

# Lecture 11

Bridge Measurements-3

# SCHERING BRIDGE

- The Schering bridge, one of the most important ac bridges, used for the measurement of
  - *Capacitors*
  - *Insulating properties, i.e., for phase angles very nearly 90°.*
  - *Dissipation Factor*
- The dissipation factor of a capacitor is the ratio of its resistance (equivalent series resistance) to its capacitive reactance
  - *Measurement of relative permittivity*
- The basic circuit arrangement is shown in Fig. 5-14

$$ESR = \frac{\sigma}{\epsilon \omega^2 C}$$

$\sigma$  is the dielectric's bulk conductivity,  
 $\omega$  is the angular frequency of the AC current  $i$ ,  
 $\epsilon$  is the lossless permittivity of the dielectric, and  
 $C$  is the lossless capacitance

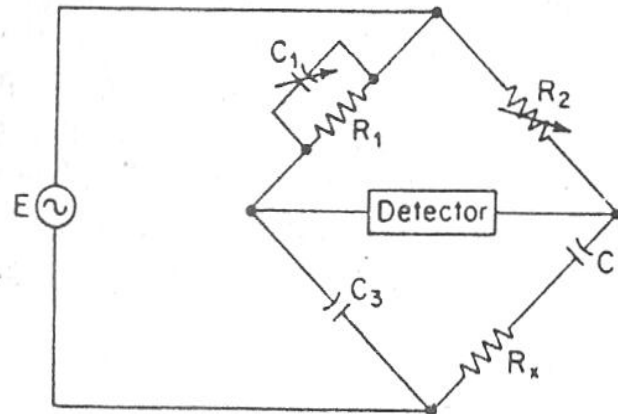


Figure 5-14 Schering bridge for the measurement of capacitance.

# SCHERING BRIDGE

- Arm 1 now contains a parallel combination of a resistor and a capacitor, and the standard arm contains only a capacitor.
- The standard capacitor is usually high-quality mica capacitor for general measurement work or an air capacitor for insulation measurements.
- A good-quality mica capacitor has very low losses (no resistance)
  - therefore has a phase angle of approximately  $90^\circ$ .
- The balance conditions require that the sum of the phase angles of arms 1 and 4 equals the sum of the phase angles of arms 2 and 3.

# SCHERING BRIDGE

- The standard- capacitor is in arm 3, the sum of the phase angles of arm 2 and arm 3 will be  $0^\circ + 90^\circ = 90^\circ$ .
- In order to obtain the  $90^\circ$  phase angle needed for balance, the sum of the angles of arm 1 and arm 4 must equal  $90^\circ$ .
- It is necessary to give arm 1 a small capacitive angle by connecting small capacitor  $C_1$  in parallel with resistor  $R_1$ .
- The balance equations are derived in the usual manner, and by substituting the corresponding impedance and admittance values in the general equation, we obtain.

$$Z_x = Z_2 Z_3 Y_1$$

# SCHERING BRIDGE

- $$R_x - \frac{j}{\omega C_x} = R_2 \left( \frac{-j}{\omega C_3} \right) \left( \frac{1}{R_1} + j\omega C_1 \right)$$

and expanding

- $$R_x - \frac{j}{\omega C_x} = \frac{R_2 C_1}{C_3} - \frac{j R_2}{\omega C_3 R_1} \quad (5-40)$$

- Equating the real terms and the imaginary terms, we find that

- $$R_x = R_2 \frac{C_1}{C_3} \quad (5-41)$$

- $$C_x = C_3 \frac{R_1}{R_2} \quad (5-42)$$

# SCHERING BRIDGE

- Variables chosen for the balance adjustment are capacitor  $C_1$  and resistor  $R_2$ .
- *Question arises how the **quality** of a capacitor is defined???*
- The power factor (PF) of a series *RC combination is defined as the cosine of the phase angle of the circuit.*
- *Therefore the PF of the unknown arm equals  $PF = R_x/Z_x$ .*
- For phase angles very close to  $90^\circ$ , the reactance is almost equal to the impedance and we can approximate the power factor to

- $$PF = \frac{R_x}{X_x} = \omega C_x R_x \quad (5-43)$$

# SCHERING BRIDGE

- The dissipation factor of a series RC circuit is defined as the **cotangent** of the phase angle and therefore, by definition, the dissipation factor

