Lecture 10

Bridge Measurements-2

GUARDED WHEATSTONE BRIDGE

- The measurement of extremely high resistances involves leakage current. For example
  - the insulation resistance of a cable or
  - the leakage resistance of a capacitor
  - often on the order of several thousands of megohms
- Ordinary DC Wheatstone bridge can not be used.
- Major Problem is the leakage occurring over and around the specimen or over the binding post by which component is attached with the instrument
- These currents are undesired because
  - Enter the measuring ckt & effect the accuracy of measurement
  - are also dependent on certain environmental factors e.g. humidity

- Guard circuit are used to block the path of these currents.
- The principle of a simple guard circuit in the R_x arm of a Wheatstone bridge is explained with Fig. 5-7.

- Without a guard circuit, leakage current I_1 *along the insulated surface of the binding post adds to current* I_x through the component under measurement
  - to produce a total circuit current considerably larger than the actual device current

Guard Circuits

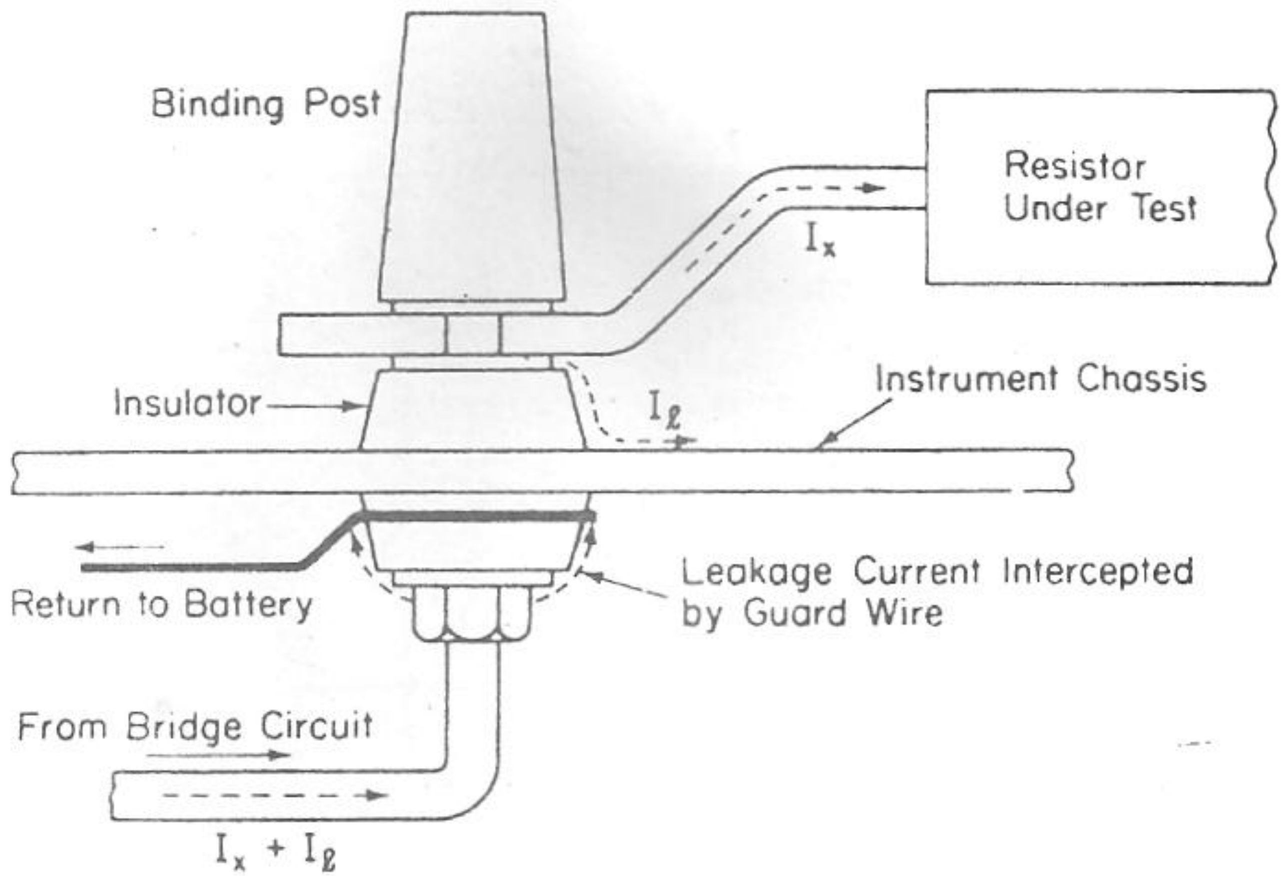


Figure 5-7 Simple guard wire on the R_x terminal of a guarded Wheatstone bridge eliminates surface leakage.

Guard Circuits

- **A guard wire,** completely surrounding the surface of the insulated post,
  - intercepts this leakage current and returns it to the battery.
