CHAPTER 1

1.1. Given the vectors $\mathbf{M} = -10\mathbf{a}_x + 4\mathbf{a}_y - 8\mathbf{a}_z$ and $\mathbf{N} = 8\mathbf{a}_x + 7\mathbf{a}_y - 2\mathbf{a}_z$, find:

a) a unit vector in the direction of $-\mathbf{M} + 2\mathbf{N}$.

$-\mathbf{M} + 2\mathbf{N} = 10\mathbf{a}_x - 4\mathbf{a}_y + 8\mathbf{a}_z + 16\mathbf{a}_x + 14\mathbf{a}_y - 4\mathbf{a}_z = (26, 10, 4)$

Thus

$$\mathbf{a} = \frac{(26, 10, 4)}{|(26, 10, 4)|} = \underline{(0.92, 0.36, 0.14)}$$

b) the magnitude of $5\mathbf{a}_x + \mathbf{N} - 3\mathbf{M}$:

$(5, 0, 0) + (8, 7, -2) - (-30, 12, -24) = (43, -5, 22)$, and $|(43, -5, 22)| = \underline{48.6}$.

c) $|\mathbf{M}||2\mathbf{N}|(\mathbf{M} + \mathbf{N})$:

$|(-10, 4, -8)||(16, 14, -4)|(-2, 11, -10) = (13.4)(21.6)(-2, 11, -10)$

$= \underline{(-580.5, 3193, -2902)}$

1.2. Given three points, $A(4, 3, 2)$, $B(-2, 0, 5)$, and $C(7, -2, 1)$:

a) Specify the vector $\mathbf{A}$ extending from the origin to the point A.

$$\mathbf{A} = (4, 3, 2) = \underline{4\mathbf{a}_x + 3\mathbf{a}_y + 2\mathbf{a}_z}$$

b) Give a unit vector extending from the origin to the midpoint of line AB.

The vector from the origin to the midpoint is given by

$\mathbf{M} = (1/2)(\mathbf{A} + \mathbf{B}) = (1/2)(4 - 2, 3 + 0, 2 + 5) = (1, 1.5, 3.5)$

The unit vector will be

$$\mathbf{m} = \frac{(1, 1.5, 3.5)}{|(1, 1.5, 3.5)|} = \underline{(0.25, 0.38, 0.89)}$$

c) Calculate the length of the perimeter of triangle ABC:

Begin with $\mathbf{AB} = (-6, -3, 3)$, $\mathbf{BC} = (9, -2, -4)$, $\mathbf{CA} = (3, -5, -1)$.

Then

$$|\mathbf{AB}| + |\mathbf{BC}| + |\mathbf{CA}| = 7.35 + 10.05 + 5.91 = \underline{23.32}$$

1.3. The vector from the origin to the point A is given as $(6, -2, -4)$, and the unit vector directed from the origin toward point B is $(2, -2, 1)/3$. If points A and B are ten units apart, find the coordinates of point B.

With $\mathbf{A} = (6, -2, -4)$ and $\mathbf{B} = \frac{1}{3}B(2, -2, 1)$, we use the fact that $|\mathbf{B} - \mathbf{A}| = 10$, or

$|(6 - \frac{2}{3}B)\mathbf{a}_x - (2 - \frac{2}{3}B)\mathbf{a}_y - (4 + \frac{1}{3}B)\mathbf{a}_z| = 10$

Expanding, obtain

$36 - 8B + \frac{4}{9}B^2 + 4 - \frac{8}{3}B + \frac{4}{9}B^2 + 16 + \frac{8}{3}B + \frac{1}{9}B^2 = 100$

or $B^2 - 8B - 44 = 0$. Thus $B = \frac{8 \pm \sqrt{64 - 176}}{2} = 11.75$ (taking positive option) and so

$$\mathbf{B} = \frac{2}{3}(11.75)\mathbf{a}_x - \frac{2}{3}(11.75)\mathbf{a}_y + \frac{1}{3}(11.75)\mathbf{a}_z = \underline{7.83\mathbf{a}_x - 7.83\mathbf{a}_y + 3.92\mathbf{a}_z}$$

1.4. given points $A(8, -5, 4)$ and $B(-2, 3, 2)$, find:

a) the distance from A to B.

$$|\mathbf{B} - \mathbf{A}| = |(-10, 8, -2)| = \underline{12.96}$$

b) a unit vector directed from A towards B. This is found through

$$\mathbf{a}_{AB} = \frac{\mathbf{B} - \mathbf{A}}{|\mathbf{B} - \mathbf{A}|} = \underline{(-0.77, 0.62, -0.15)}$$

c) a unit vector directed from the origin to the midpoint of the line AB.

$$\mathbf{a}_{0M} = \frac{(\mathbf{A} + \mathbf{B})/2}{|(\mathbf{A} + \mathbf{B})/2|} = \frac{(3, -1, 3)}{\sqrt{19}} = \underline{(0.69, -0.23, 0.69)}$$

d) the coordinates of the point on the line connecting A to B at which the line intersects the plane $z = 3$. Note that the midpoint, $(3, -1, 3)$, as determined from part c happens to have z coordinate of 3. This is the point we are looking for.

1.5. A vector field is specified as $\mathbf{G} = 24xy\mathbf{a}_x + 12(x^2 + 2)\mathbf{a}_y + 18z^2\mathbf{a}_z$. Given two points, $P(1, 2, -1)$ and $Q(-2, 1, 3)$, find:

a) $\mathbf{G}$ at P: $\mathbf{G}(1, 2, -1) = \underline{(48, 36, 18)}$

b) a unit vector in the direction of $\mathbf{G}$ at Q: $\mathbf{G}(-2, 1, 3) = (-48, 72, 162)$, so

$$\mathbf{a}_G = \frac{(-48, 72, 162)}{|(-48, 72, 162)|} = \underline{(-0.26, 0.39, 0.88)}$$

