NOTES

Subject

Sr. No:	Date	Topic
		ENGINEERING ELECTROMAGNETICS by William H. Hayt Jr
chapters	Mid	1, 2, 3, 4
	Final	8, 9, 10, 11
		Electromagnetic Field Theory Sir Gulzar.

25/9/19

Vector

Any physical quantity having magnitude and direction.

Area will be considered vector eg the inner and outer side of a wall.

Represented by bold letters in the book.
mathematically $\vec{A} = A\vec{a}$

$\vec{A}$ = vector $A \rightarrow$ magnitude
$\vec{a}$ = unit vector of magnitude 1 in the direction of vector $\vec{A}$.

Multiplication of scalar by vector results in a vector.

For unit vector. $\vec{a} = \dfrac{\vec{A}}{A}$

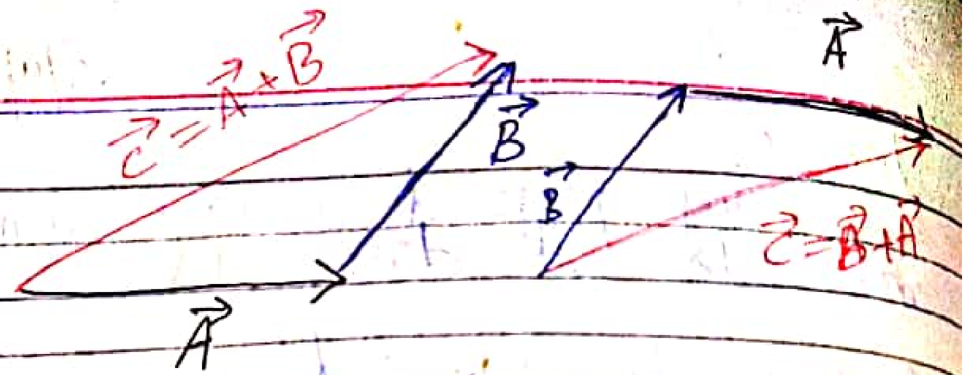
The notation for unit vector is small a.

eg $\vec{a} = \dfrac{\vec{B}}{B}$ $\vec{a} = \dfrac{\vec{C}}{C}$ etc

Graphically represented by arrow.
length $\rightarrow$ magnitude $\rightarrow$ direction

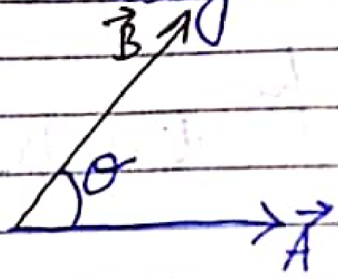
Addition of vectors
results in a vector quantity.
Obeys commutative law. $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

Graphically added by head to tail rule



Scalar Product Of Vectors

Also known as dot product.
Results in a scalar quantity
Obeys commutative law. $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$



$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A} = AB\cos\theta$

let two similar vectors (same mag and direction)

$\vec{A}\cdot\vec{A} = A \times A \times 1$ $\longrightarrow \vec{B} = \vec{A}$

$\vec{A}\cdot\vec{A} = A^2$ $\longrightarrow \vec{A}$

$\theta = 0^\circ$

$A = \sqrt{\vec{A}\cdot\vec{A}}$ for finding magnitude.

Similarly $B = \sqrt{\vec{B}\cdot\vec{B}}$ $C = \sqrt{\vec{C}\cdot\vec{C}}$ etc.

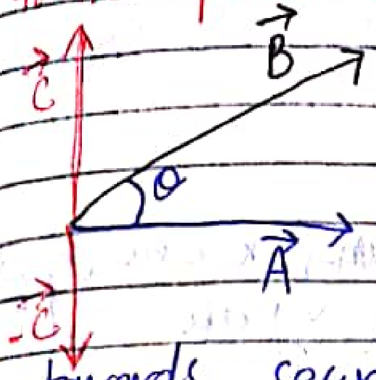
Vector product

Also known as cross product.
Results in a vector quantity.
Does not obey commutative law.

$\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$ $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

Whenever the tails of two vectors are connected together → they define a plane

The resultant vector will always be normal to the plane containing $\vec{A}$ and $\vec{B}$. → direction to be determined by right hand rule.



$$\vec{C} = \vec{A} \times \vec{B}$$

Hold right hand along first vector, curl the fingers towards second vector such that face faces second vector → thumb indicates direction of resultant.

$$\vec{C} = \vec{A} \times \vec{B} = AB \sin\theta \, \vec{a}_n$$

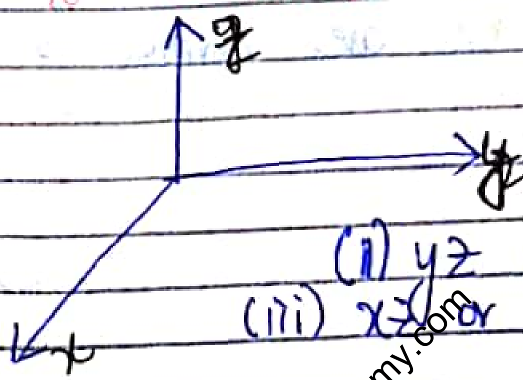
normal unit vector

$$\vec{C} = \vec{B} \times \vec{A} = -AB \sin\theta \, \vec{a}_n$$

Rectangular Cordinate System

- 3 coordinates (axes) ⇒ x, y, z
- → Normal to each other.

Chose x by your choice → move $90°$ in the counter clockwise direction to chose y
→ $90°$ counter clockwise to chose z.



Three planes:

(i) xy plane. $z=0$ plane ⤵ z

(ii) yz or $x=0$ plane

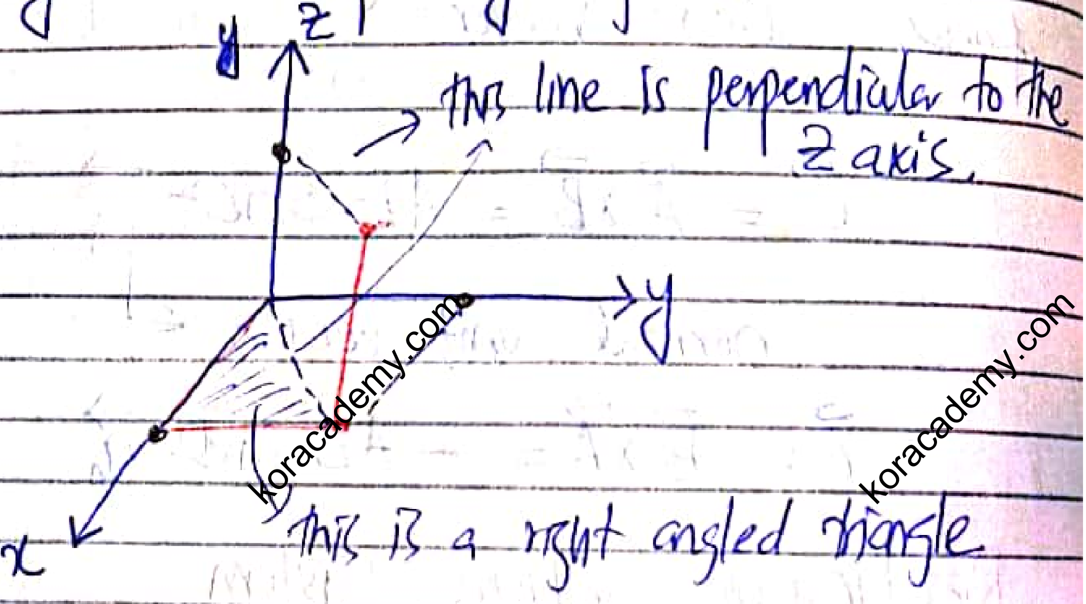
(iii) xz or $y=0$ plane. ⤵ x

↳ constant y plane

Any line in the $z=0$ plane will be normal to z axis
Similarly for x and y axis and planes

$P(x, y, z)$ in space?

Start from origin. Line along x axis equal to x coordinate. Now along y axis equal to y, and finally for z.



Unit vectors

3 in rectangular coordinate system.

Along x axis $\vec{a}_x$ Along y axis $\vec{a}_y$
Along z axis $\vec{a}_z$

These 3 unit vectors are normal to one another ie mutually orthogonal.

Scalar product

$$\vec{a}_x \cdot \vec{a}_y = \vec{a}_y \cdot \vec{a}_z = \vec{a}_x \cdot \vec{a}_z = 0$$

$$\vec{a}_x \cdot \vec{a}_x = 1 \quad \vec{a}_y \cdot \vec{a}_y = 1 \quad \vec{a}_z \cdot \vec{a}_z = 1$$

Cross product

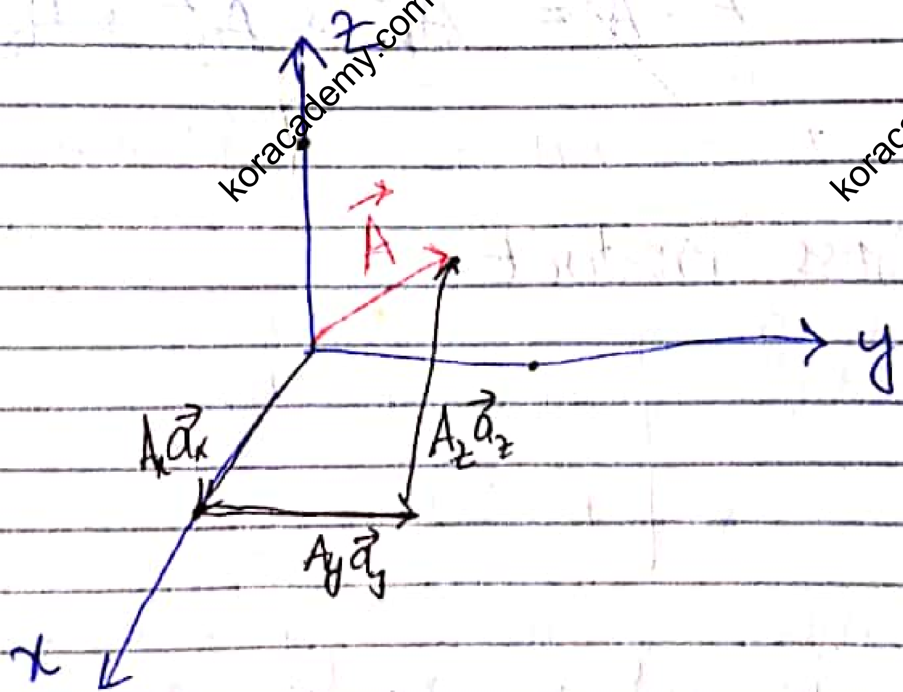
$\vec{a}_x \times \vec{a}_y = \vec{a}_z$ $\quad \vec{a}_y \times \vec{a}_z = \vec{a}_x$

