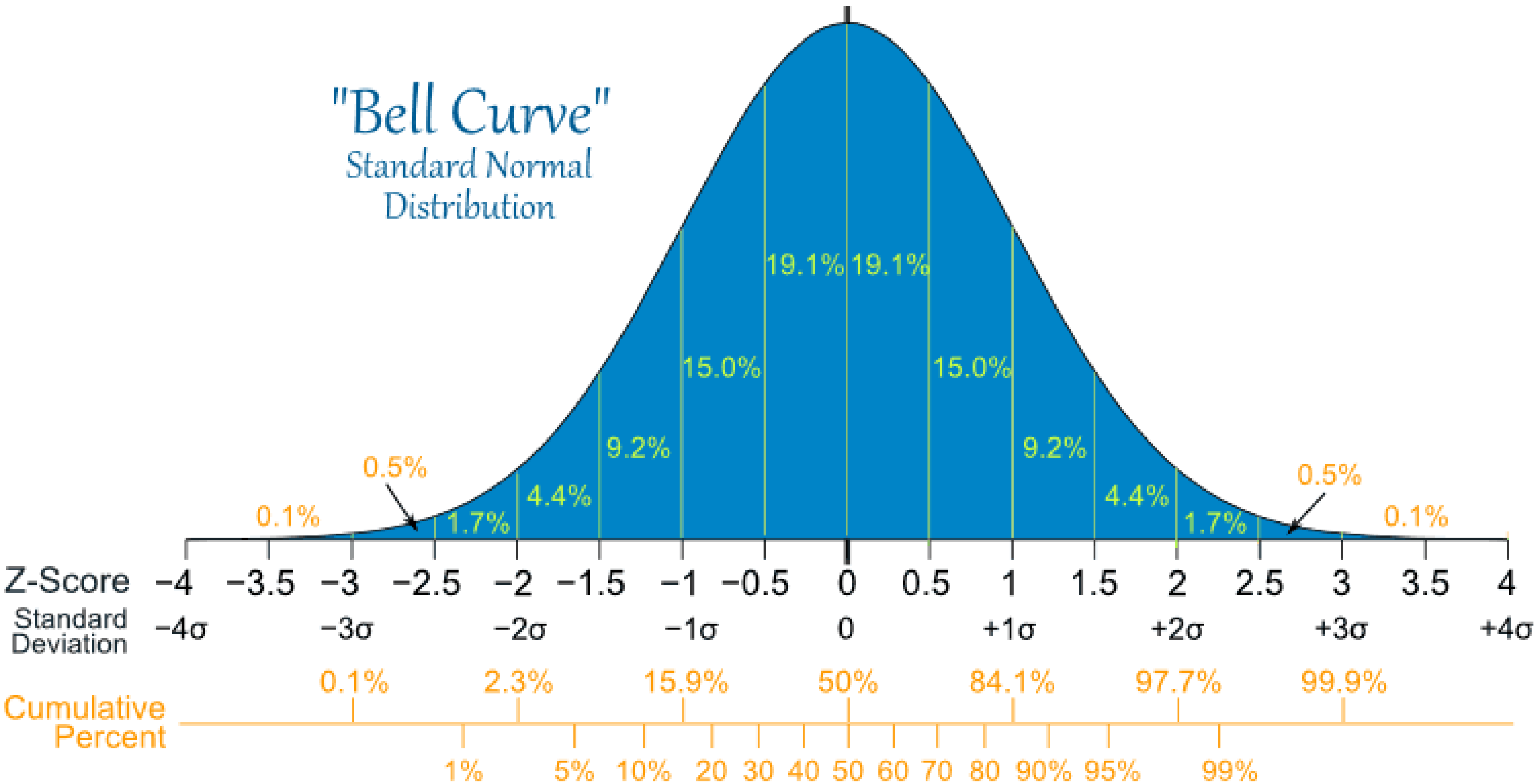
$$D = \frac{|d_1| + |d_2| + |d_3| + ...|d_n|}{n} = \frac{\sum |d|}{n} \qquad (1\text{-}3)$$

Standard Deviation

- **Root-mean-square** deviation or standard deviation of an *infinite number* of data is the square root of the sum of all the individual deviations squared, divided by the number of readings.
- Expressed mathematically as