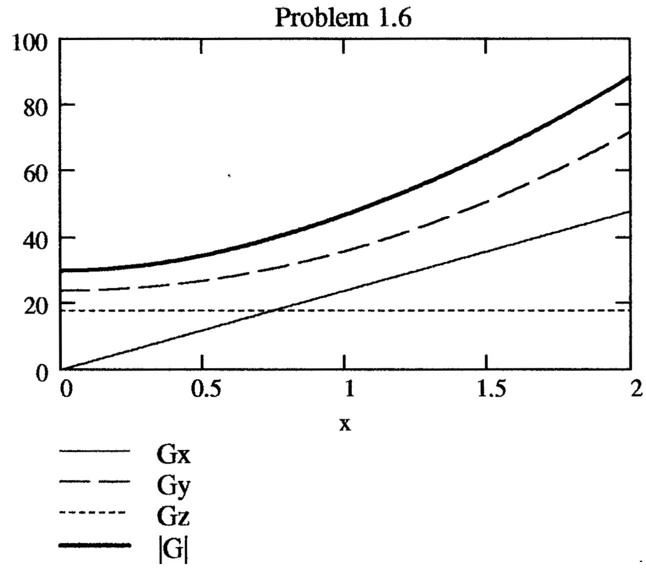
c) a unit vector directed from Q toward P:

$$\mathbf{a}_{QP} = \frac{\mathbf{P} - \mathbf{Q}}{|\mathbf{P} - \mathbf{Q}|} = \frac{(3, -1, 4)}{\sqrt{26}} = \underline{(0.59, 0.20, -0.78)}$$

d) the equation of the surface on which $|\mathbf{G}| = 60$: We write $60 = |(24xy, 12(x^2 + 2), 18z^2)|$, or $10 = |(4xy, 2x^2 + 4, 3z^2)|$, so the equation is

$$\underline{100 = 16x^2y^2 + 4x^4 + 16x^2 + 16 + 9z^4}$$

1.6. For the **G** field in Problem 1.5, make sketches of G_x, G_y, G_z and $|\mathbf{G}|$ along the line $y = 1$, $z = 1$, for $0 \leq x \leq 2$. We find $\mathbf{G}(x, 1, 1) = (24x, 12x^2 + 24, 18)$, from which $G_x = 24x$, $G_y = 12x^2 + 24$, $G_z = 18$, and $|\mathbf{G}| = 6\sqrt{4x^4 + 32x^2 + 25}$. Plots are shown below.



1.7. Given the vector field $\mathbf{E} = 4zy^2 \cos 2x\mathbf{a}_x + 2zy \sin 2x\mathbf{a}_y + y^2 \sin 2x\mathbf{a}_z$ for the region $|x|$, $|y|$, and $|z|$ less than 2, find:

a) the surfaces on which $E_y = 0$. With $E_y = 2zy \sin 2x = 0$, the surfaces are 1) the plane $\underline{z = 0}$, with $|x| < 2, |y| < 2$; 2) the plane $\underline{y = 0}$, with $|x| < 2, |z| < 2$; 3) the plane $\underline{x = 0}$, with $|y| < 2, |z| < 2$; 4) the plane $\underline{x = \pi/2}$, with $|y| < 2, |z| < 2$.

b) the region in which $E_y = E_z$: This occurs when $2zy \sin 2x = y^2 \sin 2x$, or on the plane $\underline{2z = y}$, with $|x| < 2, |y| < 2, |z| < 1$.

c) the region in which $\mathbf{E} = 0$: We would have $E_x = E_y = E_z = 0$, or $zy^2 \cos 2x = zy \sin 2x = y^2 \sin 2x = 0$. This condition is met on the plane $\underline{y = 0}$, with $|x| < 2, |z| < 2$.

1.8. Two vector fields are $\mathbf{F} = -10\mathbf{a}_x + 20x(y-1)\mathbf{a}_y$ and $\mathbf{G} = 2x^2y\mathbf{a}_x - 4\mathbf{a}_y + z\mathbf{a}_z$. For the point $P(2, 3, -4)$, find:

a) $|\mathbf{F}|$: **F** at $(2, 3, -4) = (-10, 80, 0)$, so $\underline{|\mathbf{F}| = 80.6}$.

b) $|\mathbf{G}|$: **G** at $(2, 3, -4) = (24, -4, -4)$, so $\underline{|\mathbf{G}| = 24.7}$.

c) a unit vector in the direction of $\mathbf{F} - \mathbf{G}$: $\mathbf{F} - \mathbf{G} = (-10, 80, 0) - (24, -4, -4) = (-34, 84, 4)$. So

$$\mathbf{a} = \frac{\mathbf{F} - \mathbf{G}}{|\mathbf{F} - \mathbf{G}|} = \frac{(-34, 84, 4)}{90.7} = \underline{(-0.37, 0.92, 0.04)}$$

d) a unit vector in the direction of $\mathbf{F} + \mathbf{G}$: $\mathbf{F} + \mathbf{G} = (-10, 80, 0) + (24, -4, -4) = (14, 76, -4)$. So

$$\mathbf{a} = \frac{\mathbf{F} + \mathbf{G}}{|\mathbf{F} + \mathbf{G}|} = \frac{(14, 76, -4)}{77.4} = \underline{(0.18, 0.98, -0.05)}$$

1.9. A field is given as

$$\mathbf{G} = \frac{25}{(x^2+y^2)}(x\mathbf{a}_x + y\mathbf{a}_y)$$

Find:

a) a unit vector in the direction of $\mathbf{G}$ at $P(3, 4, -2)$: Have $\mathbf{G}_p = 25/(9+16) \times (3, 4, 0) = 3\mathbf{a}_x + 4\mathbf{a}_y$, and $|\mathbf{G}_p| = 5$. Thus $\mathbf{a}_G = \underline{(0.6, 0.8, 0)}$.

b) the angle between $\mathbf{G}$ and $\mathbf{a}_x$ at P: The angle is found through $\mathbf{a}_G \cdot \mathbf{a}_x = \cos\theta$. So $\cos\theta = (0.6, 0.8, 0) \cdot (1, 0, 0) = 0.6$. Thus $\underline{\theta = 53^\circ}$.

c) the value of the following double integral on the plane $y = 7$:

$$\int_0^4 \int_0^2 \mathbf{G} \cdot \mathbf{a}_y dz dx$$

$$\int_0^4 \int_0^2 \frac{25}{x^2+y^2}(x\mathbf{a}_x + y\mathbf{a}_y) \cdot \mathbf{a}_y dz dx = \int_0^4 \int_0^2 \frac{25}{x^2+49} \times 7 \, dz dx = \int_0^4 \frac{350}{x^2+49} dx$$

$$= 350 \times \frac{1}{7}\left[\tan^{-1}\left(\frac{4}{7}\right) - 0\right] = \underline{26}$$

1.10. Use the definition of the dot product to find the interior angles at A and B of the triangle defined by the three points $A(1, 3, -2)$, $B(-2, 4, 5)$, and $C(0, -2, 1)$:

a) Use $\mathbf{R}_{AB} = (-3, 1, 7)$ and $\mathbf{R}_{AC} = (-1, -5, 3)$ to form $\mathbf{R}_{AB} \cdot \mathbf{R}_{AC} = |\mathbf{R}_{AB}||\mathbf{R}_{AC}|\cos\theta_A$. Obtain $3 + 5 + 21 = \sqrt{59}\sqrt{35}\cos\theta_A$. Solve to find $\underline{\theta_A = 65.3^\circ}$.

b) Use $\mathbf{R}_{BA} = (3, -1, -7)$ and $\mathbf{R}_{BC} = (2, -6, -4)$ to form $\mathbf{R}_{BA} \cdot \mathbf{R}_{BC} = |\mathbf{R}_{BA}||\mathbf{R}_{BC}|\cos\theta_B$. Obtain $6 + 6 + 28 = \sqrt{59}\sqrt{56}\cos\theta_B$. Solve to find $\underline{\theta_B = 45.9^\circ}$.