$\vec{a}_z \times \vec{a}_x = \vec{a}_y$

$\vec{a}_y \times \vec{a}_x = -\vec{a}_z$ $\quad \vec{a}_z \times \vec{a}_y = -\vec{a}_x$ $\quad \vec{a}_x \times \vec{a}_z = -\vec{a}_y$

Vector in 3D

$$\vec{A} = A_x \vec{a}_x + A_y \vec{a}_y + A_z \vec{a}_z$$



Addition mathematically

$$\vec{A} = A_x \vec{a}_x + A_y \vec{a}_y + A_z \vec{a}_z$$
$$\vec{B} = B_x \vec{a}_x + B_y \vec{b}_y + B_z \vec{b}_z$$

$$\vec{A} + \vec{B} = (A_x + B_x)\vec{a}_x + (A_y + B_y)\vec{a}_y + (A_z + B_z)\vec{a}_z$$

Scalar Product

$$\vec{A}\cdot\vec{B} = A_x\vec{a}_x B_x\vec{a}_x + A_x\vec{a}_x B_y\vec{a}_y + A_x B_z \vec{a}_x\vec{a}_z$$
$$+ A_y\vec{a}_y B_x\vec{a}_x + A_y\vec{a}_y B_y\vec{a}_y + A_y\vec{a}_y B_z\vec{b}_z$$
$$+ A_z\vec{a}_z B_x\vec{a}_x + A_z\vec{a}_z B_y\vec{b}_y + A_z\vec{a}_z B_z\vec{a}_z$$

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$$

If $B = A$

$$\vec{A}\cdot\vec{A} = A_x^2 + A_y^2 + A_z^2$$

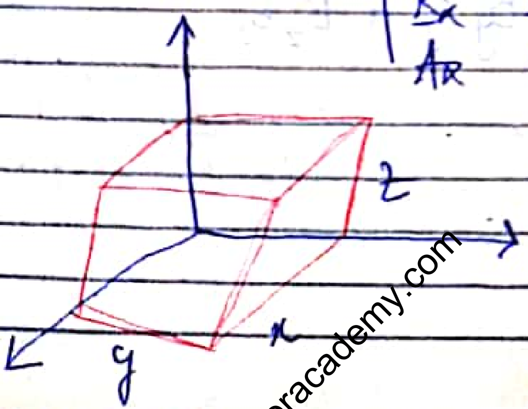
$$A = \sqrt{\vec{A}\cdot\vec{A}} = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Cross Product.

$$\vec{A}\times\vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Exchange any two rows or columns in determinant
→ we have a negative sign.

$$\vec{B}\times\vec{A} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ B_x & B_y & B_z \\ A_x & A_y & A_z \end{vmatrix}$$

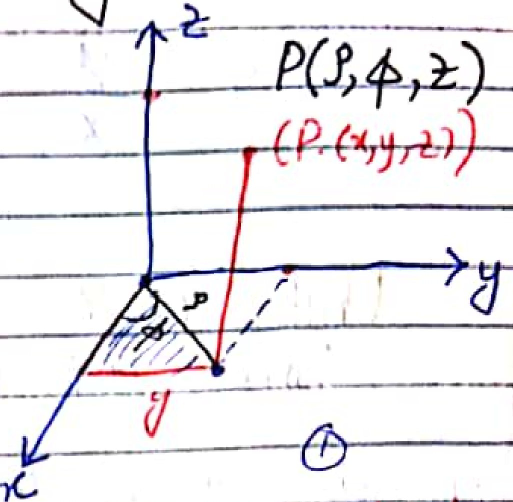


$-\alpha \le x \le \alpha$
$-\alpha \le y \le \alpha$
$-\alpha \le z \le \alpha$

Lecture 2 30/9/18

Cylindrical Coordinate System :

$P(x,y,z)$



Line represented by ρ, making an angle ϕ with the x axis.

↳ It is perpendicular to z axis.

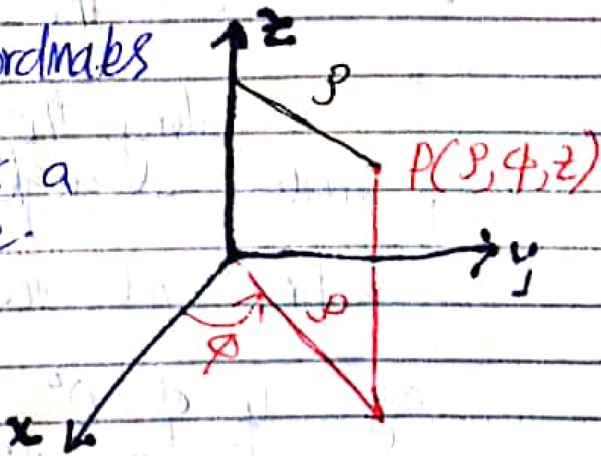
↳ ρ is the radius of the imaginary cylinder.

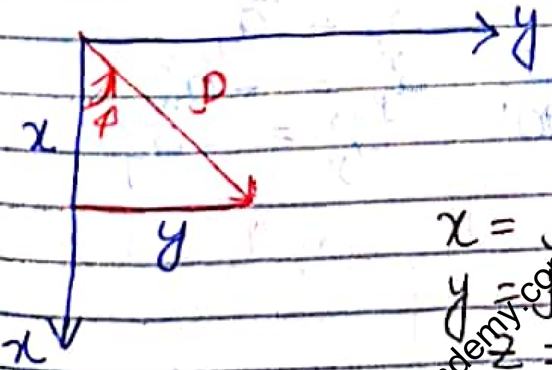
The z coordinate is same in both the rectangular and cylindrical system.

ϕ is considered in the counter clockwise direction.

$\rho, \phi, z \rightarrow$ cylindrical cordinates

The shaded triangle is a right angled triangle.





Radial line ρ is always in the $z = 0$ plane.

$x = \rho \cos\phi$

$y = \rho \sin\phi$

$z = z$

Cylindrical from rectangular

Pythogras theorem;

$$\rho = \sqrt{x^2 + y^2}$$

$$\tan\phi = \frac{y}{x} \Rightarrow \phi = \tan^{-1}\frac{y}{x}$$

$$z = z$$

Unit Vectors:

Three unit vectors $\vec{a}_\rho, \vec{a}_\phi, \vec{a}_z$

$\vec{a}_z$ will always be along the z axis and perpendicular to the plane contaning $\vec{a}_\rho$ and $\vec{a}_\phi$

→ They are mutually perpendicular.

$\vec{a}_\phi$ and $\vec{a}_\rho$ will be located somewhere in the $z=0$ plane.

$\vec{a}_\rho$ is along the radial line ρ and it makes an angle ϕ w.r.t x axis.

→ first unit vector.

eg if $\phi = 0°$ $\quad \vec{a}_\rho = \vec{a}_x$

$\phi = 90°$ $\quad \vec{a}_\rho = \vec{a}_y$

$\phi = 180°$ $\quad \vec{a}_\rho = -\vec{a}_x$

$\phi = 270°$ $\quad \vec{a}_\rho = -\vec{a}_y$

→ Its location is not fixed and depend upon the angle ϕ!

$\vec{a}_\phi$ is in the $z=0$ plane and it makes an angle of $90° + \phi$ w.r.t x axis.

$\vec{a}_z$ is along the z axis.

If $\phi = 0°$ $\Rightarrow$

$$\vec{a}_\rho = \vec{a}_x$$
$$\vec{a}_\phi = \vec{a}_y$$
$$\vec{a}_z = \vec{a}_z$$

If $\phi = 90°$ $\Rightarrow$

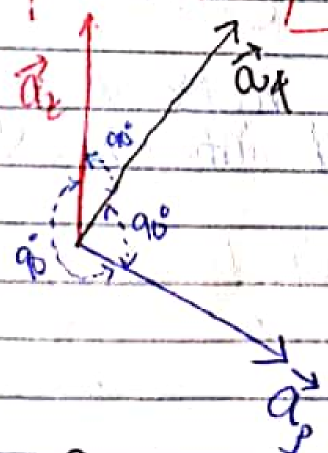
$$\vec{a}_\rho = \vec{a}_y$$
$$\vec{a}_\phi = -\vec{a}_x$$
$$\vec{a}_z = \vec{a}_z$$

If $\phi = 180°$ $\Rightarrow$

$$\vec{a}_\rho = -\vec{a}_x$$
$$\vec{a}_\phi = -\vec{a}_y$$

If $\phi = 270°$ $\Rightarrow$

$$\vec{a}_\rho = -\vec{a}_y$$
$$\vec{a}_\phi = \vec{a}_x$$



Scalar Product of Unit Vectors.

$$\vec{a}_\rho \cdot \vec{a}_\phi = \vec{a}_\phi \cdot \vec{a}_\rho = \vec{a}_\rho \cdot \vec{a}_z = \vec{a}_z \cdot \vec{a}_\rho$$
$$= \vec{a}_\phi \cdot \vec{a}_z = \vec{a}_z \cdot \vec{a}_\phi = 0$$

Two unit vectors along same line $= 1$

$$\vec{a}_\rho \cdot \vec{a}_\rho = \vec{a}_\phi \cdot \vec{a}_\phi = \vec{a}_z \cdot \vec{a}_z = 1$$

Cross Product

let $\phi = 0°$ $\vec{a}_\rho \times \vec{a}_\phi = \vec{a}_z$ And $\vec{a}_\phi \times \vec{a}_\rho = -\vec{a}_z$

$\vec{a}_\phi \times \vec{a}_z = \vec{a}_\rho$ And $\vec{a}_z \times \vec{a}_\phi = -\vec{a}_\rho$

$\vec{a}_z \times \vec{a}_\rho = \vec{a}_\phi$ And $\vec{a}_\rho \times \vec{a}_z = -\vec{a}_\phi$

If two unit vectors along the same line $= 0$

$$\vec{a}_\rho \times \vec{a}_\rho = \vec{a}_\phi \times \vec{a}_\phi = \vec{a}_z \times \vec{a}_z = 0$$