  - The guard must be carefully placed so that the leakage current always meets some portion of the guard wire and is prevented from entering the bridge circuit.
- In the schematic diagram of Fig. 5-8 the guard around the R_x binding post,
  - indicated by a small circle around the terminal,
  - does not touch any part of the bridge circuitry and is connected directly to the battery terminal.
- The principle of the guard wire on the binding post can be applied to any internal part of the bridge circuit
  - we speak of a guarded Wheatstone bridge when leakage affects the measurement.

Three-Terminal Resistance

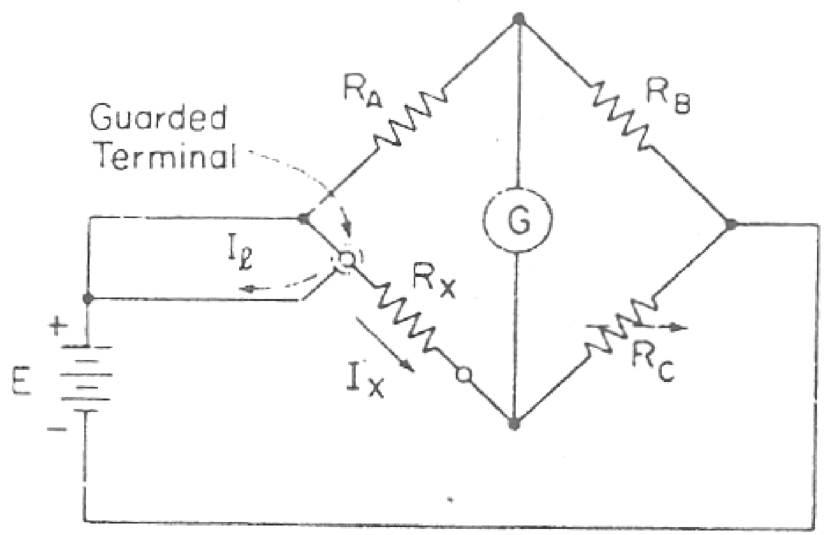


Figure 5-8 Guarded terminal returns leakage current to the battery.

Three-Terminal Resistance

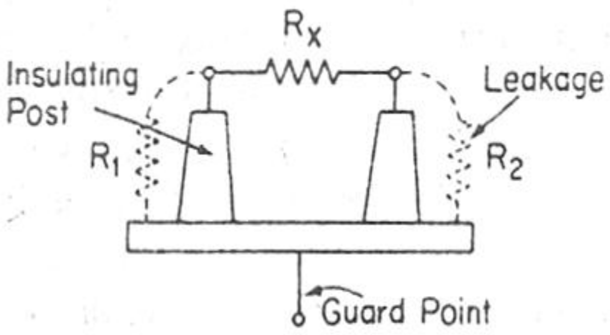
- To avoid the effects of leakage currents external to the bridge circuitry,
  - the junction of ratio arms R_A and R_B is usually brought out as a separate guard terminal on the front panel of the instrument.
- This terminal can be used to connect a so-called *three-terminal resistance, as shown in Fig. 5-9.*
- *The high is mounted on two insulating posts that are fastened to a metal plate.*
- *The two main terminals of the resistor are connected to the R_x terminals of the bridge in the usual manner.*
- *The third terminal of the resistor is the common point of resistances R_1 and R_2, which represent the leakage paths from the main terminals along the insulating posts to the metal plate, or guard.*