- $$D = \frac{R_x}{X_x} = \omega C_x R_x \quad (5-44)$$

- Incidentally, the quality of a coil is defined by  $Q = X_L/R_L$ ,
- *the dissipation factor,  $D$ , is the reciprocal of the quality factor,  $Q$ , and therefore  $D = 1/Q$ .*
- *The dissipation factor tells us something about the quality of a capacitor; i.e.,*
  - how close the phase angle of the capacitor is to the ideal value of  $90^\circ$ .
- By substituting the value of  $C_x$  in Eq. (5-42) and of  $R_x$  in Eq. (5-41) into the expression for the dissipation factor, we obtain

# SCHERING BRIDGE

$$D = \omega R_1 C_1 \quad (5-45)$$

- If resistor  $R_1$  in the Schering bridge of Fig. 5-14 has a fixed value, the dial of capacitor  $C_1$  may be calibrated directly in dissipation factor  $D$ .
- This is the usual practice in a Schering bridge.
  - Notice that the term  $\omega$  appears in the expression for the dissipation factor [Eq. (5-45)].
  - This means that the calibration of the  $C_1$  dial holds for only one particular frequency at which the dial is calibrated.
- A different frequency can be used, provided that a correction is made by multiplying the  $C_1$  dial reading by the ratio of the two frequencies.



# UNBALANCE CONDITIONS

- It sometimes happens that an ac bridge cannot be balanced at all simply because one of the stated balance conditions cannot be met (*Sec. 5-5.*)
- Consider for example, the circuit of Fig. 5-16,

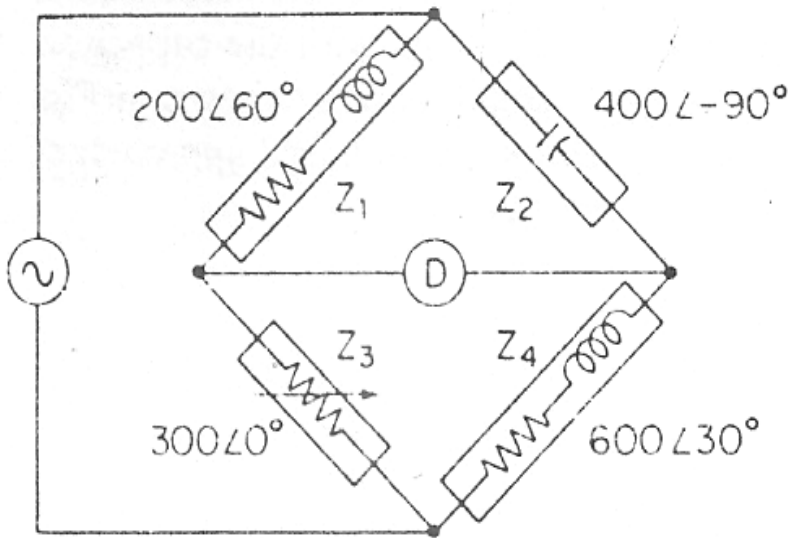


Figure 5-16 Ac bridge that cannot be balanced.

# UNBALANCE CONDITIONS

- $Z_1$  and  $Z_4$  are inductive elements (positive phase angles),  $Z_2$  is a pure capacitance ( $-90^\circ$  phase angle), and  $Z_3$  is a variable resistance (zero phase angle).
- The resistance of  $R_3$  needed to obtain bridge balance can be determined by applying the first balance condition (magnitudes) and we find that
- $$R_3 = \frac{Z_1 Z_4}{Z_2} = \frac{200 \times 600}{400} = 300 \Omega$$
- Hence adjusting  $R_3$  to a value of 300 will satisfy the first condition

# UNBALANCE CONDITIONS

- Considering the second balance condition (phase angles) yields the following
- situation:  $\theta_1 + \theta_4 = +60^\circ + 30^\circ = 90^\circ$   
 $\theta_2 + \theta_3 = -90^\circ + 0^\circ = -90^\circ$
- Obviously,  $\theta_1 + \theta_4 \neq \theta_2 + \theta_3$ , and the second condition is not satisfied.
- In this case, bridge balance cannot be obtained.
- An interesting illustration of a bridge balancing problem is given in Example 5-5
  - where minor adjustments to one or more of the bridge arms result in a situation where balance can be obtained.

### EXAMPLE 5-5

Consider the circuit of Fig. 5-17(a) and determine whether or not the bridge is in complete balance. If not, show two ways in which it can be made to balance and specify numerical values for any additional components. Assume that bridge arm 4 is the unknown that cannot be modified.