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \ldots\ldots d_n^2}{n}} = \sqrt{\frac{\sum d_i^2}{n}} \qquad (1\text{-}4)$$

Standard Deviation



Standard Deviation

- For finite number of data/readings the mathematical expression is

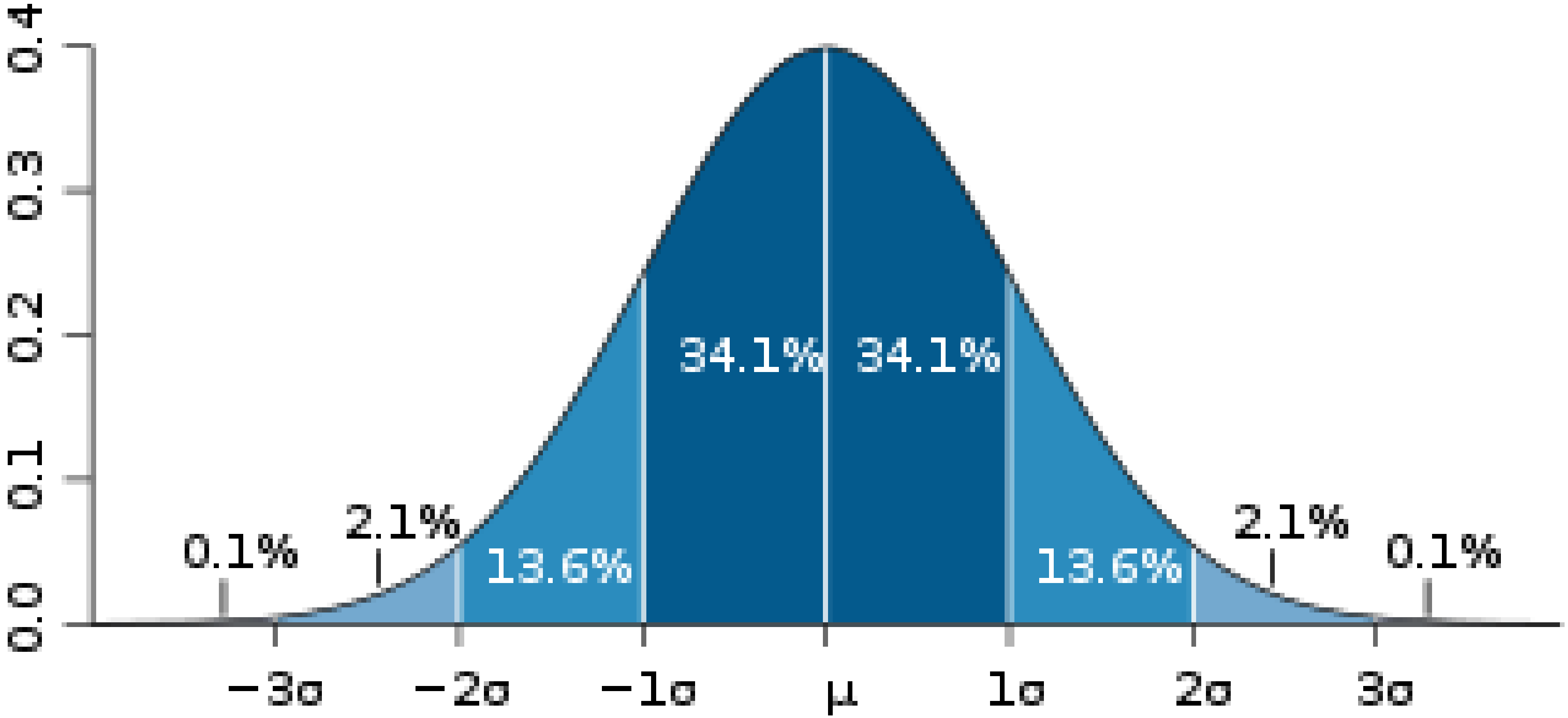
$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \ldots\ldots d_n^2}{n-1}} = \sqrt{\frac{\sum d_i^2}{n-1}} \qquad (1\text{-}5)$$

Another expression for essentially the same quantity is the variance or mean square deviation, which is the same as the standard deviation except that the square root is not extracted.