1.11. Given the points $M(0.1, -0.2, -0.1)$, $N(-0.2, 0.1, 0.3)$, and $P(0.4, 0, 0.1)$, find:

a) the vector $\mathbf{R}_{MN}$: $\mathbf{R}_{MN} = (-0.2, 0.1, 0.3) - (0.1, -0.2, -0.1) = \underline{(-0.3, 0.3, 0.4)}$.

b) the dot product $\mathbf{R}_{MN} \cdot \mathbf{R}_{MP}$: $\mathbf{R}_{MP} = (0.4, 0, 0.1) - (0.1, -0.2, -0.1) = (0.3, 0.2, 0.2)$. $\mathbf{R}_{MN} \cdot \mathbf{R}_{MP} = (-0.3, 0.3, 0.4) \cdot (0.3, 0.2, 0.2) = -0.09 + 0.06 + 0.08 = \underline{0.05}$.

c) the scalar projection of $\mathbf{R}_{MN}$ on $\mathbf{R}_{MP}$:

$$\mathbf{R}_{MN} \cdot \mathbf{a}_{RMP} = (-0.3, 0.3, 0.4) \cdot \frac{(0.3, 0.2, 0.2)}{\sqrt{0.09 + 0.04 + 0.04}} = \frac{0.05}{\sqrt{0.17}} = \underline{0.12}$$

d) the angle between $\mathbf{R}_{MN}$ and $\mathbf{R}_{MP}$:

$$\theta_M = \cos^{-1}\left(\frac{\mathbf{R}_{MN} \cdot \mathbf{R}_{MP}}{|\mathbf{R}_{MN}||\mathbf{R}_{MP}|}\right) = \cos^{-1}\left(\frac{0.05}{\sqrt{0.34}\sqrt{0.17}}\right) = \underline{78^\circ}$$

1.12. Given points $A(10, 12, -6)$, $B(16, 8, -2)$, $C(8, 1, -4)$, and $D(-2, -5, 8)$, determine:

a) the vector projection of $\mathbf{R}_{AB} + \mathbf{R}_{BC}$ on $\mathbf{R}_{AD}$: $\mathbf{R}_{AB} + \mathbf{R}_{BC} = \mathbf{R}_{AC} = (8, 1, 4) - (10, 12, -6) = (-2, -11, 10)$ Then $\mathbf{R}_{AD} = (-2, -5, 8) - (10, 12, -6) = (-12, -17, 14)$. So the projection will be:

$$(\mathbf{R}_{AC} \cdot \mathbf{a}_{RAD})\mathbf{a}_{RAD} = \left[(-2, -11, 10) \cdot \frac{(-12, -17, 14)}{\sqrt{629}}\right] \frac{(-12, -17, 14)}{\sqrt{629}} = \underline{(-6.7, -9.5, 7.8)}$$

b) the vector projection of $\mathbf{R}_{AB} + \mathbf{R}_{BC}$ on $\mathbf{R}_{DC}$: $\mathbf{R}_{DC} = (8, -1, 4) - (-2, -5, 8) = (10, 6, -4)$. The projection is:

$$(\mathbf{R}_{AC} \cdot \mathbf{a}_{RDC})\mathbf{a}_{RDC} = \left[(-2, -11, 10) \cdot \frac{(10, 6, -4)}{\sqrt{152}}\right] \frac{(10, 6, -4)}{\sqrt{152}} = \underline{(-8.3, -5.0, 3.3)}$$

c) the angle between $\mathbf{R}_{DA}$ and $\mathbf{R}_{DC}$: Use $\mathbf{R}_{DA} = -\mathbf{R}_{AD} = (12, 17, -14)$ and $\mathbf{R}_{DC} = (10, 6, -4)$. The angle is found through the dot product of the associated unit vectors, or:

$$\theta_D = \cos^{-1}(\mathbf{a}_{RDA} \cdot \mathbf{a}_{RDC}) = \cos^{-1}\left(\frac{(12, 17, -14) \cdot (10, 6, -4)}{\sqrt{629}\sqrt{152}}\right) = \underline{26^\circ}$$

1.13. a) Find the vector component of $\mathbf{F} = (10, -6, 5)$ that is parallel to $\mathbf{G} = (0.1, 0.2, 0.3)$:

$$\mathbf{F}_{||G} = \frac{\mathbf{F} \cdot \mathbf{G}}{|\mathbf{G}|^2}\mathbf{G} = \frac{(10, -6, 5) \cdot (0.1, 0.2, 0.3)}{0.01 + 0.04 + 0.09}(0.1, 0.2, 0.3) = \underline{(0.93, 1.86, 2.79)}$$

b) Find the vector component of $\mathbf{F}$ that is perpendicular to $\mathbf{G}$:

$$\mathbf{F}_{pG} = \mathbf{F} - \mathbf{F}_{||G} = (10, -6, 5) - (0.93, 1.86, 2.79) = \underline{(9.07, -7.86, 2.21)}$$

c) Find the vector component of $\mathbf{G}$ that is perpendicular to $\mathbf{F}$:

$$\mathbf{G}_{pF} = \mathbf{G} - \mathbf{G}_{||F} = \mathbf{G} - \frac{\mathbf{G} \cdot \mathbf{F}}{|\mathbf{F}|^2}\mathbf{F} = (0.1, 0.2, 0.3) - \frac{1.3}{100 + 36 + 25}(10, -6, 5) = \underline{(0.02, 0.25, 0.26)}$$

1.14. The four vertices of a regular tetrahedron are located at $O(0, 0, 0)$, $A(0, 1, 0)$, $B(0.5\sqrt{3}, 0.5, 0)$, and $C(\sqrt{3}/6, 0.5, \sqrt{2/3})$.

a) Find a unit vector perpendicular (outward) to the face ABC: First find

$$\begin{aligned}\mathbf{R}_{BA} \times \mathbf{R}_{BC} &= [(0, 1, 0) - (0.5\sqrt{3}, 0.5, 0)] \times [(\sqrt{3}/6, 0.5, \sqrt{2/3}) - (0.5\sqrt{3}, 0.5, 0)] \\ &= (-0.5\sqrt{3}, 0.5, 0) \times (-\sqrt{3}/3, 0, \sqrt{2/3}) = (0.41, 0.71, 0.29)\end{aligned}$$

The required unit vector will then be:

$$\frac{\mathbf{R}_{BA} \times \mathbf{R}_{BC}}{|\mathbf{R}_{BA} \times \mathbf{R}_{BC}|} = \underline{(0.47, 0.82, 0.33)}$$

b) Find the area of the face ABC:

$$\text{Area} = \frac{1}{2}|\mathbf{R}_{BA} \times \mathbf{R}_{BC}| = \underline{0.43}$$