Representation of Vector In Cylindrical (3D) system

A three dimensional vector in any coordinate system can be resolved into three components;

The first component will be along $\vec{a}_\rho$
" second " " " $\vec{a}_\phi$
" Third " " " $\vec{a}_z$

$$\vec{A} = A_\rho \vec{a}_\rho + A_\phi \vec{a}_\phi + A_z \vec{a}_z$$

Addition of Vectors:

$$\vec{A} = A_\rho \vec{a}_\rho + A_\phi \vec{a}_\phi + A_z \vec{a}_z$$

$$\vec{B} = B_\rho \vec{a}_\rho + B_\phi \vec{a}_\phi + B_z \vec{a}_z$$

The ϕ should be same for both vectors in order to be added in cylindrical system

↳ otherwise we will have to transform them first into rectangular coordinate system and then convert the result back to cylindrical.

$$\vec{A}+\vec{B} = (A_\rho + B_\rho)\vec{a}_\rho + (A_\phi + B_\phi)\vec{a}_\phi + (A_z + B_z)\vec{a}_z$$

Scalar Product

ϕ must be same for $\vec{A}$ and $\vec{B}$.

$$\vec{A}\cdot\vec{B} = A_\rho B_\rho + A_\phi B_\phi + A_z B_z$$

let $\vec{A}=\vec{B}$

$$\vec{A}\cdot\vec{A} = A_\rho A_\rho + A_\phi A_\phi + A_z A_z$$

$$A = \sqrt{\vec{A}\cdot\vec{A}} = \sqrt{A_\rho^2 + A_\phi^2 + A_z^2}$$

↳ magnitude

Cross Product

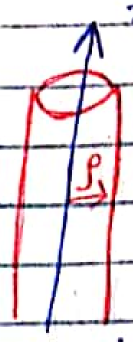
Same condition → ϕ

$$\vec{A}\times\vec{B} = \begin{vmatrix} \vec{a}_\rho & \vec{a}_\phi & \vec{a}_z \\ A_\rho & A_\phi & A_z \\ B_\rho & B_\phi & B_z \end{vmatrix}$$

Similarly

$$\vec{B}\times\vec{A} = \begin{vmatrix} \vec{a}_\rho & \vec{a}_\phi & \vec{a}_z \\ B_\rho & B_\phi & B_z \\ A_\rho & A_\phi & A_z \end{vmatrix}$$

We can construct a cylinder with the help of ρ, ϕ, z.



3 surfaces in cylinder

Limits

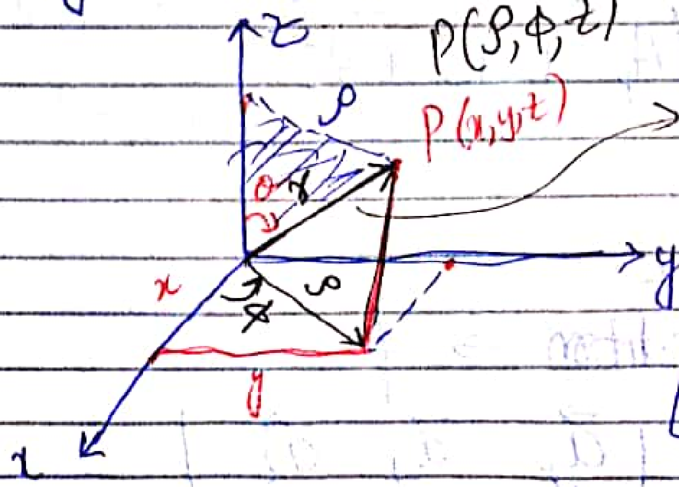
$0 \le \rho \le \infty$
$0 \le \phi \le 2\pi$
$-\infty \le z \le \infty$

If ϕ changes from 0 to $\pi/2$ → we get 1/4th of the cylinder
0 to π → 1/2 portion of cylinder

Spherical Coordinate System

↳ we assume that space is in the form of sphere having radius r.

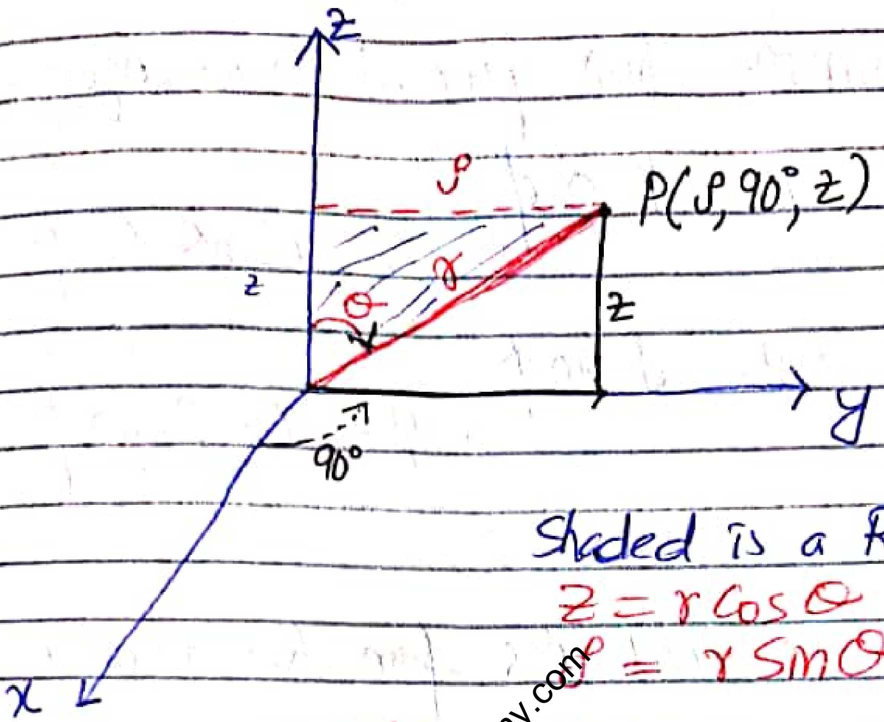
Rectangular $P(x, y, z)$ cylindrical



→ spherical system
↳ radial line r
↳ making angle θ w.r.t z axis
↳ Rissolving it into its cupnts

The shaded triangle is a right angled triangle.

$P(\rho, 90^\circ, z)$



Shaded is a R.A.T

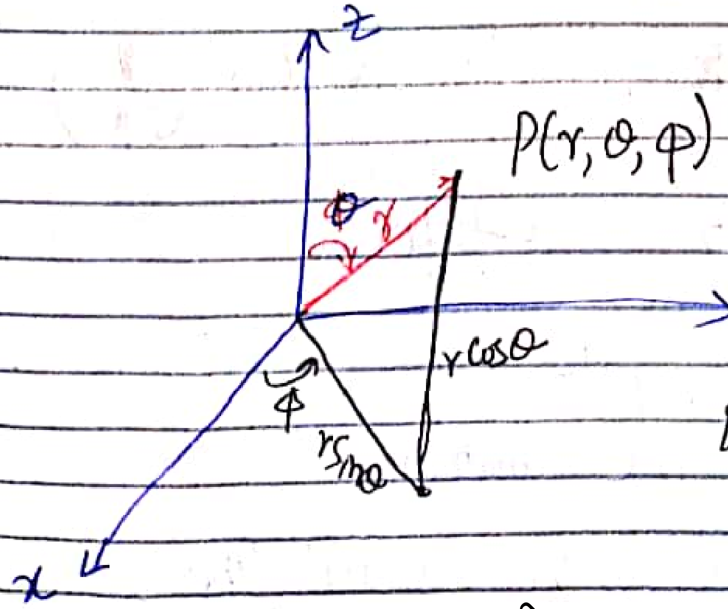
$z = r\cos\theta$

$\rho = r\sin\theta$

$P(r, \theta, \phi)$

line in $z=0$ plane such that its length is $r\sin\theta$ and it makes angle ϕ w.r.t x axis.

From the end of this line, draw another line parallel to z axis with length $r\cos\theta$.



Limits

$0 \le r \le \infty$

$0 \le \theta \le \pi$

$0 \le \phi \le 2\pi$

$\rho = r\sin\theta \qquad z = r\cos\theta$

Transformation from spherical to rectangular

As $x = \rho\cos\phi$

$\Rightarrow x = r\sin\theta\cos\phi$

As $y = \rho\sin\phi$

$\Rightarrow y = r\sin\theta\sin\phi$

$z = r\cos\theta$

Spherical Cordinates from Rectangular

$x^2+y^2 = \rho^2$

$\Rightarrow x^2+y^2 = r^2\sin^2\theta$

$+ \quad z^2 = r^2\cos^2\theta$

$x^2+y^2+z^2 = r^2$

$r = \sqrt{x^2+y^2+z^2} \qquad \because \sin^2\theta + \cos^2\theta = 1$

$\theta = \cos^{-1}\left(\frac{z}{r}\right) \qquad \phi = \tan^{-1}\left(\frac{y}{x}\right)$

Lecture 3 02/10/19

$\vec{a}_r, \vec{a}_\phi, \vec{a}_\theta$

Normal to one another.

$\vec{a}_r$ is along the radial line r and makes an angle of θ w.r.t z axis

$\vec{a}_\phi$ is in the $z=0$ plane and it makes angle of $90° + \phi$ wrt x axis.

$\vec{a}_\theta$ is always normal to the plane containing $\vec{a}_r$ and $\vec{a}_\phi$

$\vec{a}_\theta = \vec{a}_x \leftarrow \left[\begin{array}{l} \text{If } \theta = 0° \\ \text{If } \phi = 0° \end{array}\right.$ $\quad \vec{a}_r = \vec{a}_z$, $\quad \vec{a}_\phi = \vec{a}_y$

Cross Product

$$\vec{a}_r \times \vec{a}_\theta = \vec{a}_\phi$$

$$\vec{a}_\theta \times \vec{a}_\phi = \vec{a}_r$$

$$\vec{a}_\phi \times \vec{a}_r = \vec{a}_\theta$$

eg If $\phi = 0°$ and $\theta = 90°$

$\vec{a}_r = \vec{a}_x \quad \vec{a}_\phi = \vec{a}_y \quad \vec{a}_\theta = \vec{a}_{-z}$

$= 1$

satisfies the above.