Three-Terminal Resistance

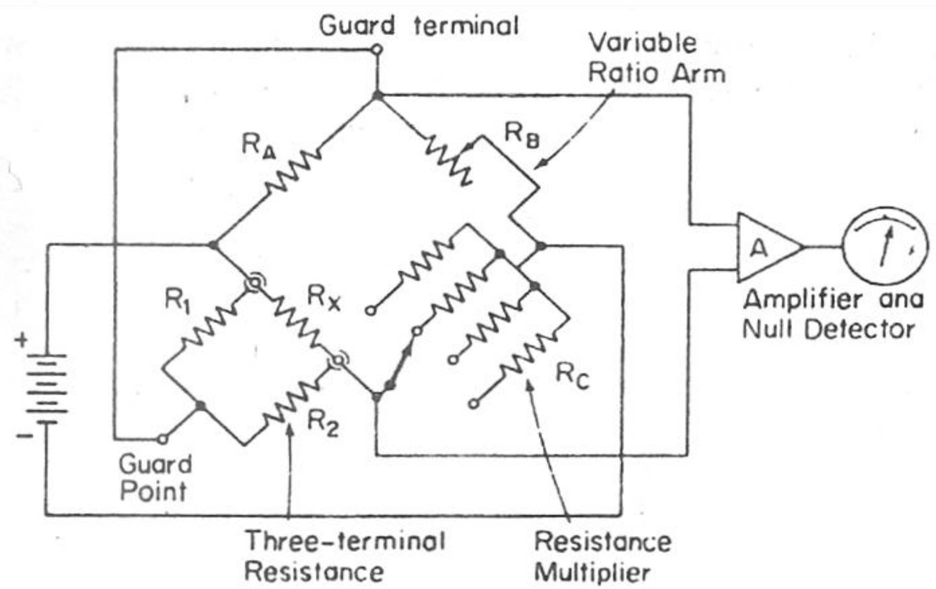
- ***The guard is connected to the guard terminal on the front panel of the bridge, as indicated in the schematic of Fig. 5-9.***
- ***This connection puts R_1 in parallel with ratio arm R_A, but since R_1 is very much larger than R_A, its shunting effect is negligible.***
- ***Similarly, leakage resistance R_2 is in parallel with the galvanometer, but the resistance of R_2 is so much higher than that of the galvanometer that the only effect is a slight reduction in galvanometer sensitivity.***
- ***The effects of external leakage paths are therefore removed by using the guard circuit on the three-terminal resistance.***
- If the guard circuit were not used, leakage resistance R_1 and R_2 would be directly across R_x and the measured value of R_x would be considerably in error.

Three-Terminal Resistance

- Assuming, for example, that the unknown is 100 M and that the leakage resistance from each terminal to the guard is also 100 M, resistance R_x would be measured as 67 M, an error of approximately 33 per cent.



(a) Three-terminal resistance



(b) Guarded bridge circuit

Figure 5-9 Three-terminal resistance, connected to a guarded high-voltage megohm bridge.

AC Bridges and their Applications

AC Bridges

1. Conditions for Bridge Balance
   – AC Bridges need a source of excitation and a null detector.
2. Source of Excitation voltage can be
   – Power line (low frequency)
   – Oscillator (high frequency)
3. Null Detector
   – Headphone
   – Amplifier with an AC Meter
   – Electron ray tube (tuning eye)