1.15. Three vectors extending from the origin are given as $\mathbf{r}_1 = (7, 3, -2)$, $\mathbf{r}_2 = (-2, 7, -3)$, and $\mathbf{r}_3 = (0, 2, 3)$. Find:

a) a unit vector perpendicular to both $\mathbf{r}_1$ and $\mathbf{r}_2$:

$$\mathbf{a}_{p12} = \frac{\mathbf{r}_1 \times \mathbf{r}_2}{|\mathbf{r}_1 \times \mathbf{r}_2|} = \frac{(5, 25, 55)}{60.6} = \underline{(0.08, 0.41, 0.91)}$$

b) a unit vector perpendicular to the vectors $\mathbf{r}_1 - \mathbf{r}_2$ and $\mathbf{r}_2 - \mathbf{r}_3$: $\mathbf{r}_1 - \mathbf{r}_2 = (9, -4, 1)$ and $\mathbf{r}_2 - \mathbf{r}_3 = (-2, 5, -6)$. So $\mathbf{r}_1 - \mathbf{r}_2 \times \mathbf{r}_2 - \mathbf{r}_3 = (19, 52, 32)$. Then

$$\mathbf{a}_p = \frac{(19, 52, 32)}{|(19, 52, 32)|} = \frac{(19, 52, 32)}{63.95} = \underline{(0.30, 0.81, 0.50)}$$

c) the area of the triangle defined by $\mathbf{r}_1$ and $\mathbf{r}_2$:

$$\text{Area} = \frac{1}{2}|\mathbf{r}_1 \times \mathbf{r}_2| = \underline{30.3}$$

d) the area of the triangle defined by the heads of $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$:

$$\text{Area} = \frac{1}{2}|(\mathbf{r}_2 - \mathbf{r}_1) \times (\mathbf{r}_2 - \mathbf{r}_3)| = \frac{1}{2}|(-9, 4, -1) \times (-2, 5, -6)| = \underline{32.0}$$

1.16. Describe the surfaces defined by the equations:

a) $\mathbf{r} \cdot \mathbf{a}_x = 2$, where $\mathbf{r} = (x, y, z)$: This will be the plane $\underline{x = 2}$.

b) $|\mathbf{r} \times \mathbf{a}_x| = 2$: $\mathbf{r} \times \mathbf{a}_x = (0, z, -y)$, and $|\mathbf{r} \times \mathbf{a}_x| = \underline{\sqrt{z^2 + y^2} = 2}$. This is the equation of a cylinder, centered on the x axis, and of radius 2.

1.17. Point $A(-4, 2, 5)$ and the two vectors, $\mathbf{R}_{AM} = (20, 18, -10)$ and $\mathbf{R}_{AN} = (-10, 8, 15)$, define a triangle.

a) Find a unit vector perpendicular to the triangle: Use

$$\mathbf{a}_p = \frac{\mathbf{R}_{AM} \times \mathbf{R}_{AN}}{|\mathbf{R}_{AM} \times \mathbf{R}_{AN}|} = \frac{(350, -200, 340)}{527.35} = \underline{(0.664, -0.379, 0.645)}$$

The vector in the opposite direction to this one is also a valid answer.

b) Find a unit vector in the plane of the triangle and perpendicular to $\mathbf{R}_{AN}$:

$$\mathbf{a}_{AN} = \frac{(-10, 8, 15)}{\sqrt{389}} = (-0.507, 0.406, 0.761)$$

Then

$$\mathbf{a}_{pAN} = \mathbf{a}_p \times \mathbf{a}_{AN} = (0.664, -0.379, 0.645) \times (-0.507, 0.406, 0.761) = \underline{(-0.550, -0.832, 0.077)}$$

The vector in the opposite direction to this one is also a valid answer.

c) Find a unit vector in the plane of the triangle that bisects the interior angle at A: A non-unit vector in the required direction is $(1/2)(\mathbf{a}_{AM} + \mathbf{a}_{AN})$, where

$$\mathbf{a}_{AM} = \frac{(20, 18, -10)}{|(20, 18, -10)|} = (0.697, 0.627, -0.348)$$

1.17c. (continued) Now

$$\frac{1}{2}(\mathbf{a}_{AM} + \mathbf{a}_{AN}) = \frac{1}{2}[(0.697, 0.627, -0.348) + (-0.507, 0.406, 0.761)] = (0.095, 0.516, 0.207)$$

Finally,

$$\mathbf{a}_{bis} = \frac{(0.095, 0.516, 0.207)}{|(0.095, 0.516, 0.207)|} = \underline{(0.168, 0.915, 0.367)}$$

1.18. Given points $A(\rho = 5, \phi = 70^\circ, z = -3)$ and $B(\rho = 2, \phi = -30^\circ, z = 1)$, find:

a) unit vector in cartesian coordinates at A toward B: $A(5\cos 70^\circ, 5\sin 70^\circ, -3) = A(1.71, 4.70, -3)$, In the same manner, $B(1.73, -1, 1)$. So $\mathbf{R}_{AB} = (1.73, -1, 1) - (1.71, 4.70, -3) = (0.02, -5.70, 4)$ and therefore

$$\mathbf{a}_{AB} = \frac{(0.02, -5.70, 4)}{|(0.02, -5.70, 4)|} = \underline{(0.003, -0.82, 0.57)}$$

b) a vector in cylindrical coordinates at A directed toward B: $\mathbf{a}_{AB} \cdot \mathbf{a}_\rho = 0.003\cos 70^\circ - 0.82\sin 70^\circ = -0.77$. $\mathbf{a}_{AB} \cdot \mathbf{a}_\phi = -0.003\sin 70^\circ - 0.82\cos 70^\circ = -0.28$. Thus

$$\mathbf{a}_{AB} = \underline{-0.77\mathbf{a}_\rho - 0.28\mathbf{a}_\phi + 0.57\mathbf{a}_z}$$

.

c) a unit vector in cylindrical coordinates at B directed toward A:
Use $\mathbf{a}_{BA} = (-0, 003, 0.82, -0.57)$. Then $\mathbf{a}_{BA} \cdot \mathbf{a}_\rho = -0.003\cos(-30^\circ) + 0.82\sin(-30^\circ) = -0.43$, and $\mathbf{a}_{BA} \cdot \mathbf{a}_\phi = 0.003\sin(-30^\circ) + 0.82\cos(-30^\circ) = 0.71$. Finally,

$$\mathbf{a}_{BA} = \underline{-0.43\mathbf{a}_\rho + 0.71\mathbf{a}_\phi - 0.57\mathbf{a}_z}$$