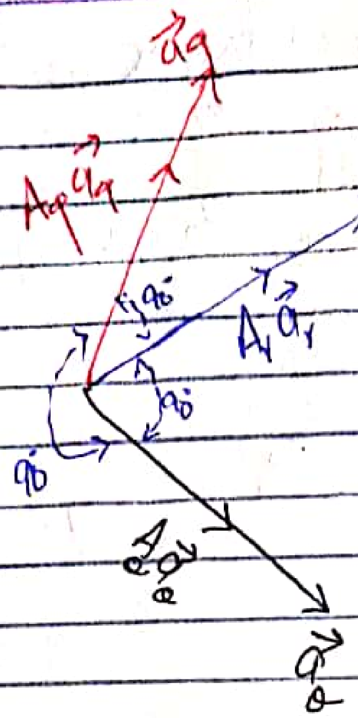
eg If $\theta = 0°$ $\phi = 90°$

$\vec{a}_r = \vec{a}_z \quad \vec{a}_\phi = -\vec{a}_x \quad \vec{a}_\theta = \vec{a}_y$

satisfies

The position of these three unit vectors depend on values of θ and ϕ.

Scalar Product



$$\vec{a}_r \cdot \vec{a}_\theta = \vec{a}_\theta \cdot \vec{a}_\phi =$$
$$\vec{a}_\phi \cdot \vec{a}_r = 0$$

$$\vec{a}_r \cdot \vec{a}_r = \vec{a}_\theta \cdot \vec{a}_\theta = \vec{a}_\phi \cdot \vec{a}_\phi =$$

Cross Product

$$\vec{a}_r \times \vec{a}_\theta = \vec{a}_\phi$$
$$\vec{a}_\theta \times \vec{a}_r = -\vec{a}_\phi$$

$$\vec{a}_\theta \times \vec{a}_\phi = \vec{a}_r \qquad \vec{a}_\phi \times \vec{a}_r = \vec{a}_\theta$$
$$\vec{a}_\phi \times \vec{a}_\theta = -\vec{a}_r \qquad \vec{a}_r \times \vec{a}_\phi = -\vec{a}_\theta$$

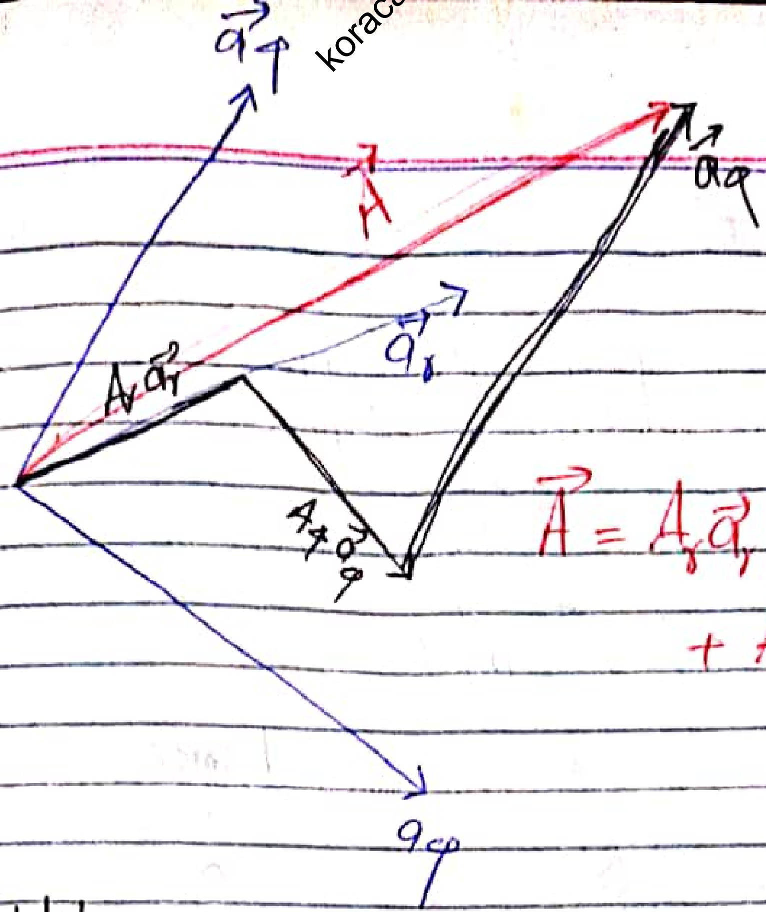
$$\vec{a}_r \times \vec{a}_r = \vec{a}_\theta \times \vec{a}_\theta = \vec{a}_\phi \times \vec{a}_\phi = 0$$

Three Dimensional Vector In Spherical Cordinate System.

Can be resolved into three component

(i) component in direction of $\vec{a}_r$
(ii) " " $\vec{a}_\phi$
(iii) " " " $\vec{a}_\theta$

Addition of these 3 components according to head & tail rule will give you the position of the vector



$\vec{A} = A_r\vec{a}_r + A_\theta\vec{a}_\theta + A_\phi\vec{a}_\phi$

Addition. Product.

Two conditions for addition, scalar, cross

(i) θ must be same for both. If not satisfied

(ii) ϕ must be same also. → rectangular → resultant → spherical

$\vec{A} = A_r\vec{a}_r \quad A_\theta\vec{a}_\theta \quad A_\phi\vec{a}_\phi$

$\vec{B} = B_r\vec{a}_r \quad B_\theta\vec{a}_\theta \quad B_\phi\vec{a}_\phi$

$\vec{A}+\vec{B} = \vec{B}+\vec{A} = (A_r+B_r)\vec{a}_r + (A_\theta+B_\theta)\vec{a}_\theta + (A_\phi+B_\phi)\vec{a}_\phi$

Scalar Product of Vectors.

$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A} = A_rB_r + A_\theta B_\theta + A_\phi B_\phi$

Let $\vec{B} = \vec{A}$

$\vec{A}\cdot\vec{A} = A_r\cdot A_r + A_\theta A_\theta + A_\phi A_\phi$

$A = \sqrt{A_r^2 + A_\theta^2 + A_\phi^2}$

Cross Product

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{a}_r & \vec{a}_\theta & \vec{a}_\varphi \\ A_r & A_\theta & A_\varphi \\ B_r & B_\theta & B_\varphi \end{vmatrix}$$

$$\vec{B} \times \vec{A} = \begin{vmatrix} \vec{a}_r & \vec{a}_\theta & \vec{a}_\varphi \\ B_r & B_\theta & B_\varphi \\ A_r & A_\theta & A_\varphi \end{vmatrix}$$

Limits

$0 \le r \le \infty$

$0 \le \theta \le \pi$

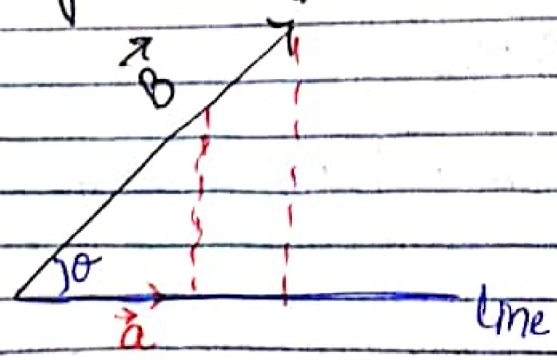
$0 \le \varphi \le 2\pi$

$\theta \quad 0 - \pi$

If θ changes from 0 to $\pi/2$ and φ changes from 0 to $\pi/2$ we have 1/4th of sphere.

$\theta = 180^\circ \quad \varphi = 180^\circ \rightarrow$ half of sphere

How to find components of a vector along any line?



The scalar component of B along the line (or unit vector $\vec{a}$) will be $B\cos\theta$.

$B\cos\theta = \vec{B} \cdot \vec{a}$

→ Scalar projection of $\vec{B}$ onto $\vec{a}$

→ The x, y and z components of vector $\vec{B}$:

$B_x = \vec{B} \cdot \vec{a}_x \qquad B_y = \vec{B} \cdot \vec{a}_y \qquad B_z = \vec{B} \cdot \vec{a}_z$

In rectangular cordinate system;

$$\vec{B} = B_x \vec{a}_x + B_y \vec{a}_y + B_z \vec{a}_z$$

Similarly for cylindrical cordinate system;

Scalar component of B along a_ρ: $B_\rho = \vec{B} \cdot \vec{a}_\rho$

$$B_\phi = \vec{B} \cdot \vec{a}_\phi \qquad B_z = \vec{B} \cdot \vec{a}_z$$

$$\text{so } \vec{B} = B_\rho \vec{a}_\rho + B_\phi \vec{a}_\phi + B_z \vec{a}_z$$

Also for spherical.

$$B_r = \vec{B} \cdot \vec{a}_r \qquad B_\theta = \vec{B} \cdot \vec{a}_\theta \qquad B_\phi = \vec{B} \cdot \vec{a}_\phi$$

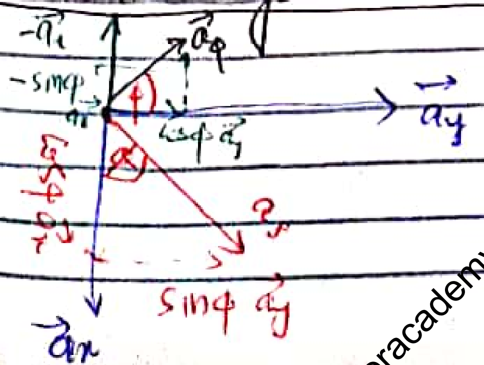
$$\text{so } \vec{B} = B_r \vec{a}_r + B_\theta \vec{a}_\theta + B_\phi \vec{a}_\phi$$

Vector Component of $\vec{B}$ along $\vec{a}$

$$= B \cos\theta \, \vec{a} = (B \cdot \vec{a}) \vec{a}$$

Lecture 4 07/10/19

Transformation of Unit Vectors From Rectangular cordinate system To cylindrical



Resolving $\vec{a}_\rho$ into its components.

$$\vec{a}_\rho = \cos\phi \, \vec{a}_x + \sin\phi \, \vec{a}_y + 0 \cdot \vec{a}_z \quad \rightarrow (1)$$

(x, y, z)

$$\boxed{\vec{a}_\rho \cdot \vec{a}_x = \cos\phi \quad \vec{a}_\rho \cdot \vec{a}_y = \sin\phi \quad \vec{a}_\rho \cdot \vec{a}_z = 0}$$

$\vec{a}_\phi$ makes angle $90° + \phi$ with x axis and ϕ with y axis.