Conditions for Bridge Balance

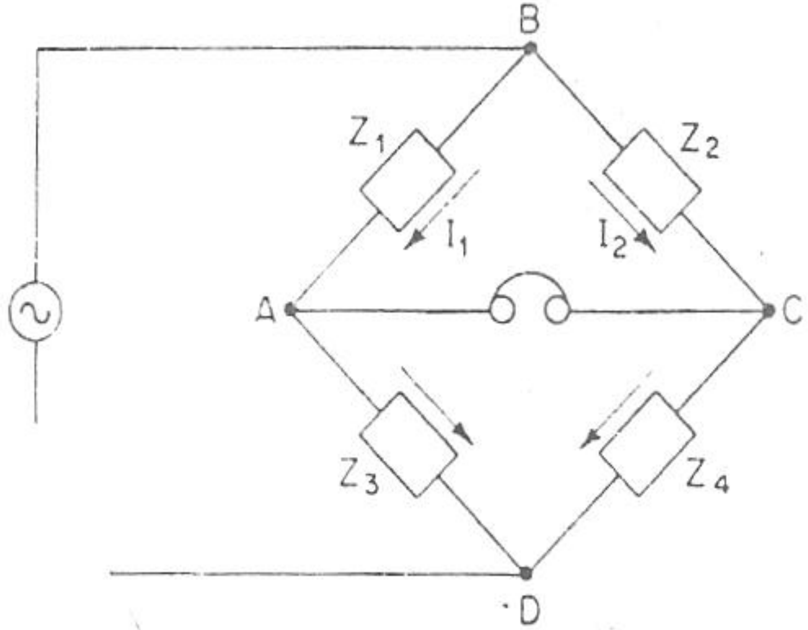


Figure 5-10 General form of the ac bridge.

Conditions for Bridge Balance…..

- The general form of an ac bridge in Fig. 5-10.
- The four bridge arms Z_1, Z_2, Z_3, and Z_4 are indicated as unspecified impedances and the detector is represented by headphones.
- For balance condition the voltage drop from B to A equals the voltage drop from B to C, in both magnitude and phase.
- In complex notation we can write

$$\mathbf{E_{BA} = E_{BC}} \quad \textbf{or} \quad \mathbf{I_1 Z_1 = I_2 Z_2} \qquad (5\text{-}18)$$

- For zero detector current (the balance condition), the currents are

Conditions for Bridge Balance…..

- $I_1 = \dfrac{E}{Z_1 + Z_3}$ (5-19)
- and $I_2 = \dfrac{E}{Z_2 + Z_4}$ (5-20)
- Substitution of Eqs. (5-19) and (5-20) into Eq. (5-18) yields

  $\mathbf{Z_1 Z_4 = Z_2\, Z_3}$ (5-21)
- or when using admittances instead of impedances.

  $\mathbf{Y_1\, Y_4 = Y_2\, Y_3}$ (5-22)
- Equation (5-21) is the general equation for balance of the ac bridge.

Conditions for Bridge Balance…..

- Equation (5-22) can be used to advantage when dealing with parallel components in bridge arms.
- Equation (5-21) states
  - the product of impedances of one pair of opposite arms must equal the product of impedances of the other pair of opposite arms,
  - with the impedances expressed in complex notation.
- If the impedance is written in the form ***Z*** = Z /, θ
  - where ***Z*** represents the magnitude and the phase angle of the complex impedance, Eq. (5-21) can be written in the form
    - $(Z_1 \angle \theta_1)(aZ_4 \angle \theta_4) = (Z_2 \angle \theta_2)(Z_3 \angle \theta_3)$ (5-23)

Conditions for Bridge Balance….

- Eq. (5-23) can also be written as
- $Z_1 Z_4 \angle(\theta_1 + \theta_4) = Z_2 Z_3 \angle(\theta_2 + \theta_3)$ (5-24)
- Equation (5-24) shows that two conditions must be met simultaneously when balancing an ac bridge.

  $Z_1 Z_4 = Z_2 Z_3$ (5-25)

  – *The products of the magnitudes of the opposite arms must be equal.*

  – Phase angles of the impedances satisfy the relationship

- $\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$ (5-26)

  – *The sum of the phase angles of the opposite arms must be equal.*

Application of the Balance Equations

- The two balance conditions expressed in Eqs. (5-25) and (5-26) can be applied when the impedances of the bridge arms are given in polar form, with both magnitude and phase angle.
- In the usual case, however, the component values of the bridge arms are given, and the problem is solved by writing the balance equation in complex notation.