1.19 a) Express the field $\mathbf{D} = (x^2 + y^2)^{-1}(x\mathbf{a}_x + y\mathbf{a}_y)$ in cylindrical components and cylindrical variables: Have $x = \rho\cos\phi$, $y = \rho\sin\phi$, and $x^2 + y^2 = \rho^2$. Therefore

$$\mathbf{D} = \frac{1}{\rho}(\cos\phi\mathbf{a}_x + \sin\phi\mathbf{a}_y)$$

Then

$$D_\rho = \mathbf{D} \cdot \mathbf{a}_\rho = \frac{1}{\rho}\left[\cos\phi(\mathbf{a}_x \cdot \mathbf{a}_\rho) + \sin\phi(\mathbf{a}_y \cdot \mathbf{a}_\rho)\right] = \frac{1}{\rho}\left[\cos^2\phi + \sin^2\phi\right] = \frac{1}{\rho}$$

and

$$D_\phi = \mathbf{D} \cdot \mathbf{a}_\phi = \frac{1}{\rho}\left[\cos\phi(\mathbf{a}_x \cdot \mathbf{a}_\phi) + \sin\phi(\mathbf{a}_y \cdot \mathbf{a}_\phi)\right] = \frac{1}{\rho}[\cos\phi(-\sin\phi) + \sin\phi\cos\phi] = 0$$

Therefore

$$\underline{\mathbf{D} = \frac{1}{\rho}\mathbf{a}_\rho}$$

1.19b. Evaluate $\mathbf{D}$ at the point where $\rho = 2$, $\phi = 0.2\pi$, and $z = 5$, expressing the result in cylindrical and cartesian coordinates: At the given point, and in cylindrical coordinates, $\underline{\mathbf{D} = 0.5\mathbf{a}_\rho}$. To express this in cartesian, we use

$$\mathbf{D} = 0.5(\mathbf{a}_\rho \cdot \mathbf{a}_x)\mathbf{a}_x + 0.5(\mathbf{a}_\rho \cdot \mathbf{a}_y)\mathbf{a}_y = 0.5\cos 36^\circ \mathbf{a}_x + 0.5\sin 36^\circ \mathbf{a}_y = \underline{0.41\mathbf{a}_x + 0.29\mathbf{a}_y}$$

1.20. Express in cartesian components:

a) the vector at $A(\rho = 4, \phi = 40^\circ, z = -2)$ that extends to $B(\rho = 5, \phi = -110^\circ, z = 2)$: We have $A(4\cos 40^\circ, 4\sin 40^\circ, -2) = A(3.06, 2.57, -2)$, and $B(5\cos(-110^\circ), 5\sin(-110^\circ), 2) = B(-1.71, -4.70, 2)$ in cartesian. Thus $\mathbf{R}_{AB} = \underline{(-4.77, -7.30, 4)}$.

b) a unit vector at B directed toward A: Have $\mathbf{R}_{BA} = (4.77, 7.30, -4)$, and so

$$\mathbf{a}_{BA} = \frac{(4.77, 7.30, -4)}{|(4.77, 7.30, -4)|} = \underline{(0.50, 0.76, -0.42)}$$

c) a unit vector at B directed toward the origin: Have $\mathbf{r}_B = (-1.71, -4.70, 2)$, and so $-\mathbf{r}_B = (1.71, 4.70, -2)$. Thus

$$\mathbf{a} = \frac{(1.71, 4.70, -2)}{|(1.71, 4.70, -2)|} = \underline{(0.32, 0.87, -0.37)}$$

1.21. Express in cylindrical components:

a) the vector from $C(3, 2, -7)$ to $D(-1, -4, 2)$:
$C(3, 2, -7) \rightarrow C(\rho = 3.61, \phi = 33.7^\circ, z = -7)$ and
$D(-1, -4, 2) \rightarrow D(\rho = 4.12, \phi = -104.0^\circ, z = 2)$.
Now $\mathbf{R}_{CD} = (-4, -6, 9)$ and $R_\rho = \mathbf{R}_{CD} \cdot \mathbf{a}_\rho = -4\cos(33.7) - 6\sin(33.7) = -6.66$. Then $R_\phi = \mathbf{R}_{CD} \cdot \mathbf{a}_\phi = 4\sin(33.7) - 6\cos(33.7) = -2.77$. So $\mathbf{R}_{CD} = \underline{-6.66\mathbf{a}_\rho - 2.77\mathbf{a}_\phi + 9\mathbf{a}_z}$

b) a unit vector at D directed toward C:
$\mathbf{R}_{CD} = (4, 6, -9)$ and $R_\rho = \mathbf{R}_{DC} \cdot \mathbf{a}_\rho = 4\cos(-104.0) + 6\sin(-104.0) = -6.79$. Then $R_\phi = \mathbf{R}_{DC} \cdot \mathbf{a}_\phi = 4[-\sin(-104.0)] + 6\cos(-104.0) = 2.43$. So $\mathbf{R}_{DC} = -6.79\mathbf{a}_\rho + 2.43\mathbf{a}_\phi - 9\mathbf{a}_z$
Thus $\mathbf{a}_{DC} = \underline{-0.59\mathbf{a}_\rho + 0.21\mathbf{a}_\phi - 0.78\mathbf{a}_z}$

c) a unit vector at D directed toward the origin: Start with $\mathbf{r}_D = (-1, -4, 2)$, and so the vector toward the origin will be $-\mathbf{r}_D = (1, 4, -2)$. Thus in cartesian the unit vector is $\mathbf{a} = (0.22, 0.87, -0.44)$. Convert to cylindrical:
$a_\rho = (0.22, 0.87, -0.44) \cdot \mathbf{a}_\rho = 0.22\cos(-104.0) + 0.87\sin(-104.0) = -0.90$, and
$a_\phi = (0.22, 0.87, -0.44) \cdot \mathbf{a}_\phi = 0.22[-\sin(-104.0)] + 0.87\cos(-104.0) = 0$, so that finally, $\mathbf{a} = \underline{-0.90\mathbf{a}_\rho - 0.44\mathbf{a}_z}$.

1.22. A field is given in cylindrical coordinates as

$$\mathbf{F} = \left[\frac{40}{\rho^2 + 1} + 3(\cos\phi + \sin\phi)\right]\mathbf{a}_\rho + 3(\cos\phi - \sin\phi)\mathbf{a}_\phi - 2\mathbf{a}_z$$

where the magnitude of $\mathbf{F}$ is found to be:

$$|\mathbf{F}| = \sqrt{\mathbf{F} \cdot \mathbf{F}} = \left[\frac{1600}{(\rho^2 + 1)^2} + \frac{240}{\rho^2 + 1}(\cos\phi + \sin\phi) + 22\right]^{1/2}$$