↳ Resolving it into its components;

$$\vec{a}_\phi = -\sin\phi\,\vec{a}_x + \cos\phi\,\vec{a}_y + 0 \cdot \vec{a}_z \rightarrow (2)$$

$$\boxed{\vec{a}_\phi \cdot \vec{a}_x = -\sin\phi \quad \vec{a}_\phi \cdot \vec{a}_y = \cos\phi \quad \vec{a}_\phi \cdot \vec{a}_z = 0}$$

The third unit vector is along z axis.

$$\vec{a}_z = 0\vec{a}_x + 0\vec{a}_y + 1\vec{a}_z \rightarrow (3)$$

In matrices format

$$\begin{bmatrix} \vec{a}_\rho \\ \vec{a}_\phi \\ \vec{a}_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \vec{a}_x \\ \vec{a}_y \\ \vec{a}_z \end{bmatrix}$$

Transformation of Vector from Rectangular To Cylindrical

Consider a 3D vector;

$$\vec{A} = A_x \vec{a}_x + A_y \vec{a}_y + A_z \vec{a}_z$$

$A_\rho = \vec{A}\cdot\vec{a}_\rho = (A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z)\cdot\vec{a}_\rho$

$A_\rho = \cos\phi . A_x + \sin\phi . A_y + 0 . A_z \rightarrow (1)$

$A_\phi = \vec{A}\cdot\vec{a}_\phi = (A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z)\cdot\vec{a}_\phi$

$A_\phi = -\sin\phi . A_x + \cos\phi . A_y + 0 A_z \rightarrow (2)$

$A_z = A_z = 0 A_x + 0 A_y + A_z \rightarrow (3)$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Transformation of A Vector from Cylindrical To Rectangular

Consider a vector in the C.C.S

$\vec{A} = A_\rho\vec{a}_\rho + A_\phi\vec{a}_\phi + A_z\vec{a}_z$

$A_x = \vec{A}\cdot\vec{a}_x = (A_\rho\vec{a}_\rho + A_\phi\vec{a}_\phi + A_z\vec{a}_z)\cdot\vec{a}_x$

$A_x = \cos\phi A_\rho - \sin\phi . A_\phi + 0 . A_z \rightarrow (1)$

$A_y = \vec{A}\cdot\vec{a}_y = (A_\rho\vec{a}_\rho + A_\phi\vec{a}_\phi + A_z\vec{a}_z)$

$A_y = \sin\phi . A_\rho + \cos\phi . A_\phi + 0 A_z \rightarrow (2)$

$A_z = 0 A_\rho + 0 A_\phi + A_z \rightarrow (3)$

In matrix form

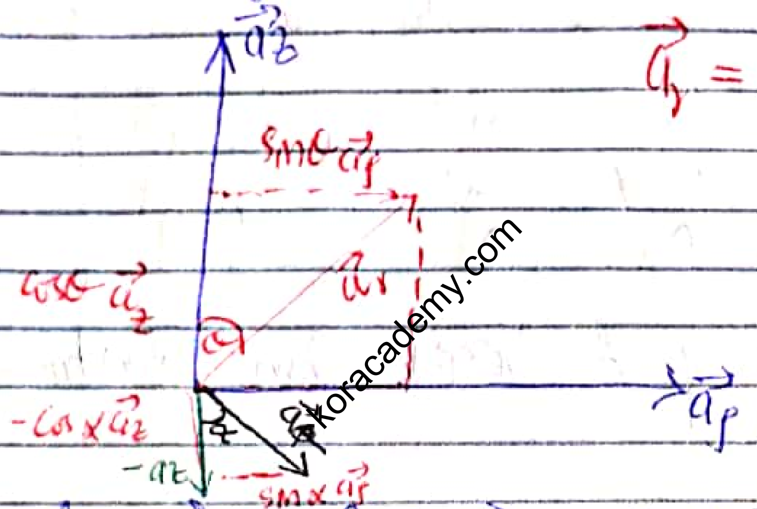
$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

Transformation of Unit Vectors from Rectangular to spherical Coordinate system

$\vec{a}_z$ is always along z axis.

$\vec{a}_\rho$ makes 90° w.r.t z axis.

The first unit vector of S.C.S is $\vec{a}_r$ making an angle θ w.r.t z axis.



$$\vec{a}_r = \sin\theta\,\vec{a}_\rho + \cos\theta\,\vec{a}_z$$

As $\vec{a}_\rho = \cos\phi\,\vec{a}_x + \sin\phi\,\vec{a}_y$

$$\Rightarrow \vec{a}_r = \sin\theta\cos\phi\,\vec{a}_x + \sin\theta\sin\phi\,\vec{a}_y + \cos\theta\,\vec{a}_z \rightarrow (A)$$

$$\boxed{\begin{aligned} \vec{a}_r\cdot\vec{a}_x &= \sin\theta\cos\phi \\ \vec{a}_r\cdot\vec{a}_y &= \sin\theta\sin\phi \\ \vec{a}_r\cdot\vec{a}_z &= \cos\theta \end{aligned}}$$

$$\boxed{\begin{aligned} \vec{a}_\theta\cdot\vec{a}_x &= \cos\theta\cos\phi \\ \vec{a}_\theta\cdot\vec{a}_y &= \cos\theta\sin\phi \\ \vec{a}_\theta\cdot\vec{a}_z &= -\sin\theta \\ \vec{a}_\phi\cdot\vec{a}_x &= -\sin\phi \\ \vec{a}_\phi\cdot\vec{a}_y &= \cos\phi \\ \vec{a}_\phi\cdot\vec{a}_z &= 0 \end{aligned}}$$

The second unit vector $\vec{a}_\theta$ will be perpendicular to $\vec{a}_r$ making angle α with -ve z axis.

$$\theta + 90^\circ + \alpha = 180^\circ$$

$$\Rightarrow \alpha = 90^\circ - \theta$$

$$\vec{a}_\theta = \sin\alpha\,\vec{a}_\rho - \cos\alpha\,\vec{a}_z$$

$$\vec{a}_\theta = \cos\theta\,\vec{a}_\rho - \sin\theta\,\vec{a}_z$$

putting value of $\vec{a}_\rho$

$$\vec{a}_\theta = \cos\theta\cos\phi\,\vec{a}_x + \cos\theta\sin\phi\,\vec{a}_y - \sin\theta\,\vec{a}_z \rightarrow (B)$$

$$\vec{a}_\phi = -\sin\phi\,\vec{a}_x + \cos\phi\,\vec{a}_y + 0\cdot\vec{a}_z \rightarrow (C)$$

In matrix form

$$\begin{bmatrix} \vec{a}_r \\ \vec{a}_\theta \\ \vec{a}_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} \vec{a}_x \\ \vec{a}_y \\ \vec{a}_z \end{bmatrix}$$

Lecture 5 09/10/19

Transformation of A Vector from Rectangular To Spherical Coordinate system

$$\vec{A} = A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z \rightarrow (A)$$

$$A_r = \vec{A}\cdot\vec{a}_r = (A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z)\cdot\vec{a}_r$$

$$A_r = \sin\theta\cos\phi\,A_x + \sin\theta\sin\phi\,A_y + \cos\theta\,A_z \rightarrow (1)$$

$$A_\theta = \vec{A}\cdot\vec{a}_\theta = (A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z)\cdot\vec{a}_\theta$$

$$A_\theta = \cos\theta\cos\phi\,A_x + \cos\theta\sin\phi\,A_y - \sin\theta\,A_z \rightarrow (2)$$

$$A_\phi = \vec{A}\cdot\vec{a}_\phi = (A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z)\cdot\vec{a}_\phi$$

$$A_\phi = -\sin\phi\,A_x + \cos\phi\,A_y + 0\,A_z \rightarrow (3)$$

The same vector (A) can be written in the spherical coordinate system as;

$$\vec{A} = A_r \vec{a_r} + A_\theta \vec{a_\theta} + A_\phi \vec{a_\phi} \rightarrow (B)$$

(1), (2), (3) in matrices format

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Transformation of A Vector from Spherical to Rectangular coordinate system!

Consider a 3D vector in S.C.S

$$\vec{A} = A_r \vec{a_r} + A_\theta \vec{a_\theta} + A_\phi \vec{a_\phi} \rightarrow (1)$$

To transform it into

$$\vec{A} = A_x \vec{a_x} + A_y \vec{a_y} + A_z \vec{a_z} \rightarrow (2)$$

we have to calculate the 3 components i.e A_x, A_y, A_z

Whenever we transform a vector from one coordinate system to another, the magnitude and direction remains the same.

$$A_x = \vec{A} \cdot \vec{a_x} = (A_r \vec{a_r} + A_\theta \vec{a_\theta} + A_\phi \vec{a_\phi}) \cdot \vec{a_x}$$

$$A_x = \sin\theta\cos\phi A_r + \cos\theta\cos\phi A_\theta - \sin\phi A_\phi \rightarrow (1)$$

$$A_y = \vec{A} \cdot \vec{a_y} = (A_r \vec{a_r} + A_\theta \vec{a_\theta} + A_\phi \vec{a_\phi}) \cdot \vec{a_y}$$

$$A_y = \sin\theta\sin\phi A_r + \cos\theta\sin\phi A_\theta + \cos\phi A_\phi \rightarrow (2)$$

$$A_z = \vec{A}\cdot\vec{a_z} = (A_r\vec{a_r} + A_\theta\vec{a_\theta} + A_\phi\vec{a_\phi})\cdot\vec{a_z}$$

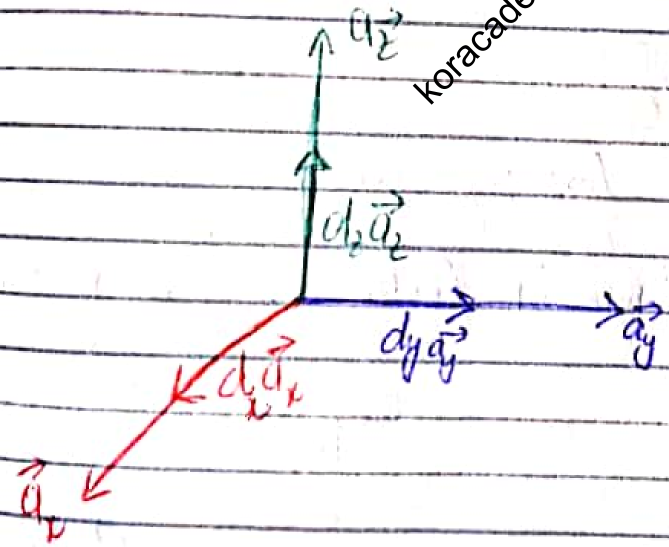
$$A_z = \cos\theta A_r - \sin\theta A_\theta + 0 A_\phi \rightarrow (3)$$

In matrices format.