**SOLUTION** Inspection of the circuit shows that the first balance condition (magnitudes) can easily be met by slightly increasing the resistance of  $R_3$ . The second balance condition requires that  $\theta_1 + \theta_4 = \theta_2 + \theta_3$  where

$$\theta_1 = -90^\circ \text{ (pure capacitance)}$$

$$\theta_2 = \theta_3 = 0^\circ \text{ (pure resistance)}$$

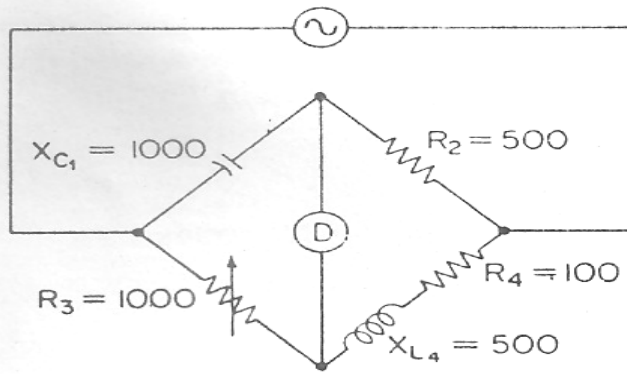
$$\theta_4 < +90^\circ \text{ (inductive impedance)}$$

Obviously, balance is not possible with the configuration of Fig. 5-17(a) because the sum of  $\theta_1$  and  $\theta_4$  will be slightly negative while  $\theta_2 + \theta_3$  will be exactly  $0^\circ$ . Balance can be restored by modifying the circuit in such a way that the phase angle condition is satisfied. There are basically two methods to accomplish this: The first option is to modify  $Z_1$  so that its phase angle is decreased to less than  $90^\circ$  (equal to  $\theta_4$ ) by placing a resistor in parallel with the capacitor. This modification results in a Maxwell bridge configuration, as shown in Fig. 5-17(b). The resistance of  $R_1$  can be determined by the standard approach of Sec. 5-6, using the admittance of arm 1, and we can write

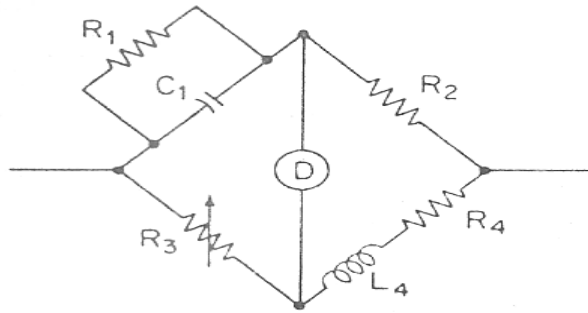
$$Y_1 = \frac{Z_4}{Z_2 Z_3}$$

where

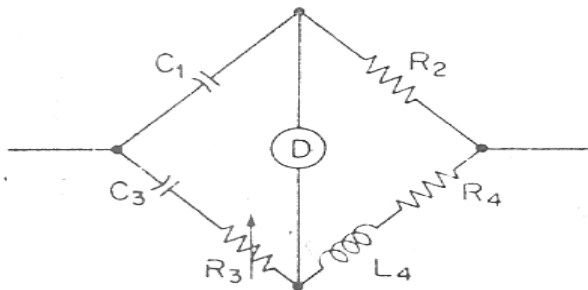
$$Y_1 = \frac{1}{R_1} + \frac{j}{1,000}$$



(a) Unbalanced condition



(b) Bridge balance is restored by adding a resistor to arm 1. (Maxwell configuration).



(c) Alternative method of restoring bridge balance, by adding a capacitor to arm 3.

Figure 5-17 Bridge balancing problem.

?

## EXAMPLE 5-5

- Substituting the known values and solving for  $R_1$ , we obtain

$$\frac{1}{R_1} + \frac{j}{1,000} = \frac{100 + j500}{500 \times 1,000}$$

- It should be noted that the addition of  $R_1$  upsets the first balance condition of the circuit (the magnitude of  $Z_1$  has changed)
  - variable resistor  $R_3$  should be adjusted to compensate for this effect.
- The second option is to modify the phase angle of arm 2 or arm 3 by adding a series capacitor, as shown in Fig. 5-17(c).
- Again writing the general balance equation, using impedances this time, we obtain

