Lecture 9

Bridge Measurements

INTRODUCTION

- A **Wheatstone bridge** is an electrical circuit used to measure an unknown electrical resistance by
  - balancing two legs of a bridge circuit,
  - one leg of which includes the unknown component.
- Its operation is similar to the original potentiometer.
- It was invented by Samuel Hunter Christie in 1833
- Improved and popularized by Sir Charles Wheatstone in 1843.
- One of the Wheatstone bridge's initial uses was for the purpose of soils analysis and comparison

INTRODUCTION……..

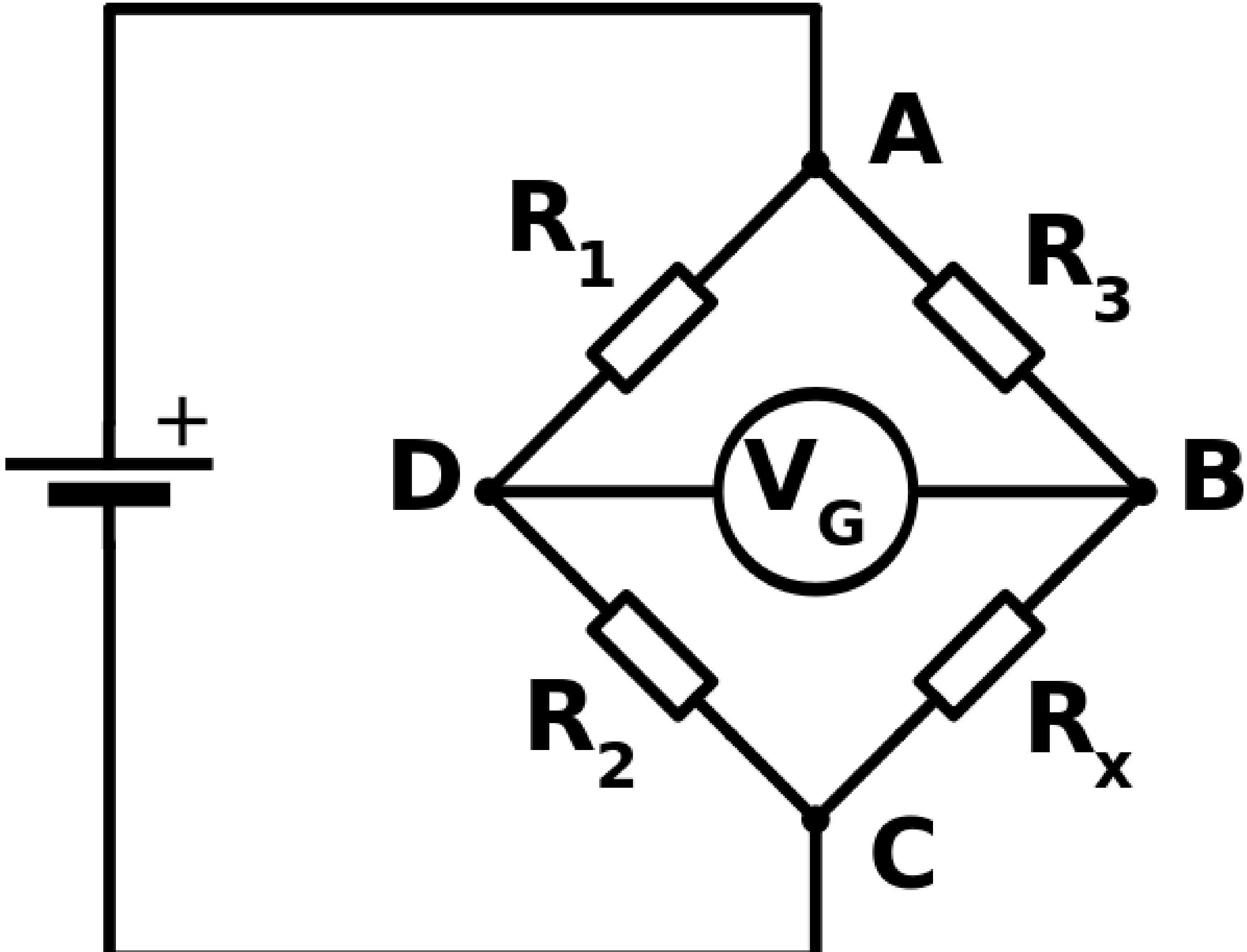
- Precision measurements of components values have been made for many years using various forms of bridges.
- The simplest form of bridge is for the purpose is called the ***Wheatstone bridge.***
- There are variations of the ***Wheatstone bridge*** for measuring very high and very low resistances.
- There is an entire group of ac bridges for measuring
  - Inductance
  - Capacitance,
  - Admittance
  - Conductance
  - any of the impedance parameters.
- The bridge circuit still forms the backbone of some measurements and for the interfacing of transducers.