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

Differential Volume In Rectangular Cordinate system

Start with the unit vectors.



Consider a very small vector along x axis (dl)

$$dl = d_x\vec{a_x}$$

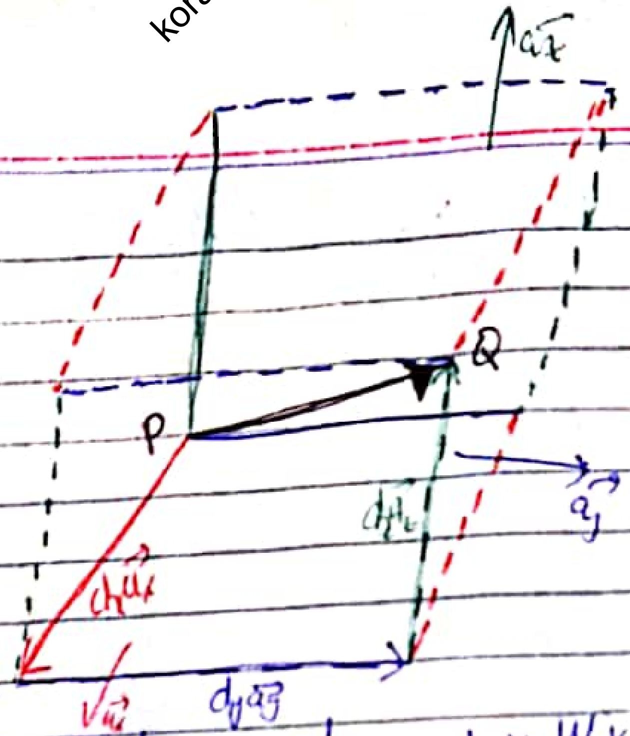
Similarly along y axis.

$$dl = d_y\vec{a_y}$$

Along z axis $\rightarrow$ $dl = d_z\vec{a_z}$

Consider the magnitudes of these vectors and we construct a differential volume with them.

$$\vec{dl} = d_x\vec{a_x} + d_y\vec{a_y} + d_z\vec{a_z}$$



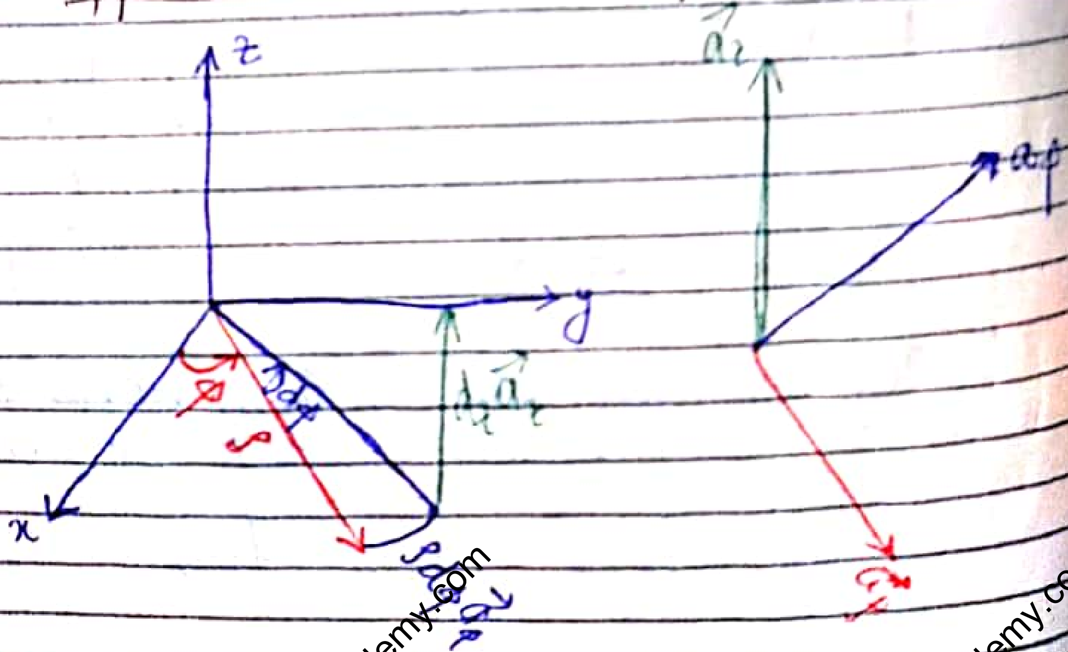
Differential volume $= dv = L \times W \times H$

$dv = dx.dy.dz$

$$\boxed{\begin{aligned} \vec{dS} &= dy.dz\,\vec{a_x} \\ \vec{dS} &= dx.dz\,\vec{a_y} \\ \vec{dS} &= dx.dy\,\vec{a_z} \end{aligned}}$$

Let a vector in the box; components

Differential Volume In Cylindrical C-System



Consider a very small vector in the direction of $\vec{a_\rho}$.

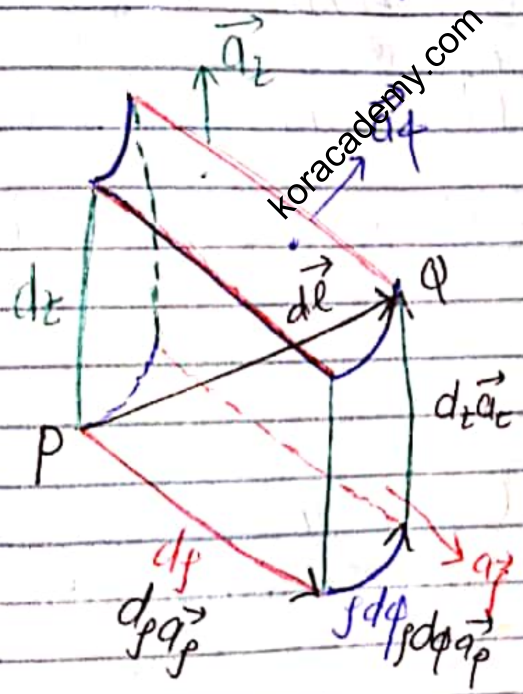
$$d\ell = d\rho\, \vec{a_\rho}$$

Rotate ρ by a small angular displacement $d\phi$
(straight line)

$$dl = \rho\, d\phi\, \vec{a_\phi}$$

$$dl = d_z\, \vec{a_z}$$

Consider the magnitudes to draw a cylinder.



Differential volume

$$dV = L \times W \times H$$

$$dV = \rho\, d\rho\, d\phi\, d_z$$

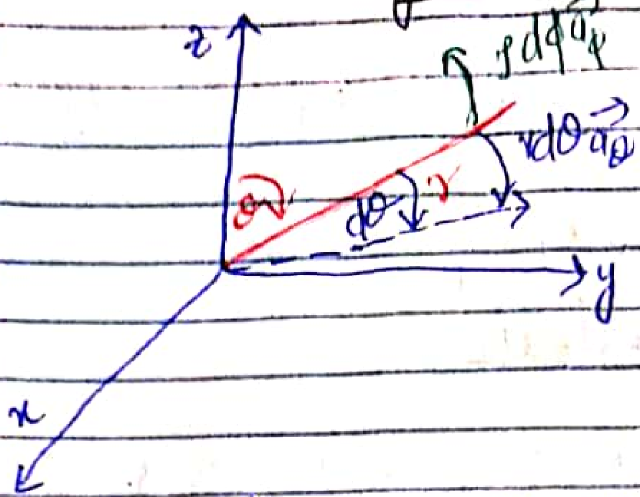
$$\boxed{\begin{aligned} d\vec{S} &= \rho\, d\phi\, d_z\, \vec{a_\rho} \\ d\vec{S} &= d\rho\, d_z\, \vec{a_\phi} \\ d\vec{S} &= \rho\, d\rho\, d\phi\, \vec{a_z} \end{aligned}}$$

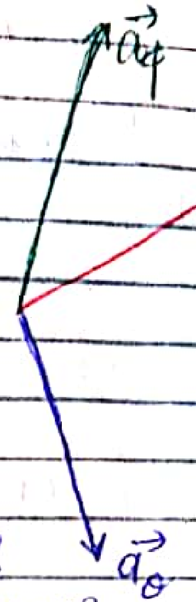
Considering a vector PQ in the cylinder.

→ components.

$$\vec{d\ell} = d\rho\, \vec{a_\rho} + \rho\, d\phi\, \vec{a_\phi} + d_z\, \vec{a_z}$$

Differential Volume In Spherical Cordinate System





first $\vec{a}_r \rightarrow$ angle θ with z axis
Second $\vec{a}_\phi \rightarrow$ angle $90° + \phi$ with x axis
third $\vec{a}_\theta \rightarrow$ perpendicular to plane of first two

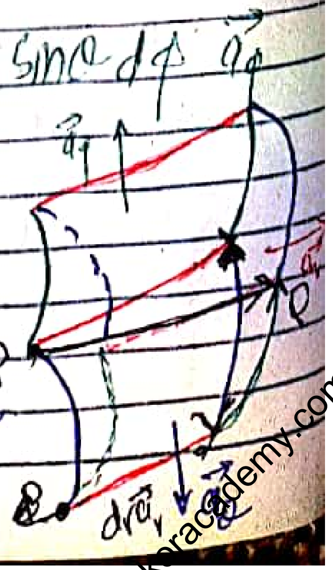
Consider a very small vector along the direction of unit vectors.

$$dl = dr\,\vec{a}_r$$

Rotate radial line r through a very small angular displacement $(d\theta) \rightarrow$ the arc of length is $r d\theta$.

$$dl = r\,d\theta\,\vec{a}_\theta$$

$$dl = \rho\,d\phi\,\vec{a}_\phi = r\sin\theta\,d\phi\,\vec{a}_\phi$$



Considering magnitudes;

Differential volume;

$dv = L \times W \times H = dr \times r\sin\theta\,d\phi \times r\,d\theta$

$$dv = r^2 \sin\theta\,d\theta\,d\phi\,dr.$$

$$d\vec{s} = r^2 \sin\theta\, d\theta\, d\phi\, \vec{a_r}$$
$$d\vec{s} = r dr\, d\theta\, \vec{a_\phi}$$
$$d\vec{s} = r\sin\theta\, dr\, d\phi\, \vec{a_\theta}$$

considering a vector inside the box; outputs

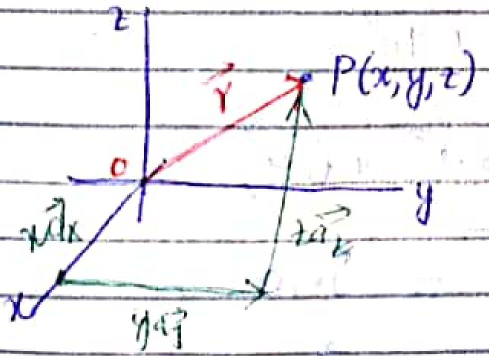
$$\vec{dl} = dr\vec{a_r} + r d\theta \vec{a_\theta} + r\sin\theta\, d\phi\, \vec{a_\phi}$$

Position Vector

A vector extending from the origin to any other point P is known as the position vector of that point P.
The position vector is denoted by $\vec{r}$.