Example 5-3

EXAMPLE 5-3

The impedances of the basic ac bridge of Fig. 5-10 are given as follows:

$$\mathbf{Z}_1 = 100\ \Omega\ \angle 80^\circ \text{ (inductive impedance)}$$

$$\mathbf{Z}_2 = 250\ \Omega \text{ (pure resistance)}$$

$$\mathbf{Z}_3 = 400\ \Omega\ \angle 30^\circ \text{ (inductive impedance)}$$

$$\mathbf{Z}_4 = \text{unknown}$$

Determine the constants of the unknown arm.

SOLUTION The first condition for bridge balance requires that

$$Z_1 Z_4 = Z_2 Z_3 \qquad (5\text{-}25)$$

Substituting the magnitudes of the known components and solving for Z_4, we obtain

$$Z_4 = \frac{Z_2 Z_3}{Z_1} = \frac{250 \times 400}{100} = 1{,}000\ \Omega$$

The second condition for bridge balance requires that the sums of the phase angles of opposite arms be equal, or

$$\theta_1 + \theta_4 = \theta_2 + \theta_3 \qquad (5\text{-}26)$$

Substituting the known phase angles and solving for θ_4, we obtain

$$\theta_4 = \theta_2 + \theta_3 - \theta_1 = 0 + 30 - 80 = -50^\circ$$

Hence the unknown impedance Z_4 can be written in polar form as

$$\mathbf{Z}_4 = 1{,}000\ \Omega\ \angle -50^\circ$$

indicating that we are dealing with a capacitive element, possibly consisting of a series combination of a resistor and a capacitor.

Application of the Balance Equations..

- The problem becomes slightly more complex when the component values of the bridge arms arc specified and the impedances are to be expressed in complex notation.
- In this case, the inductive or capacitive reactance can only be calculated when the frequency of *the excitation voltage* is known,
- See Example 5-4

EXAMPLE 5-4

The ac bridge of Fig. 5-10 is in balance with the following constants: arm AB, $R = 450\ \Omega$; arm BC, $R = 300\ \Omega$ in series with $C = 0.265\ \mu F$; arm CD, unknown; arm DA, $R = 200\ \Omega$ in series with $L = 15.9$ mH. The oscillator frequency is 1 kHz. Find the constants of arm CD.

SOLUTION The general equation for bridge balance states that

$$\mathbf{Z}_1\mathbf{Z}_4 = \mathbf{Z}_2\mathbf{Z}_3 \qquad (5\text{-}21)$$

$$\mathbf{Z}_1 = R = 450\ \Omega$$

$$\mathbf{Z}_2 = R - j/\omega C = (300 - j600)\ \Omega$$

$$\mathbf{Z}_3 = R + j\omega L = (200 + j100)\ \Omega$$

$$\mathbf{Z}_4 = \text{unknown}$$

Substituting the known values in Eq. (5-21) and solving for the unknown yields

$$\mathbf{Z}_4 = \frac{450 \times (200 + j100)}{300 - j600} = +j150\ \Omega$$

This result indicates that $\mathbf{Z}_4$ is a pure inductance with an inductive reactance of 150 Ω at a frequency of 1 kHz. Since the inductive reactance $X_L = 2\pi fL$, we solve for L and obtain $L = 23.9$ mH.