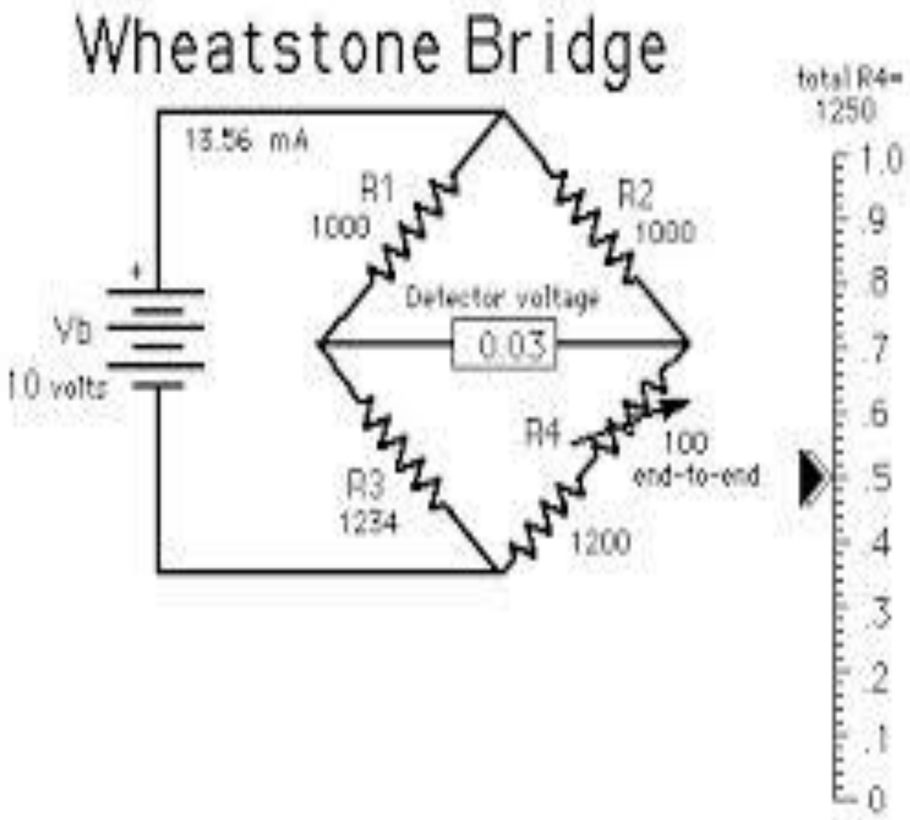
WHEATSTONE BRIDGE

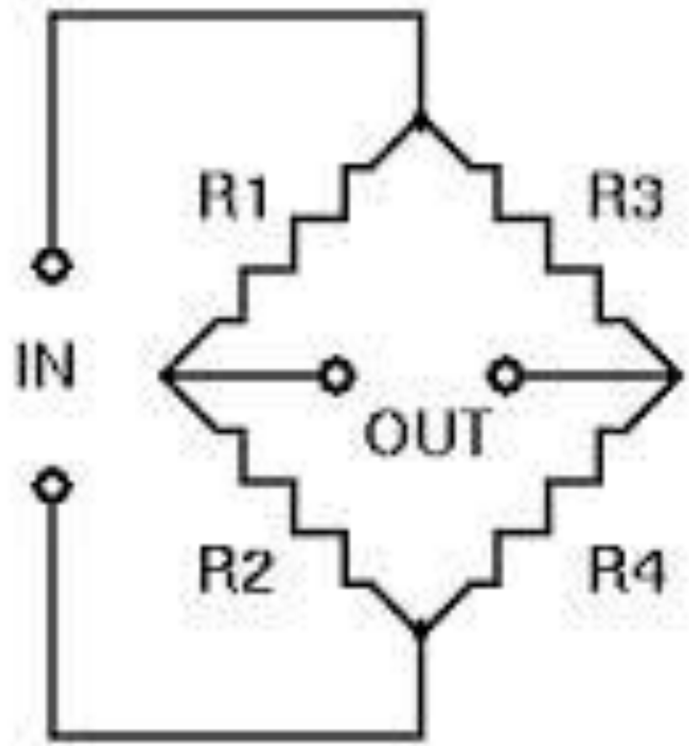
Basic Operation

- The bridge has four resistive arms, together with a source of emf (a battery) and a null detector, usually a galvanometer or other sensitive current meter.
- The current through the galvanometer depends on the potential difference between points ***c*** and ***d***.
- The bridge is said to be balanced when the potential difference across the galvanometer is 0 V so that there is no current through the galvanometer.

WEATSTONE BRIDGE




Wheatstone Bridge
13.56 mA
R1
1000
R2
1000
Detector voltage
0.03
+
Vb
1.0 volts
R3
1234
R4
100
end-to-end
1200
total R4=
1250
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0


R1
R3
IN
OUT
R2
R4

WHEATSTONE BRIDGE……

- This condition occurs when
  - the voltage from point ***c*** to point ***a*** equals the voltage from point ***d*** to point ***a***; or
  - by referring to the other battery terminal, when the voltage from point ***c*** to point ***b*** equals the voltage from point ***d*** to ***point 1'***. Hence the bridge is balanced when

$$I_1 R_1 = I_2 R_2 \qquad (5.1)$$

- If the galvanometer current is zero, the following condition also exist:
- $$I_1 = I_3 = \frac{E}{R_1 + R_3} \qquad (5.2)$$ and
-
- $$I_2 = I_4 = \frac{E}{R_1 + R_4} \qquad (5.3)$$

WHEATSTONE BRIDGE……

- Combining Eqs. (5-1), (5-2) and (5-3) and simplifying, we obtain $$R_1 R_4 = R_2 R_3 \qquad (5.4)$$
- from which $$\frac{R_1}{R_1 + R_3} = \frac{R_2}{R_2 + R_4} \qquad (5.5)$$
- Which is the famous equation for a balanced Wheatstone bridge
- Wheatstone Bridge is used for the precision measurement of Resistances ranging from fractions of an ohm to several megohms.
- The ratio control switches control the ratio arms in decade steps.
- The remaining four step switches set the resistance of the standard arm which can be expressed in term of the remaining resistors as follows: $$R_x = R_3 \frac{R_1}{R_1} \qquad (5.6)$$

WHEATSTONE BRIDGE……

- Resistor R_3 is called the standard arm of the bridge, and resistor R_2 and R_1 are called the ratio arms.
- The measurement of the unknown resistance R_x is independent of the characteristics of the calibration of the null-detecting galvanometer
  - provided that the null detector has sufficient sensitivity to indicate the balance position of the bridge
  - with the required degree of precision