Sketch $|\mathbf{F}|$:

a) vs. ϕ with $\rho = 3$: in this case the above simplifies to

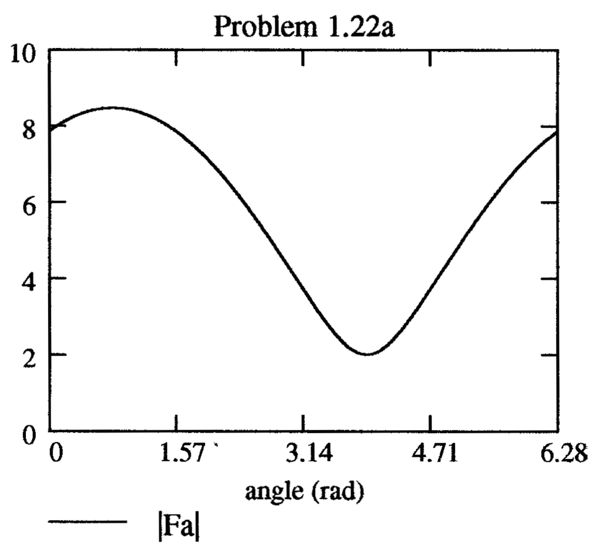
$$|\mathbf{F}(\rho = 3)| = |Fa| = [38 + 24(\cos\phi + \sin\phi)]^{1/2}$$

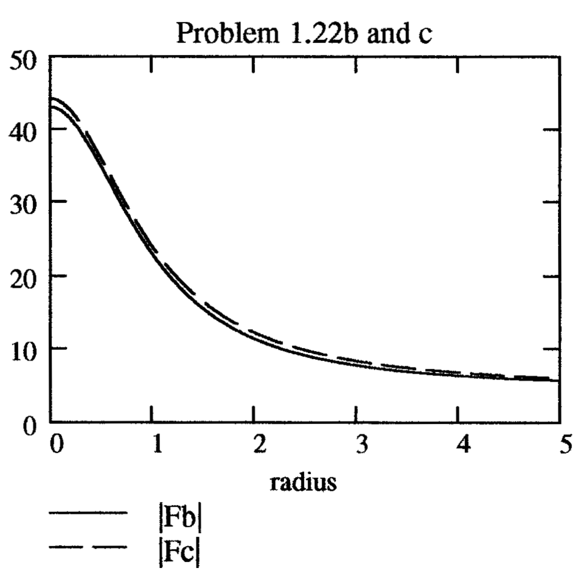
b) vs. ρ with $\phi = 0$, in which:

$$|\mathbf{F}(\phi = 0)| = |Fb| = \left[\frac{1600}{(\rho^2+1)^2} + \frac{240}{\rho^2+1} + 22\right]^{1/2}$$

c) vs. ρ with $\phi = 45^\circ$, in which

$$|\mathbf{F}(\phi = 45^\circ)| = |Fc| = \left[\frac{1600}{(\rho^2+1)^2} + \frac{240\sqrt{2}}{\rho^2+1} + 22\right]^{1/2}$$





1.23. The surfaces $\rho = 3$, $\rho = 5$, $\phi = 100^\circ$, $\phi = 130^\circ$, $z = 3$, and $z = 4.5$ define a closed surface.

a) Find the enclosed volume:

$$\text{Vol} = \int_3^{4.5} \int_{100^\circ}^{130^\circ} \int_3^5 \rho \, d\rho \, d\phi \, dz = \underline{6.28}$$

NOTE: The limits on the ϕ integration must be converted to radians (as was done here, but not shown).

b) Find the total area of the enclosing surface:

$$\text{Area} = 2 \int_{100^\circ}^{130^\circ} \int_3^5 \rho \, d\rho \, d\phi \; + \; \int_3^{4.5} \int_{100^\circ}^{130^\circ} 3 \, d\phi \, dz$$
$$+ \; \int_3^{4.5} \int_{100^\circ}^{130^\circ} 5 \, d\phi \, dz \; + \; 2 \int_3^{4.5} \int_3^5 d\rho \, dz = \underline{20.7}$$

c) Find the total length of the twelve edges of the surfaces:

$$\text{Length} = 4 \times 1.5 \; + \; 4 \times 2 \; + \; 2 \times \left[\frac{30^\circ}{360^\circ} \times 2\pi \times 3 \; + \; \frac{30^\circ}{360^\circ} \times 2\pi \times 5 \right] = \underline{22.4}$$

d) Find the length of the longest straight line that lies entirely within the volume: This will be between the points A($\rho = 3$, $\phi = 100^\circ$, $z = 3$) and B($\rho = 5$, $\phi = 130^\circ$, $z = 4.5$). Performing point transformations to cartesian coordinates, these become A($x = -0.52$, $y = 2.95$, $z = 3$) and B($x = -3.21$, $y = 3.83$, $z = 4.5$). Taking A and B as vectors directed from the origin, the requested length is

$$\text{Length} = |\mathbf{B} - \mathbf{A}| = |(-2.69, 0.88, 1.5)| = \underline{3.21}$$

1.24. At point $P(-3, 4, 5)$, express the vector that extends from P to $Q(2, 0, -1)$ in:

a) rectangular coordinates.

$$\mathbf{R}_{PQ} = \mathbf{Q} - \mathbf{P} = \underline{5\mathbf{a}_x - 4\mathbf{a}_y - 6\mathbf{a}_z}$$

Then $|\mathbf{R}_{PQ}| = \sqrt{25 + 16 + 36} = 8.8$

b) cylindrical coordinates. At P, $\rho = 5$, $\phi = \tan^{-1}(4/-3) = -53.1^\circ$, and $z = 5$. Now,

$$\mathbf{R}_{PQ} \cdot \mathbf{a}_\rho = (5\mathbf{a}_x - 4\mathbf{a}_y - 6\mathbf{a}_z) \cdot \mathbf{a}_\rho = 5\cos\phi - 4\sin\phi = 6.20$$

$$\mathbf{R}_{PQ} \cdot \mathbf{a}_\phi = (5\mathbf{a}_x - 4\mathbf{a}_y - 6\mathbf{a}_z) \cdot \mathbf{a}_\phi = -5\sin\phi - 4\cos\phi = 1.60$$

Thus

$$\mathbf{R}_{PQ} = \underline{6.20\mathbf{a}_\rho + 1.60\mathbf{a}_\phi - 6\mathbf{a}_z}$$

and $|\mathbf{R}_{PQ}| = \sqrt{6.20^2 + 1.60^2 + 6^2} = 8.8$

c) spherical coordinates. At P, $r = \sqrt{9 + 16 + 25} = \sqrt{50} = 7.07$, $\theta = \cos^{-1}(5/7.07) = 45^\circ$, and $\phi = \tan^{-1}(4/-3) = -53.1^\circ$.

$$\mathbf{R}_{PQ} \cdot \mathbf{a}_r = (5\mathbf{a}_x - 4\mathbf{a}_y - 6\mathbf{a}_z) \cdot \mathbf{a}_r = 5\sin\theta\cos\phi - 4\sin\theta\sin\phi - 6\cos\theta = 0.14$$

$$\mathbf{R}_{PQ} \cdot \mathbf{a}_\theta = (5\mathbf{a}_x - 4\mathbf{a}_y - 6\mathbf{a}_z) \cdot \mathbf{a}_\theta = 5\cos\theta\cos\phi - 4\cos\theta\sin\phi - (-6)\sin\theta = 8.62$$

$$\mathbf{R}_{PQ} \cdot \mathbf{a}_\phi = (5\mathbf{a}_x - 4\mathbf{a}_y - 6\mathbf{a}_z) \cdot \mathbf{a}_\phi = -5\sin\phi - 4\cos\phi = 1.60$$

1.24. (continued)

Thus

$$\mathbf{R}_{PQ} = \underline{0.14\mathbf{a}_r + 8.62\mathbf{a}_\theta + 1.60\mathbf{a}_\phi}$$

and $|\mathbf{R}_{PQ}| = \sqrt{0.14^2 + 8.62^2 + 1.60^2} = 8.8$

d) Show that each of these vectors has the same magnitude. Each does, as shown above.