Distance vector

A vector b/w any two points (excluding the point of origin) is known as distance vector. represented by $\vec{R}$.

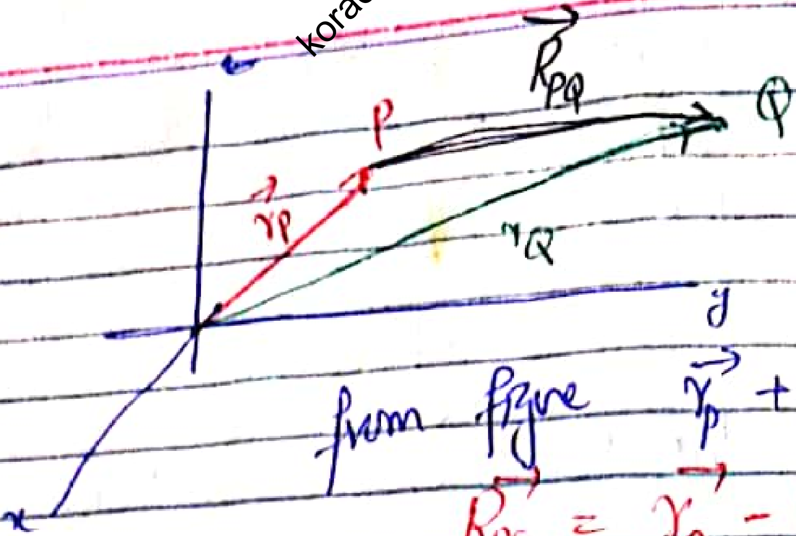


$$\vec{r_p} = x\vec{a_x} + y\vec{a_y} + z\vec{a_z}$$

If two points P and Q.

$$P(x_1, y_1, z_1) \qquad Q(x_2, y_2, z_2).$$

$$\vec{r_P} = x_1\vec{a_x} + y_1\vec{a_x} + z_1\vec{a_z}$$
$$\vec{r_Q} = x_2\vec{a_x} + y_2\vec{a_x} + z_2\vec{a_z}$$



from figure $\vec{r}_P + \vec{R}_{PQ} = \vec{r}_Q$

$\vec{R}_{PQ} = \vec{r}_Q - \vec{r}_P$

$$\vec{R}_{PQ} = (x_2-x_1)\vec{a}_x + (y_2-y_1)\vec{a}_y + (z_2-z_1)\vec{a}_z$$

$$R_{PQ} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

Lecture 6 14/10/19.

1.1) $\vec{r}_A = -\vec{a}_x - 3\vec{a}_y - 4\vec{a}_z$

$\vec{r}_B = 2\vec{a}_x + 2\vec{a}_y + 2\vec{a}_z$

$\vec{r}_C = \vec{a}_x + 3\vec{a}_y + 4\vec{a}_z$

$\vec{R}_{AB} = \vec{r}_B - \vec{r}_A = 3\vec{a}_x + 5\vec{a}_y + 6\vec{a}_z$

$r_A = \sqrt{(1)^2 + (-3)^2 + (-4)^2} = 5.10$

$\vec{a}_{r_A} = \dfrac{\vec{r}_A}{r_A} = -0.19\vec{a}_x - 0.588\,\vec{a}_y - 0.784\,\vec{a}_z$

$\vec{a}_{AB} = \dfrac{\vec{R}_{AB}}{R_{AB}} \Rightarrow R_{AB} = \sqrt{9+25+36} = 8.36$

$\vec{a}_{AB} = 0.358\,\vec{a}_x + 0.598\,\vec{a}_y + 0.717\,\vec{a}_z$

$$\vec{a}_{CA} = \frac{\vec{R}_{CA}}{R_{CA}}$$

$$\vec{R}_{CA} = \vec{r}_A - \vec{r}_C = -2\vec{a}_x - 6\vec{a}_y - 8\vec{a}_z$$

$$R_{CA} = \sqrt{4+36+64} = 10.19$$

$$\vec{a}_{CA} = -0.196\,\vec{a}_x - 0.588\,\vec{a}_y - 0.785\,\vec{a}_z$$

1.3) $A(2,5,-1)$ $B(3,-2,4)$ $C(-2,3,1)$

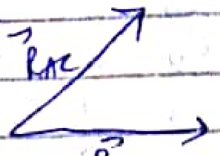
$$\vec{R}_{AB} \cdot \vec{R}_{AC} = ?$$

$$\vec{R}_{AB} = \vec{r}_B - \vec{r}_A = \vec{a}_x - 7\vec{a}_y + 5a_z$$

$$\vec{R}_{AC} = \vec{r}_C - \vec{r}_A = -4\vec{a}_x - 2\vec{a}_y + 2\vec{a}_z$$

$$\vec{R}_{AB} \cdot \vec{R}_{AC} = (1)(-4) + (-7)(-2) + (5)(2) = 20$$

θ.



$$\vec{R}_{AB} \cdot \vec{R}_{AC} = R_{AB} \times R_{AC} \times \cos\theta$$

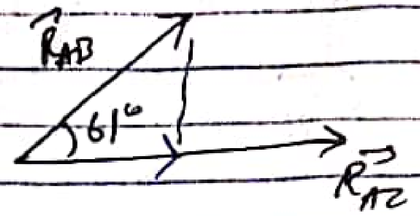
$$\Rightarrow \theta = \cos^{-1}\left(\frac{\vec{R}_{AB} \cdot \vec{R}_{AC}}{R_{AB} \times R_{AC}}\right)$$

$$R_{AB} = \sqrt{1+49+25} = 8.66$$

$$R_{AC} = \sqrt{16+4+4} = 4.89$$

$$\Rightarrow \theta = \left(\frac{20}{8.66 \times 4.89}\right) = 61.87^\circ$$

Scalar projection of R_{AB} on R_{AC}



$R_{AB}\cos\theta = 8.66 \times \cos 61.4°$

$= 4.07\ \vec{a}_{AC}$

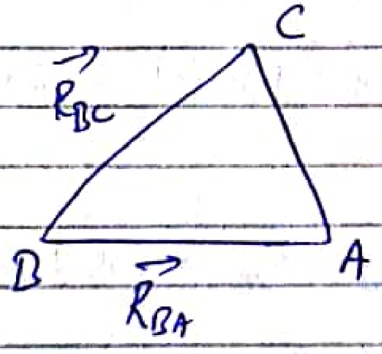
Vector projection of $\vec{R}_{AB}$ on $\vec{R}_{AC}$

$$\vec{a}_{AC} = \frac{\vec{R}_{AC}}{R_{AC}} = -0.818\vec{a}_x - 0.408\vec{a}_y + 0.408\vec{a}_z$$

1.4) Vertices of triangle:

A(2,-5,1) B(-3,2,4) C(0,3,1)

(a) $\vec{R}_{BC} \times \vec{R}_{BA}$ $\vec{R}_{BC} = \vec{r}_C - \vec{r}_B = 3\vec{a}_x + \vec{a}_y - 3\vec{a}_z$



$\vec{R}_{BA} = \vec{r}_A - \vec{r}_B = 5\vec{a}_x - 7\vec{a}_y - 3\vec{a}_z$

$$\vec{R}_n = \vec{R}_{BC} \times \vec{R}_{BA} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 3 & 1 & -3 \\ 5 & -7 & -3 \end{vmatrix}$$

$\vec{R}_n = -24\vec{a}_x - 6\vec{a}_y - 26\vec{a}_z$

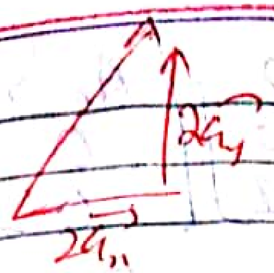
(c) $\vec{a}_n = \dfrac{\vec{R}_n}{R_n}$ $R_n = \sqrt{(-24)^2 + (-6)^2 + (-26)^2}$

$= 35.88$

$\vec{a}_n = -0.668\vec{a}_x - 0.167\vec{a}_y - 0.724\vec{a}_z$

(b) Area of triangle $= \frac{1}{2}\left|\vec{R}_{BC} \times \vec{R}_{BA}\right|$

$= \frac{1}{2}R_n$ 17.94



$A = \frac{1}{2} \times 2 \times 2 = 2$

$A = \frac{1}{2} \left| 2\vec{a_x} \times 2\vec{a_y} \right|$

$= \frac{1}{2} \left| 4\vec{a_z} \right| = \frac{1}{2} \times 4 = 2$

1.5) $A(2, 3, -1)$

(a) distance from A to origin.

$O(0,0,0)$

$$d = \sqrt{(2-0)^2 + (3-0)^2 + (-1-0)^2} = 3.74 \text{ units}$$

(b) cylindrical system $B(4, -50°, 2)$

Distance from B to O.

transform from cylindrical to rectangular

$x = \rho \cos\varphi = 2.57$

$y = \rho \sin\varphi = -3.06$ and $z = 2$

so $B(2.57, -3.06, 2)$

so distance from origin

$$d = \sqrt{(2.57-0)^2 + (-3.06-0)^2 + (2-0)^2} = 4.07 \text{ units.}$$

(c) Distance b/w A and B

$$R_{AB} = \sqrt{(2.57-2)^2 + (-3.06-3)^2 + (2+1)^2} = 6.78$$

1.6) Transform $5\vec{a_x}$ to C.C.S at $P(4, 120°, 2)$

let $\vec{A} = 5\vec{a_x}$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos 120^\circ & \sin 120^\circ & 0 \\ -\sin 120^\circ & \cos 120^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x = 5 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5\cos 120^\circ \\ -5\sin 120^\circ \\ 0 \end{bmatrix} = \begin{bmatrix} -2.5 \\ -4.33 \\ 0 \end{bmatrix}$$

$$\Rightarrow \vec{A} = -2.5\,\vec{a}_\rho - 4.33\,\vec{a}_\phi$$

1.7) $A(2,3,-1)$ Calculate the spherical coordinates of the point.