MAXWELL BRIDGE

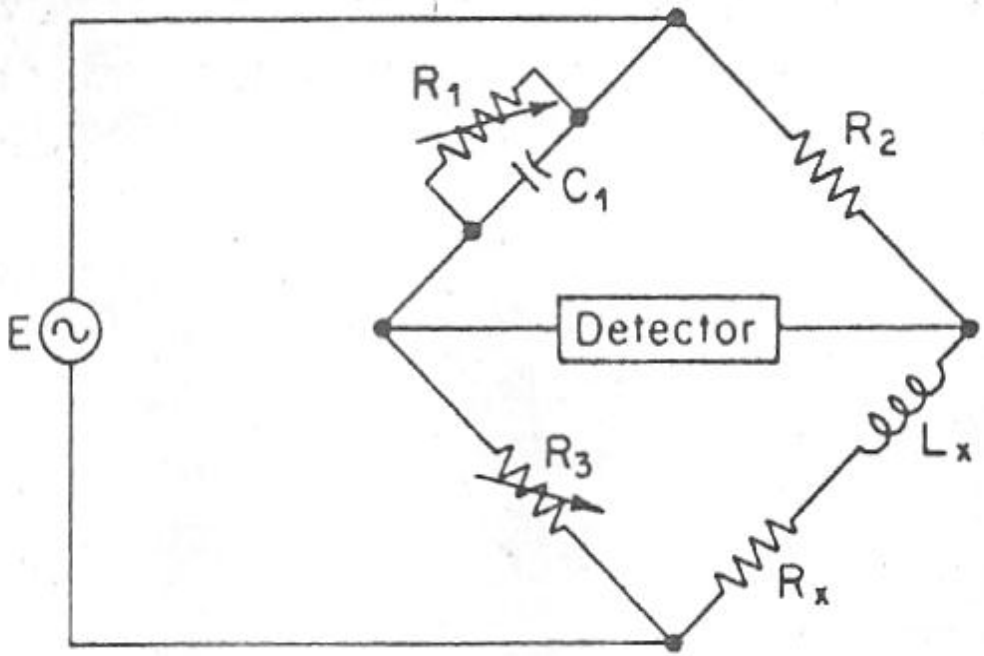


Figure 5-11 Maxwell bridge for inductance measurements.

MAXWELL BRIDGE…..

- In Maxell bridge:
  - One of the ratio arms has a resistance and a capacitance in parallel
- Easier to write the balance equations using $Y_{1,}$ the ***admittance*** of arm 1 instead of its impedance.
- Rearranging the general equation for bridge balance, as expressed in Eq. (5-21), we obtain
- where Y_1 is the admittance of arm 1. Reference to Fig. 5-11 shows that
- $Z_2 = R_2$; $Z_3 = R_3$; and $Y_1 = \frac{1}{R_1} + j\,\omega C_1$

MAXWELL BRIDGE…..

- Substitution of these values in Eq. (5-27) gives
- $Zx = Rx + j\omega Lx = R_2R_3(\frac{1}{R_1} + j\omega C_1)$ (5-28)
- Separation of the real and imaginary terms yields

$$R_x = \frac{R_2R_3}{R_1} \quad (5\text{-}29)$$

- and
- $L_x = R_2R_3C_1$ (5-30)
- where the resistances are expressed in ohms, inductance in henrys, and capacitance in farads.

MAXWELL BRIDGE…..

- The Maxwell bridge is limited to the measurement of medium-Q coils ($1 < Q < 10$).
- the second balance condition states that the sum of the phase angles of one pair of opposite arms must be equal to the sum of the phase angles of the other pair.
  - Since the phase angles of the resistive elements in arm 2 and arm 3 add up to 0^o, the sum of the angles of arm 1 and arm 4 must also add up to 0^o.
  - The phase angle of a high-Q coil will be very nearly 90° (positive),
  - which requires that the phase angle of the capacitive arm must also be very nearly 90° (negative).
- This in turn means that the resistance of R_1 must be very large indeed, which can be very impractical.
- High-Q coils are therefore measured on the Hay bridge.

MAXWELL BRIDGE…….

- The Maxwell bridge is also unsuited for the measurement of coils with a very low *Q-value ($Q < 1$) because of balance convergence problems.*
- As can be seen from the equations for R_x and L_x, adjustment for inductive balance by R_3 upsets the resistive balance by R_1 and gives the effect known as sliding balance.
- Sliding balance describes the interaction between controls, so that when we balance with R_1 and then with R_3, then go back to R_1, we find a new balance point.
- The balance point appears to move or slide toward its final point after many adjustments.
  - Interaction does not occur when R_1 and C_1 are used for the balance adjustments, but a variable capacitor is not always suitable.