- Therefore

  Variance (V) = Mean Square Deviation = σ^2

Standard Deviation



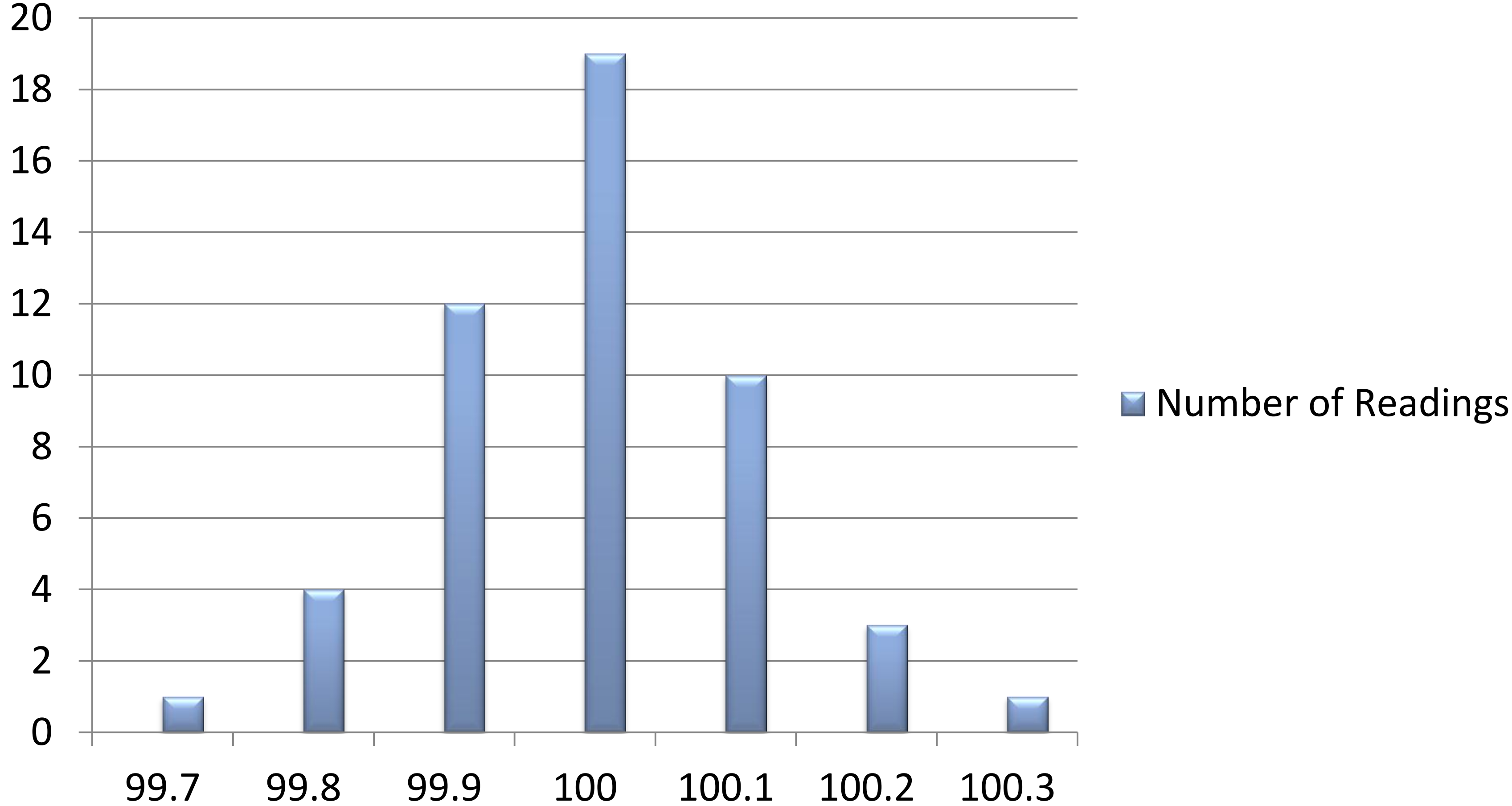
Probability of Errors

- **Normal Distribution of Errors**
- Table 1-1 shows a tabulation of 50 voltage readings that were taken at small time intervals and recorded to the nearest 0.! V. The nominal value of the measured voltage was 100.0 V.