1.25. Given point $P(r = 0.8,\ \theta = 30^\circ,\ \phi = 45^\circ)$, and

$$\mathbf{E} = \frac{1}{r^2}\left(\cos\phi\,\mathbf{a}_r + \frac{\sin\phi}{\sin\theta}\,\mathbf{a}_\phi\right)$$

a) Find $\mathbf{E}$ at P: $\mathbf{E} = \underline{1.10\mathbf{a}_\rho + 2.21\mathbf{a}_\phi}$.

b) Find $|\mathbf{E}|$ at P: $|\mathbf{E}| = \sqrt{1.10^2 + 2.21^2} = \underline{2.47}$.

c) Find a unit vector in the direction of $\mathbf{E}$ at P:

$$\mathbf{a}_E = \frac{\mathbf{E}}{|\mathbf{E}|} = \underline{0.45\mathbf{a}_r + 0.89\mathbf{a}_\phi}$$

1.26. a) Determine an expression for $\mathbf{a}_y$ in spherical coordinates at $P(r = 4,\ \theta = 0.2\pi,\ \phi = 0.8\pi)$: Use $\mathbf{a}_y \cdot \mathbf{a}_r = \sin\theta\sin\phi = 0.35$, $\mathbf{a}_y \cdot \mathbf{a}_\theta = \cos\theta\sin\phi = 0.48$, and $\mathbf{a}_y \cdot \mathbf{a}_\phi = \cos\phi = -0.81$ to obtain

$$\mathbf{a}_y = \underline{0.35\mathbf{a}_r + 0.48\mathbf{a}_\theta - 0.81\mathbf{a}_\phi}$$

b) Express $\mathbf{a}_r$ in cartesian components at P: Find $x = r\sin\theta\cos\phi = -1.90$, $y = r\sin\theta\sin\phi = 1.38$, and $z = r\cos\theta = -3.24$. Then use $\mathbf{a}_r \cdot \mathbf{a}_x = \sin\theta\cos\phi = -0.48$, $\mathbf{a}_r \cdot \mathbf{a}_y = \sin\theta\sin\phi = 0.35$, and $\mathbf{a}_r \cdot \mathbf{a}_z = \cos\theta = 0.81$ to obtain

$$\mathbf{a}_r = \underline{-0.48\mathbf{a}_x + 0.35\mathbf{a}_y + 0.81\mathbf{a}_z}$$

1.27. The surfaces $r = 2$ and 4, $\theta = 30^\circ$ and 50°, and $\phi = 20^\circ$ and 60° identify a closed surface.

a) Find the enclosed volume: This will be

$$\text{Vol} = \int_{20^\circ}^{60^\circ}\int_{30^\circ}^{50^\circ}\int_2^4 r^2\sin\theta\,dr\,d\theta\,d\phi = \underline{2.91}$$

where degrees have been converted to radians.

b) Find the total area of the enclosing surface:

$$\text{Area} = \int_{20^\circ}^{60^\circ}\int_{30^\circ}^{50^\circ}(4^2 + 2^2)\sin\theta\,d\theta\,d\phi + \int_2^4\int_{20^\circ}^{60^\circ} r(\sin 30^\circ + \sin 50^\circ)\,dr\,d\phi + 2\int_{30^\circ}^{50^\circ}\int_2^4 r\,dr\,d\theta = \underline{12.61}$$

c) Find the total length of the twelve edges of the surface:

$$\text{Length} = 4\int_2^4 dr \ + \ 2\int_{30^\circ}^{50^\circ}(4+2)\,d\theta + \int_{20^\circ}^{60^\circ}(4\sin 50^\circ + 4\sin 30^\circ + 2\sin 50^\circ + 2\sin 30^\circ)\,d\phi = \underline{17.49}$$

1.27. (continued)

d) Find the length of the longest straight line that lies entirely within the surface: This will be from $A(r = 2, \theta = 50^\circ, \phi = 20^\circ)$ to $B(r = 4, \theta = 30^\circ, \phi = 60^\circ)$ or

$$A(x = 2\sin 50^\circ \cos 20^\circ,\ y = 2\sin 50^\circ \sin 20^\circ,\ z = 2\cos 50^\circ)$$

to

$$B(x = 4\sin 30^\circ \cos 60^\circ,\ y = 4\sin 30^\circ \sin 60^\circ,\ z = 4\cos 30^\circ)$$

or finally $A(1.44, 0.52, 1.29)$ to $B(1.00, 1.73, 3.46)$. Thus $\mathbf{B} - \mathbf{A} = (-0.44, 1.21, 2.18)$ and

$$\text{Length} = |\mathbf{B} - \mathbf{A}| = \underline{2.53}$$

1.28. a) Determine the cartesian components of the vector from $A(r = 5, \theta = 110^\circ, \phi = 200^\circ)$ to $B(r = 7, \theta = 30^\circ, \phi = 70^\circ)$: First transform the points to cartesian: $x_A = 5\sin 110^\circ \cos 200^\circ = -4.42$, $y_A = 5\sin 110^\circ \sin 200^\circ = -1.61$, and $z_A = 5\cos 110^\circ = -1.71$; $x_B = 7\sin 30^\circ \cos 70^\circ = 1.20$, $y_B = 7\sin 30^\circ \sin 70^\circ = 3.29$, and $z_B = 7\cos 30^\circ = 6.06$. Now

$$\mathbf{R}_{AB} = \mathbf{B} - \mathbf{A} = \underline{5.62\mathbf{a_x} + 4.90\mathbf{a}_y + 7.77\mathbf{a}_z}$$

b) Find the spherical components of the vector at $P(2, -3, 4)$ extending to $Q(-3, 2, 5)$: First, $\mathbf{R}_{PQ} = \mathbf{Q} - \mathbf{P} = (-5, 5, 1)$. Then at P, $r = \sqrt{4+9+16} = 5.39$, $\theta = \cos^{-1}(4/\sqrt{29}) = 42.0^\circ$, and $\phi = \tan^{-1}(-3/2) = -56.3^\circ$. Now

$$\mathbf{R}_{PQ} \cdot \mathbf{a}_r = -5\sin(42^\circ)\cos(-56.3^\circ) + 5\sin(42^\circ)\sin(-56.3^\circ) + 1\cos(42^\circ) = -3.90$$

$$\mathbf{R}_{PQ} \cdot \mathbf{a}_\theta = -5\cos(42^\circ)\cos(-56.3^\circ) + 5\cos(42^\circ)\sin(-56.3^\circ) - 1\sin(42^\circ) = -5.82$$

$$\mathbf{R}_{PQ} \cdot \mathbf{a}_\phi = -(-5)\sin(-56.3^\circ) + 5\cos(-56.3^\circ) = -1.39$$