Measurement Errors

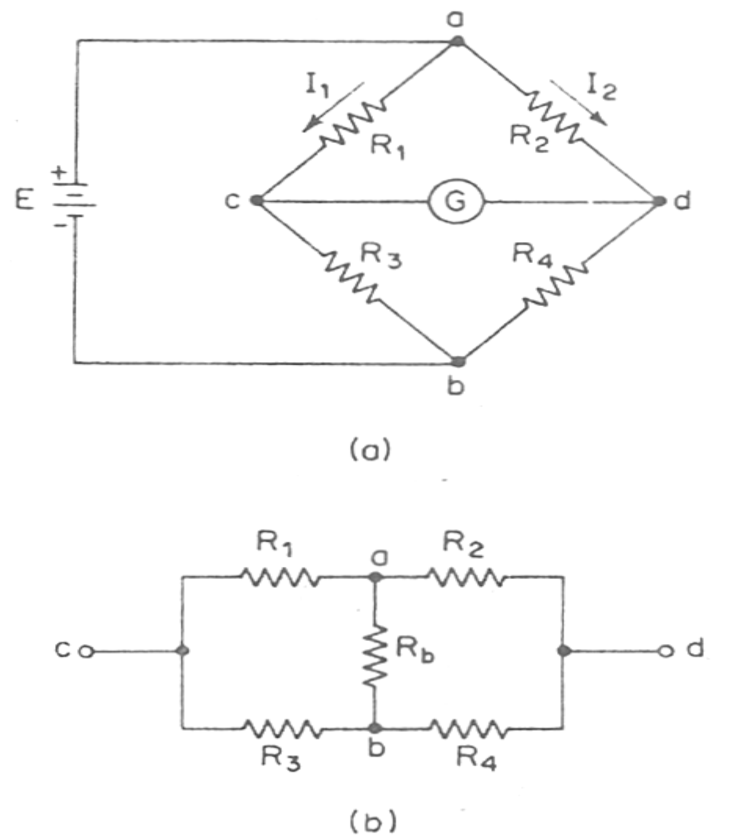
- Wheatstone bridge is widely used for precision measurement of resistance from approximately 1 to the low megohm range.
- The main source of measurement error is found in the limiting errors of the three known resistors.
- Other errors may include the following:
  - Insufficient sensitivity of the null detector.
  - Changes resistance of the bridge arms due to the heating effect of the current through the resistors.
  - Heat effect ($I^2 R$) of the bridge arm currents may change the resistance of the resistor in question.
  - The rise in temperature affects the resistance during the actual measurement and
  - excessive currents may cause a permanent change in resistance values.

Measurement Errors…..

- The power dissipation in the bridge arms must therefore be cornuted in advance,
  - particularly when low-resistance values are to be measured, and
  - the current must be limited to a safe value.
- Thermal emfs in the bridge circuit or the galvanometer circuit can also cause problems when low value resistors are being measured.
- To prevent thermal emfs, the more sensitive galvanometers sometimes have copper coils and copper suspension systems to avoid having dissimilar metals in contact with one another and generating thermal emfs.
- Errors due to the resistance of leads and contacts exterior to the actual bridge circuit play a role in the measurement of very low-resistance values.
- These errors may be reduced by using a Kelvin bridge

Thevenin Equivalent Circuit

- To determine whether or not the galvanometer has the required sensitivity to detect an unbalance condition, it is necessary to calculate the galvanometer current.
- Different galvanometers may require
  - different currents per unit deflection (current sensitivity),
  - may have a different internal resistance.
  - It is impossible to say, without prior computation, which galvanometer has the required degree of precision.
- Wheatstone Bridge will make the bridge circuit more sensitive to an unbalance condition.
- This sensitivity can be calculated by "solving" the bridge circuit for a small unbalance.
- The solution is approached by converting the Wheatstone bridge of Fig. 5-1 to its Thevenin equivalent.



$$I_1 = \frac{E}{R_1 + R_3} \text{ and } I_2 = \frac{E}{R_2 + R_4} \tag{5-7}$$

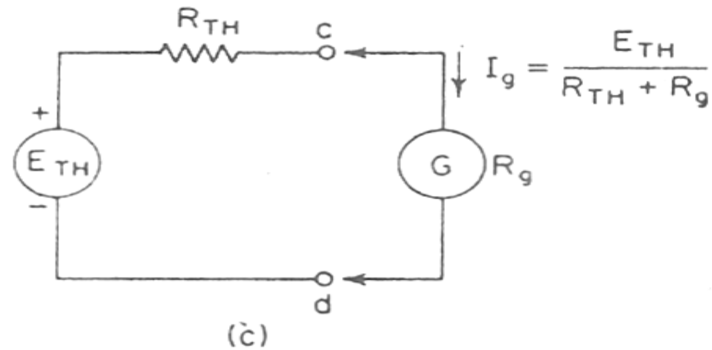


Figure 5-2 Application of Thévenin's theorem to the Wheatstone bridge. (a) Wheatstone bridge configuration. (b) Thévenin resistance looking into terminals *c* and *d*. (c) Complete Thévenin circuit, with the galvanometer connected to terminals *c* and *d*.

Thevenin Equivalent Circuit….