Probability of Errors

Voltage Reading (V)	Number of Readings
99.7 99.8 99.9 100.0 100.1 100.2 100.3	1 4 12 19 10 3 1
	50

Probability of Errors



Probability of Errors

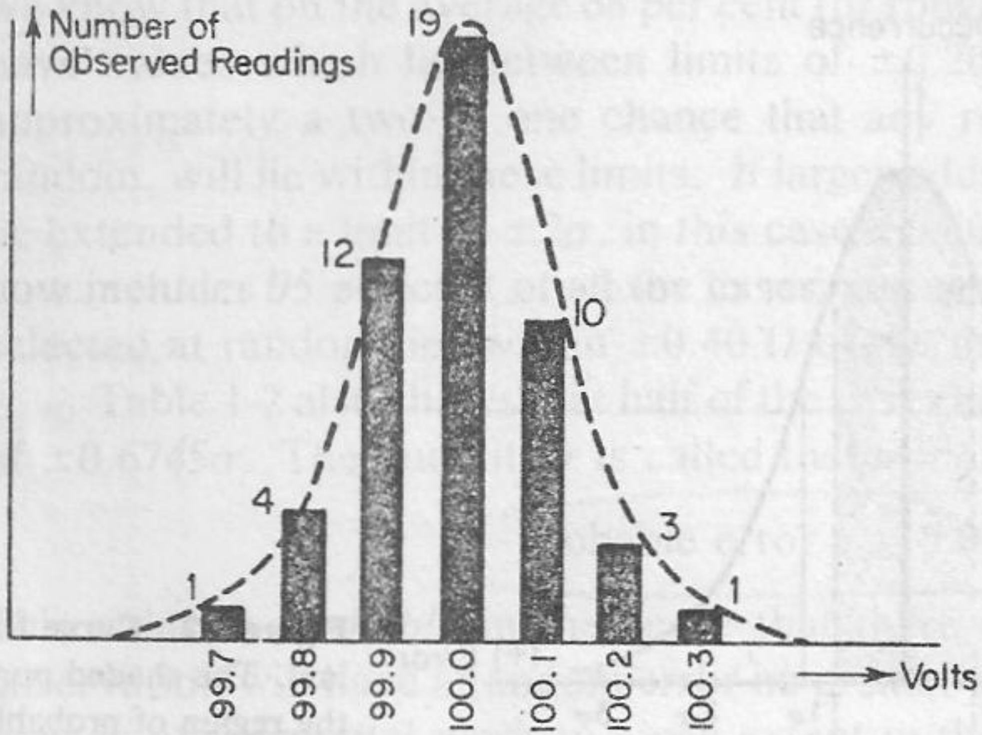


Figure 1-1 Histogram showing the frequency of occurrence of the 50 voltage readings of Table 1-1. The dashed curve represents the limiting case of the histogram when a large number of readings at small increments are taken.

The result of this series of measurements can be presented graphically in the form of a block diagram or histogram in which the number of observations is plotted against each observed voltage reading.

Probability of Errors

- This bell-shaped curve is known as a **Gaussian curve.**
- The sharper and narrower the curve, the more definitely an observer may state that the most probable value of the true reading is the central value or mean reading.

Probability of Errors

- Normal law:

a) All observations include small disturbing effects, called random errors.

b) Random errors can be positive or negative.

c) There is an equal probability of positive and negative random errors.

Probability of Errors

- The possibilities as to the form of the error distribution curve can be stated as follows

a) Small errors are more probable than large errors.

b) Large errors are very improbable.

c) There is an equal probability of plus and minus errors so that the probability of a given error will be symmetrical about the zero value.

Probability of Errors

- **Probable Error**
- The area under the Gaussian probability curve of Figure between the limits $+\infty$ and $-\infty$, represents the entire number of observations.
- The area under the curve between the $+\sigma$ and $-\sigma$ limits represents the cases that differ from the mean by no more than the standard deviation.

Probability of Errors

- Integration of the area under the curve within the $\pm \sigma$ limits gives the total number of cases within these limits.
- For normally dispersed data, following the Gaussian distribution, approximately 68 percent of all the cases lie between the limits of $+ \sigma$ and $- \sigma$ from the mean.