So finally,

$$\mathbf{R}_{PQ} = \underline{-3.90\mathbf{a}_r - 5.82\mathbf{a}_\theta - 1.39\mathbf{a}_\phi}$$

c) If $\mathbf{D} = 5\mathbf{a}_r - 3\mathbf{a}_\theta + 4\mathbf{a}_\phi$, find $\mathbf{D} \cdot \mathbf{a}_\rho$ at $M(1, 2, 3)$: First convert $\mathbf{a}_\rho$ to cartesian coordinates at the specified point. Use $\mathbf{a}_\rho = (\mathbf{a}_\rho \cdot \mathbf{a}_x)\mathbf{a}_x + (\mathbf{a}_\rho \cdot \mathbf{a}_y)\mathbf{a}_y$. At $A(1, 2, 3)$, $\rho = \sqrt{5}$, $\phi = \tan^{-1}(2) = 63.4^\circ$, $r = \sqrt{14}$, and $\theta = \cos^{-1}(3/\sqrt{14}) = 36.7^\circ$. So $\mathbf{a}_\rho = \cos(63.4^\circ)\mathbf{a}_x + \sin(63.4^\circ)\mathbf{a}_y = 0.45\mathbf{a}_x + 0.89\mathbf{a}_y$. Then

$$\begin{aligned}
&(5\mathbf{a}_r - 3\mathbf{a}_\theta + 4\mathbf{a}_\phi) \cdot (0.45\mathbf{a}_x + 0.89\mathbf{a}_y) = \\
&5(0.45)\sin\theta\cos\phi \;+\; 5(0.89)\sin\theta\sin\phi \;-\; 3(0.45)\cos\theta\cos\phi \\
&-\; 3(0.89)\cos\theta\sin\phi \;+\; 4(0.45)(-\sin\phi) \;+\; 4(0.89)\cos\phi = \underline{0.59}
\end{aligned}$$

1.29. Express the unit vector $\mathbf{a}_x$ in spherical components at the point:

a) $r = 2, \theta = 1$ rad, $\phi = 0.8$ rad: Use

$$\mathbf{a}_x = (\mathbf{a}_x \cdot \mathbf{a}_r)\mathbf{a}_r + (\mathbf{a}_x \cdot \mathbf{a}_\theta)\mathbf{a}_\theta + (\mathbf{a}_x \cdot \mathbf{a}_\phi)\mathbf{a}_\phi =$$
$$\sin(1)\cos(0.8)\mathbf{a}_r + \cos(1)\cos(0.8)\mathbf{a}_\theta + (-\sin(0.8))\mathbf{a}_\phi = \underline{0.59\mathbf{a}_r + 0.38\mathbf{a}_\theta - 0.72\mathbf{a}_\phi}$$

1.29 (continued) Express the unit vector $\mathbf{a}_x$ in spherical components at the point:

b) $x = 3$, $y = 2$, $z = -1$: First, transform the point to spherical coordinates. Have $r = \sqrt{14}$, $\theta = \cos^{-1}(-1/\sqrt{14}) = 105.5^\circ$, and $\phi = \tan^{-1}(2/3) = 33.7^\circ$. Then

$$\begin{aligned}\mathbf{a}_x &= \sin(105.5^\circ)\cos(33.7^\circ)\mathbf{a}_r + \cos(105.5^\circ)\cos(33.7^\circ)\mathbf{a}_\theta + (-\sin(33.7^\circ))\mathbf{a}_\phi \\ &= \underline{0.80\mathbf{a}_r - 0.22\mathbf{a}_\theta - 0.55\mathbf{a}_\phi}\end{aligned}$$

c) $\rho = 2.5$, $\phi = 0.7$ rad, $z = 1.5$: Again, convert the point to spherical coordinates. $r = \sqrt{\rho^2 + z^2} = \sqrt{8.5}$, $\theta = \cos^{-1}(z/r) = \cos^{-1}(1.5/\sqrt{8.5}) = 59.0^\circ$, and $\phi = 0.7\,\text{rad} = 40.1^\circ$. Now

$$\begin{aligned}\mathbf{a}_x &= \sin(59^\circ)\cos(40.1^\circ)\mathbf{a}_r + \cos(59^\circ)\cos(40.1^\circ)\mathbf{a}_\theta + (-\sin(40.1^\circ))\mathbf{a}_\phi \\ &= \underline{0.66\mathbf{a}_r + 0.39\mathbf{a}_\theta - 0.64\mathbf{a}_\phi}\end{aligned}$$

1.30. Given $A(r = 20, \theta = 30^\circ, \phi = 45^\circ)$ and $B(r = 30, \theta = 115^\circ, \phi = 160^\circ)$, find:

a) $|\mathbf{R}_{AB}|$: First convert A and B to cartesian: Have $x_A = 20\sin(30^\circ)\cos(45^\circ) = 7.07$, $y_A = 20\sin(30^\circ)\sin(45^\circ) = 7.07$, and $z_A = 20\cos(30^\circ) = 17.3$. $x_B = 30\sin(115^\circ)\cos(160^\circ) = -25.6$, $y_B = 30\sin(115^\circ)\sin(160^\circ) = 9.3$, and $z_B = 30\cos(115^\circ) = -12.7$. Now $\mathbf{R}_{AB} = \mathbf{R}_B - \mathbf{R}_A = (-32.6, 2.2, -30.0)$, and so $|\mathbf{R}_{AB}| = \underline{44.4}$.

b) $|\mathbf{R}_{AC}|$, given $C(r = 20, \theta = 90^\circ, \phi = 45^\circ)$. Again, converting C to cartesian, obtain $x_C = 20\sin(90^\circ)\cos(45^\circ) = 14.14$, $y_C = 20\sin(90^\circ)\sin(45^\circ) = 14.14$, and $z_C = 20\cos(90^\circ) = 0$. So $\mathbf{R}_{AC} = \mathbf{R}_C - \mathbf{R}_A = (7.07, 7.07, -17.3)$, and $|\mathbf{R}_{AC}| = \underline{20.0}$.

c) the distance from A to C on a great circle path: Note that A and C share the same r and ϕ coordinates; thus moving from A to C involves only a change in θ of 60°. The requested arc length is then

$$\text{distance} = 20 \times \left[60\left(\frac{2\pi}{360}\right)\right] = \underline{20.9}$$