- Since we are interested in the current through the galvanometer, the Thevenin equivalent circuit is determined by looking into galvanometer terminals ***c*** and ***d*** in Fig. 5.1
- Two steps must be taken to find the Thévenin equivalent;
  - the first step involves finding the equivalent voltage appearing at terminals ***c*** and ***d*** when the galvanometer is removed from the circuit.
  - The second step involves finding the equivalent resistance looking into terminals ***c*** and ***d***, with the battery replaced by its internal resistance.
- The Thevenin, or open-circuit, voltage is found by referring to Fig. 5-2(a), and we can write
- $E_{cd} = E_{ac} — 1_1R1 — I_2R_2$

Thevenin Equivalent Circuit…..

- This is the voltage of the Thévenin generator.
- The resistance of the Thevenin equivalent circuit is found by looking back into terminals ***c*** and ***d*** and replacing the battery by its internal resistance.
- The circuit of Fig. 5-2(b) represents the Thévenin resistance.
  - Notice that the internal resistance, R_b of the battery has been included in Fig. 5-2(b).
- Converting this circuit into a more convenient form requires use of the delta-wye transformation theorem.
- In most cases, however, the extremely low internal resistance of the battery can be neglected
  - this simplifies the reduction of Fig. 5-2(a) to its Thévenin equivalent considerably.

Solve the related examples yourself

- Referring to Fig. 5-2(b), we see that a short circuit exists between points ***a*** and ***b*** when the internal resistance of the battery is assumed to be 0 Ω.
- The Thevenin resistance, looking into terminals ***c*** and ***d***, then becomes
-
- $$\text{Rth} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4} \qquad (5\text{-}8)$$
- The Thévenin equivalent of the Wheatstone bridge circuit therefore reduces to a Thevenin generator with an emf described by Eq. (5-7) and an internal resistance given by Eq. (5-8). See Fig. 5-2(c).
- When the null detector is now connected to the output terminals of the Thévenin equivalent circuit, the galvanometer current is found to be
-
- $$\text{Ig} = \frac{E_{TH}}{R_{TH} + R_g}$$
- where I_g is the galvanometer current and R_g its resistance.

Limitations

- The Wheatstone bridge is limited to the measurement of resistances ranging from a few ohms to several megohms.
- The upper limit is set by the reduction in sensitivity to unbalance, caused by high resistance values
  - in this case the equivalent Thevenin resistance becomes high,
  - thus reducing the galvanometer current.
- The lower limit is set by the resistance of the connecting leads and the contact resistance at the binding posts
- The resistance of the leads could be calculated or measured, and the final result modified
  - but contact resistance is very hard to compute or measure.
- For low-resistance measurements, therefore, the Kelvin bridge is generally the preferred instrument

KELVIN BRIDGE

- The Kelvin bridge is a modification of the Wheatstone bridge
  - provides greatly increased accuracy in the measurement of low-value resistances, generally below1 .
- Consider the bridge circuit where R_y represents the resistance of the connecting lead from R_3 to R_x.
- Two galvanometer connections are possible, to point ***n*** or to point ***m***.
- When the galvanometer is connected to point ***m***, the resistance R_y of the connecting lead is added to the unknown R_x
  - resulting in too high indication for R_x.
- When connection is made to point ***n***, R_y is added to bridge arm R_3 and the resulting measurement of R_x will be lower than it should be
  - because now the actual value of R_3 is higher than its nominal value by resistance R_y.