MAXWELL BRIDGE…..

- The usual procedure for balancing the Maxwell bridge is by
  - first adjusting R_3 for inductive balance and
  - then adjusting R_1 for resistive balance.
  - Returning to the R_3 adjustment,
  - we find that the resistive balance is being disturbed and moves to a new value.
- This process is repeated and gives slow convergence to final balance.
- For medium-*Q coils*, *the resistance effect is not pronounced, and balance is reached after a few adjustments*.

HAY BRIDGE

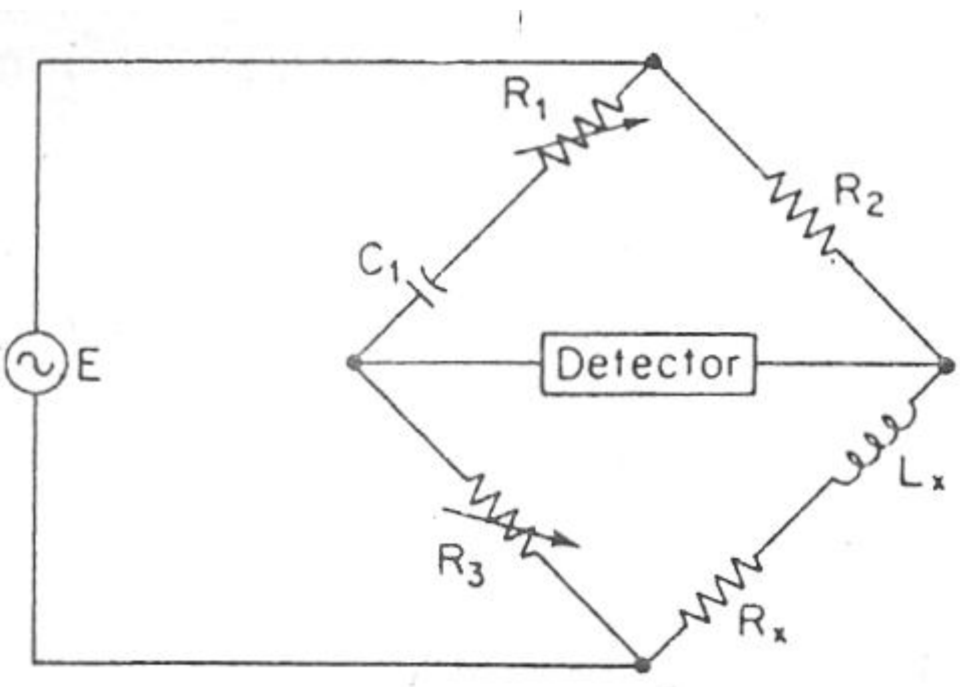


Figure 5-12 Hay bridge for inductance measurements.

HAY BRIDGE……

- The Hay bridge of Fig. 5-12 differs from the Maxwell bridge by having resistor R_1 in ***series*** with standard capacitor C_1instead of in parallel.
- for large phase angles, R_1 should have a very low value.
- The Hay circuit is therefore more convenient for measuring high-*Q coils*.
- The balance equations are again derived by substituting the values of the impedances of the bridge arms into the general equation for bridge balance.

HAY BRIDGE…..

- For the circuit of Fig. 5-12, we find that
- $Z_1 = R_1 - \frac{j}{\omega C_1}$; $Z_2 = R_2$; $Z_3 = R_3$; $Z_x = R_x + j\omega L_x$
- Substituting these values in Eq. (5-21), we get

$$\left(R_1 - \frac{j}{\omega C_1}\right)(R_x + j\omega L_x) = R_2 R_3 \qquad (5\text{-}31)$$

Which expands to

- $R_1 R_x + \frac{L_x}{C_1} - \frac{jR_x}{\omega C_1} + j\omega L_x R_1 = R_2 R_3$
- Separating the real and imaginary terms, we obtain
  - $R_1 R_x + \frac{L_x}{C_1} - \frac{jR_x}{\omega C_1} = R_2 R_3 \qquad (5\text{-}32)$

HAY BRIDGE…..