- $$Z_3 = \frac{Z_1 Z_4}{Z_2}$$

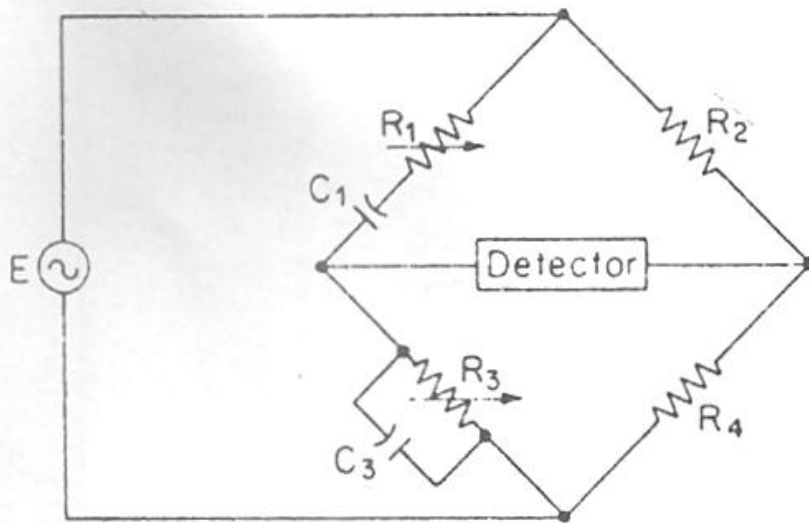
## EXAMPLE 5-5

- Substituting the component values and solving for X yields
- $1,000 - jX_c = \frac{-j1,000(100 + j500)}{500}$
- or
- $X_c = 200 \Omega$
- Here the magnitude of Z3 has increased so that the first balance condition has changed.
- A small readjustment of R1 is necessary to restore balance.

# WIEN BRIDGE

- Developed by Max Wien in 1891
- The Wien bridge is used
  - Used for precision measurement of capacitance in terms of resistance & frequency
  - for measuring frequency (mainly audio) in ac bridges
  - as a notch filter in harmonic distortion analyzer
  - as the frequency-determining element in audio and HF oscillators
  - Bridge does not require equal values of R and C
- Has a series RC combination in one arm and a parallel RC combination in the adjoining arm





**Figure 5-18** Frequency measurement with the Wien bridge.

# WIEN BRIDGE

- The impedance of arm 1 is  $Z_1 = R_1 - j/\omega C_1$  .
- *The admittance of arm 3 is  $Y_3 = 1/R_3 + j/\omega C_3$  .*
- Using the basic equation for bridge balance and substituting the appropriate values, we obtain

- $$R_2 = \left( R_1 - \frac{j}{\omega C_1} \right) R_4 \left( \frac{1}{R_3} + j\omega C_3 \right) \quad (5-46)$$

- Expanding this expression, we get

- $$R_2 = \frac{R_1 R_4}{R_3} + j\omega C_3 R_1 R_4 - \frac{jR_4}{\omega C_1 R_3} + \frac{R_4 C_3}{C_1} \quad (5-47)$$

- Equating the real terms, we obtain

- $$R_2 = \frac{R_1 R_4}{R_3} + \frac{R_4 C_3}{C_1} \quad (5-48)$$

# WIEN BRIDGE

- $$\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1} \quad (5-49)$$

- Equating the imaginary terms, we obtain

- $$\omega C_3 R_1 R_4 = \frac{R_4}{\omega C_1 R_3} \quad (5-50)$$

solving for  $f$ , we get

- $$f = \frac{1}{2\pi \sqrt{C_1 C_3 R_1 R_3}} \quad (5-51)$$

- The two conditions for bridge balance now result in an expression determining the required resistance ratio,  $R_2/R_4$  and
- another expression determining the frequency of the applied voltage