KELVIN BRIDGE

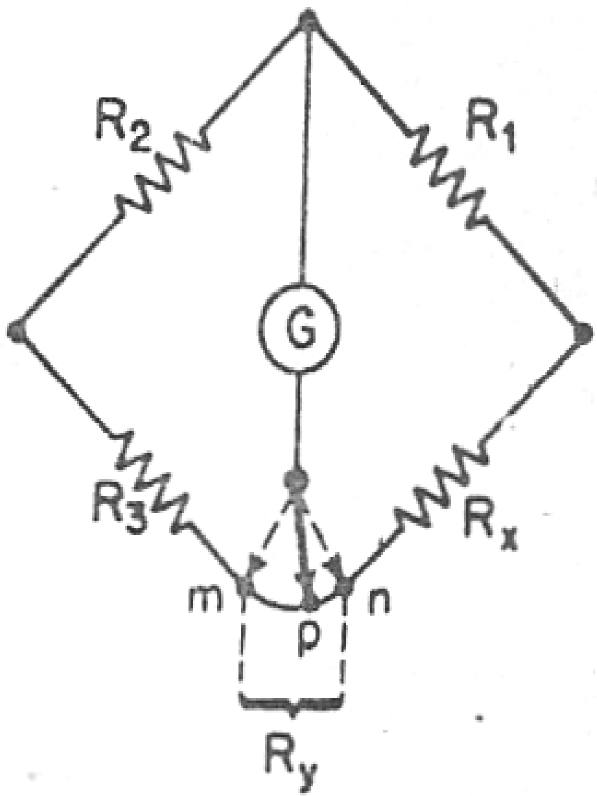


Figure 5-4 Wheatstone bridge circuit, showing resistance R_y of the lead from point *m* to point *n*.

KELVIN BRIDGE…..

- If the galvanometer is connected to a point p, in between the two points ***m*** and ***n***, in such a way that the ratio of the resistances from *n* to ***p*** and from *m* to *p* equals the ratio of resistors R_1 and R_2, we can write $$\frac{R_{np}}{R_{mp}} = \frac{R_1}{R_2} \qquad (5\text{-}10)$$

- The balance equation for the bridge yields
- $$R_x + R_{np} = \frac{R_1}{R_2}(R_3 + R_{mp}) \qquad (5\text{-}11)$$

- Substituting Eq. (5-10) into Eq. (5-11), we obtain
- $$R_x + \left(\frac{R_1}{R_1 + R_2}\right) R_y = \left[R_3 + \left(\frac{R_2}{R_1 + R_2}\right) R_y\right] \qquad (5\text{-}12)$$

- which reduces to $$R_x = \frac{R_1}{R_2} R_3 \qquad (5\text{-}13)$$

KELVIN BRIDGE…..

- Equation (5-13) is the usual balance equation developed for the Wheatstone bridge
  - it indicates that the effect of the resistance of the connecting lead from point ***n*** to point ***m*** has been eliminated by connecting the galvanometer to the intermediate position ***p***.
- This development forms the basis for construction of the Kelvin double bridge, commonly known as the Kelvin bridge.

Kelvin Double Bridge

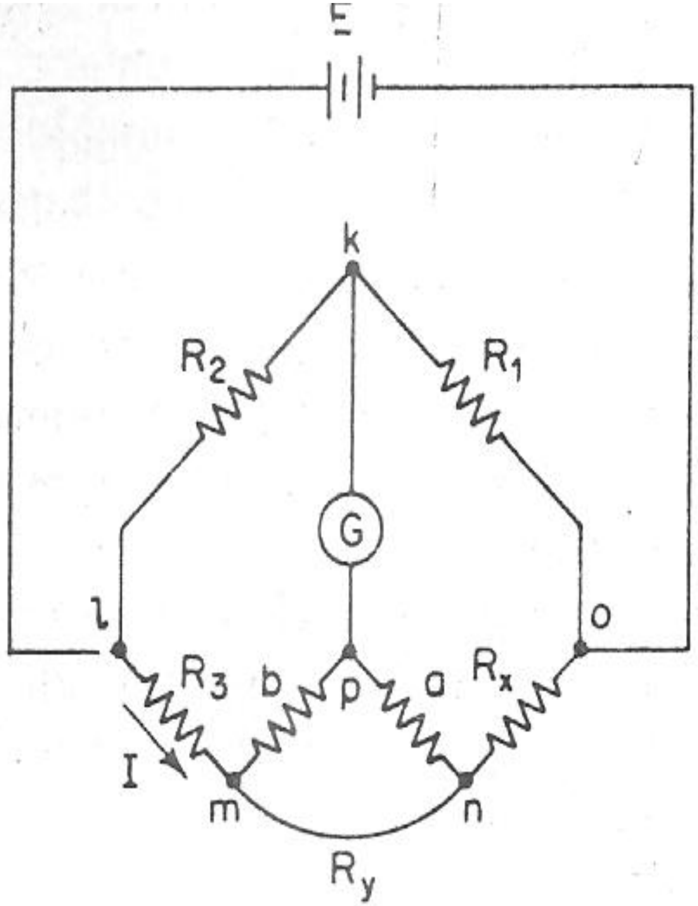


Figure 5-5 Basic Kelvin double bridge circuit.