- And
- $$\frac{R_x}{\omega C_1} = \omega L_x R_1$$ (5-33)
- Both Eq. (5-32) and Eq. (5.33) contain L_x and R_x, and we must solve these equations simultaneously.
- This yields $R_x = \frac{\omega^2 C_1^2 R_1 R_2 R_3}{1+\omega^2 C_1^2 R_1^2}$ (5-34)
- $$L_x = \frac{R_2 R_3 C_1}{1+\omega^2 C_1^2 R_1^2}$$ (5-35)

- These expressions for the unknown inductance and resistance both contain the angular velocity
- it therefore appears that the frequency of the voltage source must be known accurately.

HAY BRIDGE…..

- That this is not true when a high-Q coil is being measured follows from the following considerations:
  - the sum of the opposite sets of phase angles must be equal,
  - the inductive phase angle must be equal to the capacitive phase angle, since the resistive angles are zero.
  - Figure 5-13 shows that the tangent of the inductive phase angle equals
- $$\tan\theta_L = \frac{X_L}{R} = \frac{\omega L_x}{R_x} = Q \qquad (5\text{-}36)$$
- and that of the capacitive phase angle is
- $$\tan\theta_L = \tan\theta_c \; or \; Q = \frac{1}{\omega C_1 R_1} \qquad (5\text{-}37)$$
- When the two phase angles are equal, their tangents are also equal and we can write
- $$\tan\theta_L = \tan\theta_c \; or \; Q = \frac{1}{\omega C_1 R_1} \qquad (5\text{-}38)$$

HAY BRIDGE….

- Returning now to the term (1 ± w2ClR1) which appears in Eqs. (5-34) and (5-35),we find that, after submitting Eq. (5-38) in the expression for 4, Eq. (5-35) reduces to
- $$Lx = \frac{R_2 R_3 C_1}{1 + (1/Q)^2} \qquad (5\text{-}39)$$
- For a value of *Q greater than ten, the term (1/Q)*^{2} *will be smaller than 1/100 and can be neglected. Equation (5-35) therefore reduces to the expression derived for the Maxwell bridge,*
  - $L_x = R_2R_3C_1$

HAY BRIDGE….

- The Hay bridge is used in *Inductors, a Q greater than 10.*
- *For Q- values smaller than 100, the term (1/Q)^2 becomes important and cannot be neglected.*
- *In this case, the Max-well bridge is more suitable.*

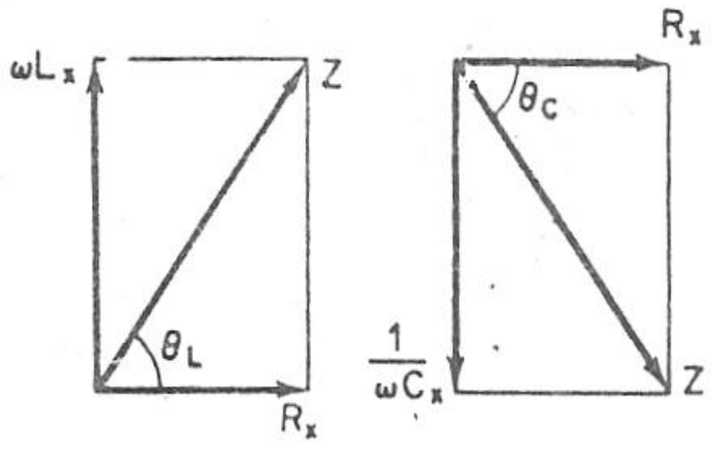


Figure 5-13 Impedance triangles illustrate inductive and capacitive phase angles.

Thank you