# WIEN BRIDGE

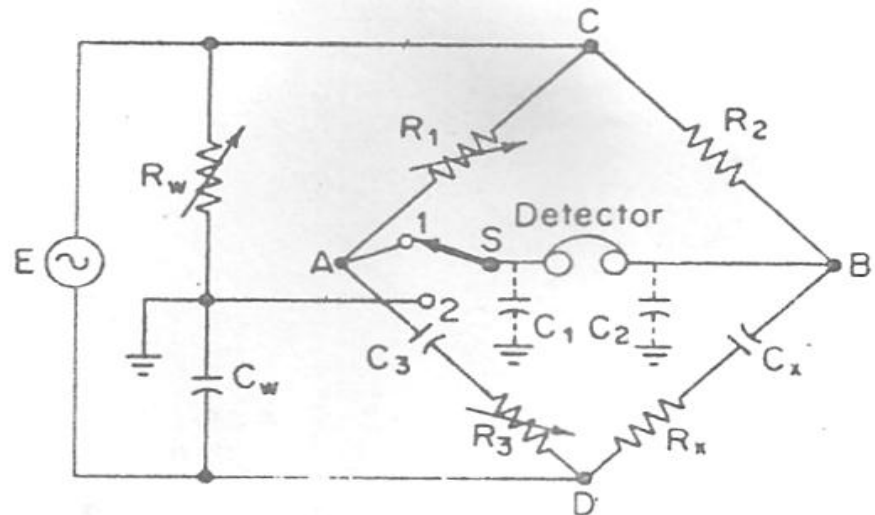
- In most Wien bridge circuits, the components are chosen such that  $R_1 = R_3$  and  $C_1 = C_3$ .
- This reduces Eq. (5-49) to  $R_2/R_4 = 2$  and Eq. (5-51) to
- $$f = \frac{1}{2\pi RC} \quad (5-52)$$
- This is the general expression for the frequency of the Wien bridge.
- In a practical bridge, capacitors  $C_1$  and  $C_3$  are fixed capacitors, and resistors  $R_1$  and  $R_3$  are variable resistors controlled by a common shaft.
  - Provided now that  $R_2 = 2R_4$ ,
  - the bridge may be used as a frequency determining device balanced by a single control.
- This control may be calibrated directly in terms of frequency.

# WIEN BRIDGE

- Because of its frequency sensitivity, the Wien bridge may be difficult to balance (unless the waveform of the applied voltage is purely sinusoidal).
- Since the bridge is not balanced for any harmonics present in the applied voltage, these harmonics will sometimes produce an output voltage masking the true balance point.

# WAGNER GROUND CONNECTION

- The discussion so far has assumed that the four bridge arms consist of **simple lumped** impedances which do not interact in any way.
- In practice, however, **stray capacitances** exist between the various bridge elements and ground, and also between the bridge arms themselves.



# WAGNER GROUND CONNECTION

- These stray capacitances shunt the bridge arms and cause measurement errors,
  - particularly at the higher frequencies or
  - when small capacitors or large inductors are measured.
- One way to control stray capacitances is by shielding the arms and connecting the shields to ground.
- This does not eliminate the capacitances but at least makes them constant in value, and they can therefore be compensated.

# WAGNER GROUND CONNECTION....

- One of the most widely used methods for eliminating some of the effects of capacitance in a bridge circuit is the Wagner ground connection.
- This circuit eliminates the troublesome capacitance which exists between the detector terminals and ground.
- Figure 5-19(a) shows the circuit of a capacitance bridge.
  - where  $C_1$  and  $C_2$  represent these stray capacitances.



# WAGNER GROUND CONNECTION...

- The oscillator is removed from its usual ground connection and bridged by a series combination of resistor  $R_w$  and capacitor  $C_w$ .
- The junction of  $R_w$  and  $C_w$  is grounded and is called the Wagner ground connection.
- The procedure for initial adjustment of the bridge is as follows:
  - The detector is connected to point 1, and  $R_1$  is adjusted for null or minimum sound in the headphones.
- The switch is then thrown to position 2, which connects the detector to the Wagner ground point.
- Resistor  $R_w$ , is now' adjusted for minimum sound.

# WAGNER GROUND CONNECTION....

- When the switch is thrown to position 1 again.
  - some unbalance will probably be shown.
- Resistors  $R_1$  and  $R_3$  are then adjusted for minimum detector response, and the switch is again thrown to position 2.
- A few adjustments of  $R_w$  and  $R_1$  (and  $R_3$ ) may be necessary before a null is reached on both switch positions.
- When null is finally obtained, points 1 and 2 are at the same potential, and this is ground potential.
- Stray capacitances  $C_1$  and  $C_2$  are then effectively shorted out and have no effect on normal bridge balance.

# WAGNER GROUND CONNECTION....

- There are also capacitances from points C and D to ground,
  - but the addition of the Wagner ground point eliminates them from the detector circuit,
  - since current through these capacitances will enter through the Wagner ground connection.
- capacitances across the bridge arms are not eliminated by the Wagner ground connection
  - they will still affect the accuracy of the measurement.
- The idea of the Wagner ground can also be applied to other bridges, as long as care is taken that the grounding arms duplicate the impedance of one pair of bridge arms across which they are connected.
- Since the addition of the Wagner ground connection does not affect the balance conditions, the procedure for measurement remains unaltered.

**Thank you**