Kelvin Double Bridge….

- The term *double* bridge is used because the circuit contains a second set of ratio arms, as shown in the schematic diagram of Fig. 5-5.
- This second set of arms, labeled ***a*** and ***b*** in the diagram, connects the galvanometer to a point ***p*** at the appropriate potential between ***m*** and ***n***
- it eliminates the effect of the yoke resistance R_y.
- An initially established condition is that the resistance ratio of ***a*** and ***b*** is the same as the ratio of R_1 and R_2.
- The galvanometer indication will be zero when the potential at ***k*** equal the potential at ***p***, or when $E_{kl} = E_{lmp}$ where

Kelvin Double Bridge….

- $E_{kl} = \frac{R_2}{R_1+R_2}E = \frac{R_2}{R_1+R_2}I[R_3+R_x+\frac{(a+b)R_y}{a+b+R_y}]$ (5-14)
- and $E_{Imp} = I\{R_3 + \frac{b}{a+b}[\frac{(a+b)R_y}{a+b+R_y}]\}$ (5-15)
- We can solve for ***R1*** by equating $\boldsymbol{E_{kl}}$, and $\boldsymbol{E_{Imp}}$ in the following manner: $$\frac{R_2}{R_1+R_2}I[R_3+R_x+\frac{(a+b)R_y}{a+b+R_y}]=[R_3+\frac{b}{a+b}\frac{(a+b)R_y}{a+b+R_y}]$$
- Or simplifying, we get
- $$R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} = \frac{R_1+R_2}{R_2}[R_3+\frac{bR_y}{a+b+R_y}]$$
-
- and expanding the right-hand member yields
- $$R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} = \frac{R_1R_3}{R_2}R_3 + \frac{R_1+R_2}{R_2}\cdot\frac{bR_y}{a+\mathbf{b}+R_y}$$

Kelvin Double Bridge….

- Solving for $\boldsymbol{R_x}$ yields

- $R_x = \frac{R_1 R_3}{R_2} = \frac{R_1 R_3}{R_2} R_3 + \frac{R_1 + R_2}{R_2} . \frac{bR_y}{a + \mathbf{b} + R_y}$ so that
- $R_x = \frac{R_1 R_3}{R_2} + \frac{R_1}{R_2} . \frac{bB_y}{a + b + R_y} \left(\frac{R_1}{R_2} - \frac{a}{b} \right)$ (5-16)

- Using the initially established condition that $a/b = R_1/R_2$, we see that Eq. (5-16) reduces to the well-known relationship $R_x = R_3 \frac{R_1}{R_2}$ (5-17)

- Equation (5-17) is the usual working equation for the Kelvin bridge.
- It indicates that the resistance of the yoke has no effect on the measurement, provided that the two sets of ratio arms have equal resistance ratios.

Applications

- The Kelvin bridge is used for measuring very low resistances, from approximately 1 to as low as 0.00001 .
- Figure 5-6 shows the simplified circuit diagram of a commercial Kelvin bridge capable of measuring resistances from 10 to 0.00001 .
- In this bridge, resistance R_3 of Eq. (5-17) is represented by the variable standard resistor in Fig. 5-6.
- The ratio arms (R_1 and R_2) can usually be switched in a number of decade steps.
- Contact potential drops in the measuring circuit may cause large errors

Applications…..

- To reduce this effect, the standard resistor consists of nine steps of 0.001 each, plus a calibrated manganin bar of 0.0011 with a sliding contact.
- The total resistance of the ***R3*** arm therefore amounts to 0.0101 and is variable in steps of 0.001 plus fractions of 0.0011 by the sliding contact.
- When both contacts are switched to select the suitable value of standard resistor, the voltage drop between the ratio-arm connection points is changed
  - but the total resistance around the battery circuit is unchanged.
- This arrangement places any contact resistance in series with the relatively high-resistance values of the ratio arms
  - the contact resistance has negligible effect.

Applications…..