Probability of Errors

- Corresponding values of other deviations, expressed in terms of σ, are given in Table

Deviation ($\pm$) σ	**Fraction of total area included**
0.6745	0.5
1.0	0.6828
2.0	0.9546
3.0	0.9972

Probability of Errors

- Table also shows that half of the cases are included in the deviation limits of $\pm 0.6745\sigma$. The quantity *r* is called the *probable error* and is defined as
- Probable error $r = \pm 0.6745\sigma$ (1-6)

- This value is probable in the sense that there is an even chance that any one observation will have a random error no greater than $\pm r$.

EXAMPLE: 1-11

- Ten measurements of the resistance of a resistor gave 101.2Ω, 101.7Ω, 101.3Ω, 101.0Ω, 101.5Ω, 101.3Ω, 101.2Ω, 101.4Ω, 101.3Ω, and 101.1Ω. Assume that only random errors are present. Calculate (a) the arithmetic mean; (b) the standard deviation of the readings; (c) the probable error.

EXAMPLE: 1-11

SOLUTION With a large number of readings a simple tabulation of data is very convenient and avoids confusion and mistakes.

EXAMPLE: 1-11

Reading, x	Deviation	
	d	d^2
101.2	-0.1	0.01
101.7	0.4	0.16
101.3	0.0	0.00
101.0	-0.3	0.09
101.5	0.2	0.04
101.3	0.0	0.00
101.2	-0.1	0.01
101.4	0.1	0.01
101.3	0.0	0.00
101.1	-0.2	0.04
$\sum x = 1.013.0$	$\sum \lvert d \rvert = 1.4$	$\sum d^2 = 0.36$

EXAMPLE: 1-11

a. Arithmetic mean, $\bar{x} = \frac{\sum x}{n} = \frac{1{,}013.0}{10} = 101.3\Omega$

b. Standard deviation, $\sigma = \sqrt{\frac{d^2}{n-1}} = \sqrt{\frac{0.36}{9}} = 0.2\Omega$

c. Probable error = 0.6745 σ = 0.6745 x 0.2 = 0.1349Ω

LIMITING ERRORS

- In most indicating instruments the accuracy is guaranteed to a certain percentage of full-scale reading.
- The limits of these deviations from the specified values are as limiting errors or guarantee errors.
- For example, if the resistance of a resistor is given as 500Ω ± 10 percent, the manufacturer guarantees that the resistance falls between the limits 450Ω and 550Ω.

EXAMPLE 1-12

A 0-150-V voltmeter has a guaranteed accuracy of 1 percent full-scale reading. The voltage measured by this instrument is 83 V. Calculate the limiting error in percent.

SOLUTION

The magnitude of the limiting error is

$$0.01 \times 150V = 1.5V$$

The percentage error at a meter indication of 83 V is

$$\frac{1.5}{83} \times 100 \text{ percent} = 1.81 \text{ percent}$$

EXAMPLE: 1-13

The voltage generated by a circuit is equally dependent on the value of three resistors and is given by the following equation:

$$V_{out} = \frac{R_1 R_2}{R_3}$$

If the tolerance of each resistor is 0.1 percent, what is the maximum error of the generated voltage?

Solution

Using the maximum value of R1, and R2 and the minimum value for R3 results in the greatest value for Vout of

$$V_{out} = \frac{(1.001R_1)(1.001R_2)}{0.999R_3} = 1.003$$

EXAMPLE: 1-13

The lowest resulting voltage occurs when the value of R_3 is highest and R_1 and R_2 are the lowest. The resulting voltage is

$$V_{out} = \frac{(0.999R_1)(0.999R_2)}{1.003R_3} = 0.997$$

The total variation of the resultant voltage is ± 0.3 percent, which is the algebraic sum of the three tolerances.

Similarly refer to Example 1-14 for further use of above formulas.

That’s all for today
Thank you

QUESTIONs???

Quiz No.2

The power factor and the phase angle in a circuit carrying a sinusoidal current are determined by measurements of current, voltage and power. The current is read as 2.50A on 5-A ammeter, the voltage as 115 V on a 250-V voltmeter and the power as 220 W on a 500-W wattmeter. All the instruments are guaranteed accurate to within ±0.8 percent of full-scale indication. Calculate

a) The percentage accuracy to which the power factor can be guaranteed.
b) The possible error in phase angle.