- The ratio $\boldsymbol{R_1/R_2}$ should be selected such that a relatively large part of the standard resistance is used in the measuring circuit.
- In this way the value of unknown resistance R_x is determined with the largest possible number of significant figures, and the measurement accuracy is improved.

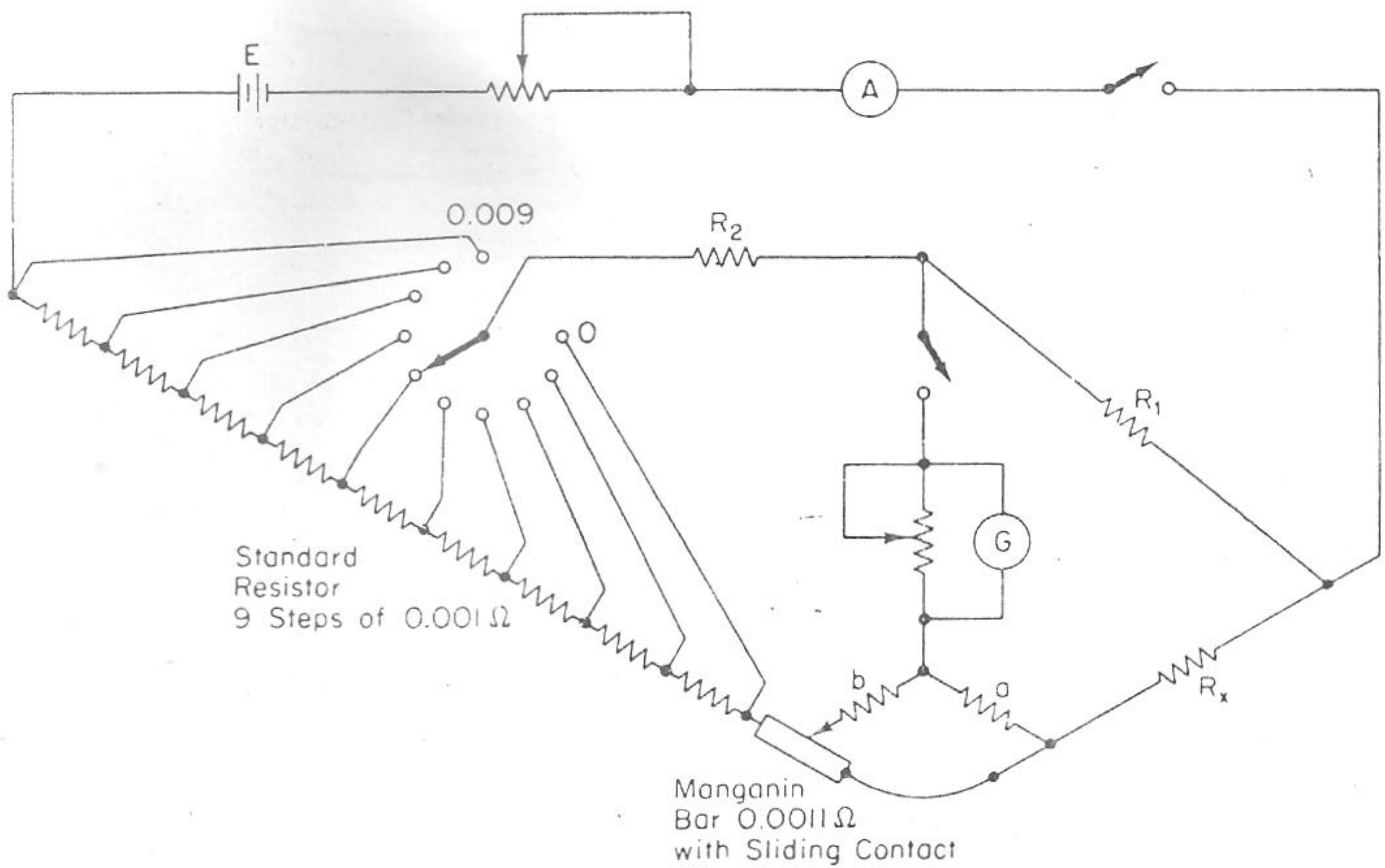


Figure 5-6 Simplified circuit of a Kelvin double bridge used for the measurement of very low resistances.

Thank you