(a) ~~distance from A to O.~~

~~$d = (2-0)^2$~~ $A(r, \theta, \phi)$

$$r = \sqrt{4+9+1} = 3.74$$

$$\theta = \cos^{-1}\left(\frac{z}{r}\right) = \cos^{-1}\left(\frac{-1}{3.74}\right) = 105.5^\circ$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{3}{2}\right) = 56.3^\circ$$

$$\Rightarrow A(3.74, 105.5^\circ, 56.3^\circ)$$

(b) S.C.R $B(4, 25^\circ, 120^\circ)$

$$x = r\sin\theta\cos\phi = -0.845$$
$$y = r\sin\theta\sin\phi = 1.46$$
$$z = r\cos\theta = 3.63$$

(c) Distance b/w A and B.

$$R_{AB} = \sqrt{(-0.845-2)^2 + (1.46-3)^2 + (3.63+1)^2}$$
$$= 5.64.$$

1.8) Transform $5\vec{a_x}$ into S.C.S at $P(4, 25°, 120°)$

let $\vec{A} = 5\vec{a_x}$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin 25 \cos 120° & \sin 25 \sin 120° & \cos 25 \\ \cos 25 \cos 120° & \cos 25 \sin 120° & -\sin 25 \\ -\sin 120° & \cos 120° & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

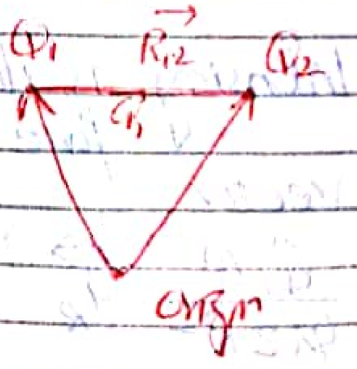
$A_r = -1.056 \quad A_\theta = -2.27 \quad A_\phi = -4.33$

$\Rightarrow \vec{A} = -1.056\vec{a_r} - 2.27\vec{a_\theta} - 4.33\vec{a_\phi}$

Chapter 2 :

Coulomb's law

Consider two charges Q_1 and Q_2 having position vectors $\vec{r_1}$ and $\vec{r_2}$. The distance vector from Q_1 to Q_2 is $\vec{R_{12}}$. The unit vector in this direction is $\vec{a_r}$



Q_1 and $Q_2 \Rightarrow$ Attraction or Repulsion

unlike ↲ ↳ like

$F \propto \frac{Q_1 Q_2}{R_2}$

$K = \frac{1}{4\pi\varepsilon}$ $\quad \varepsilon$ = permittivity

$\varepsilon = \varepsilon_0 \varepsilon_r$ $\quad \boxed{\varepsilon_0 = 8.85 \times 10^{-12}}$

$$F = \frac{\cancel{\alpha}\, Q_1 Q_2}{4\pi\varepsilon R^2}$$

Force is a vector quantity → directed along the straight line joining these two particles.

Assuming them to be like charge particles

The force acting on Q_2 due to presence of Q_1.

$$\vec{F_2} = \frac{Q_1 Q_2}{4\pi\varepsilon R^2}\,\vec{a_R}$$

As $\vec{a_R} = \dfrac{\vec{R}}{R}$ $\Rightarrow$ $\boxed{\vec{F_2} = \dfrac{Q_1 Q_2 \vec{R}}{4\pi\varepsilon R^3}}$

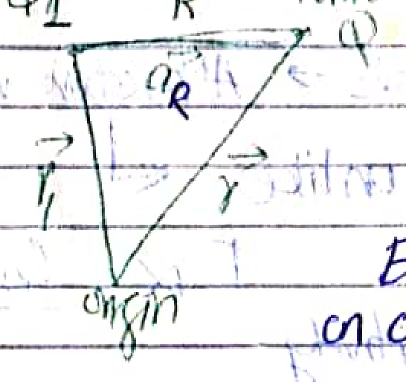
$\vec{R} = \vec{R_{12}} = \vec{r_2} - \vec{r_1}$ $\quad R = |\vec{r_2} - \vec{r_1}|$

$$\boxed{F_2 = \frac{Q_1 Q_2 (\vec{r_2} - \vec{r_1})}{4\pi\varepsilon |r_2 - r_1|^3}}$$

Similarly $F_1 = -F_2$

Electric Field Intensity Due to a Point Charge

Source Q_1 $\vec{R}$ Point P



- Intensity is electric force
- Vector

$$\vec{F} = \frac{Q_1 Q}{4\pi\varepsilon R^2}\,\vec{a_R}$$

Electric force is a force on charge of Q columbs.

Force on a charge of 1C will be electric field intensity.

↳ Force per unit positive charge

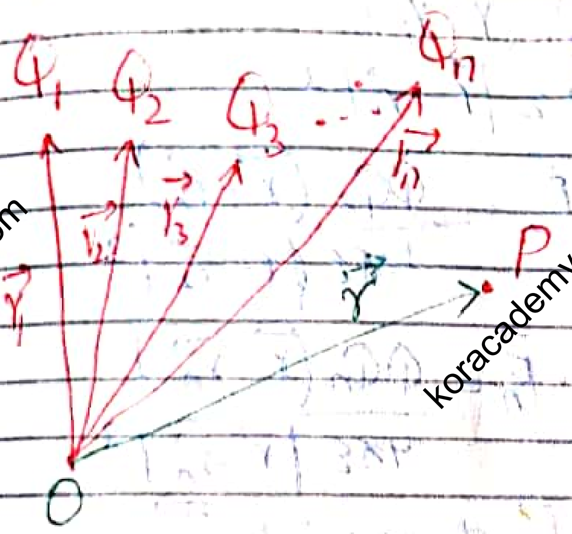
↳ Represented by E.

$$\vec{E} = \frac{\vec{F}}{Q} = \frac{Q_1}{4\pi\varepsilon R^2}\,\vec{a_R}$$

$$\vec{a}_R = \frac{\vec{R}}{R} \Rightarrow \boxed{\vec{E} = \frac{Q_1 \vec{R}}{4\pi\varepsilon R^3}}$$

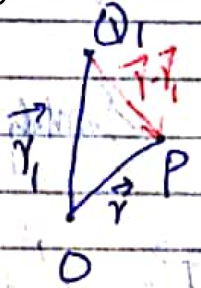
As $\vec{R} = \vec{r} - \vec{r}_1$
$R = |\vec{r} - \vec{r}_1|$ $\Rightarrow$ $\boxed{\vec{E} = \frac{Q_1(\vec{r} - \vec{r}_1)}{4\pi\varepsilon(\vec{r} - \vec{r}_1)^3}}$

Electric Field Intensity due to "n" charge particles



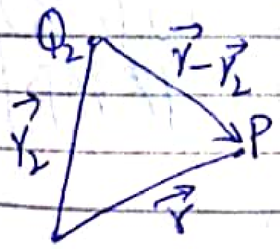
Apply superposition theorem;

$Q_2 = Q_3 = \cdots = Q_n = 0$



$$\vec{E}_1 = \frac{Q_1(\vec{r} - \vec{r}_1)}{4\pi\varepsilon|\vec{r} - \vec{r}_1|^3}$$

Now $Q_1 = Q_3 = \cdots = Q_n = 0$



$$\vec{E}_2 = \frac{Q_2(\vec{r}_1 - \vec{r}_2)}{4\pi\varepsilon|\vec{r} - \vec{r}_2|^3} \qquad \vec{E}_3 = \frac{Q_3(\vec{r} - \vec{r}_3)}{4\pi\varepsilon|\vec{r} - \vec{r}_3|^3}$$

Similarly $\vec{E}_n = \frac{Q_n(\vec{r} - \vec{r}_n)}{4\pi\varepsilon|\vec{r} - \vec{r}_n|^3}$

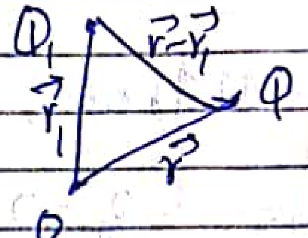
$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots + \vec{E}_n \rightarrow (A)$$

$$\vec{E} = \frac{1}{4\pi\varepsilon}\left[\frac{Q_1(\vec{r} - \vec{r}_1)}{|\vec{r} - \vec{r}_1|^3} + \frac{Q_2(\vec{r} - \vec{r}_2)}{|\vec{r} - \vec{r}_2|^3} + \cdots + \frac{Q_n(\vec{r} - \vec{r}_n)}{|\vec{r} - \vec{r}_n|^3}\right]$$

$$\vec{E} = \frac{1}{4\pi\epsilon}\sum_{i=1}^{n}\frac{Q_i(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3}$$

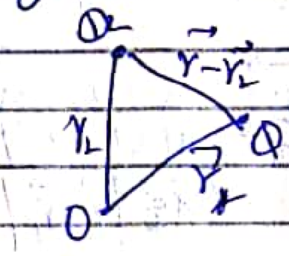
Force On Charge of Q Coulombs Due to "n" number of charges

Superposition theorem



$$\vec{F}_1 = \frac{QQ_1(\vec{r}-\vec{r}_1)}{4\pi\epsilon|\vec{r}-\vec{r}_1|^3}$$

Similarly $\vec{F}_2 = \dfrac{QQ_2(\vec{r}-\vec{r}_2)}{4\pi\epsilon|\vec{r}-\vec{r}_2|^3}$



Similarly $\vec{F}_n = \dfrac{QQ_n(\vec{r}-\vec{r}_n)}{4\pi\epsilon|\vec{r}-\vec{r}_n|^3}$

Total $\Rightarrow$ $\vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_n$

$$\vec{F} = \frac{Q}{4\pi\epsilon}\left[\frac{Q_1(\vec{r}-\vec{r}_1)}{|\vec{r}-\vec{r}_1|^3} + \frac{Q_2(\vec{r}-\vec{r}_2)}{|\vec{r}-\vec{r}_2|^3} + \cdots + \frac{Q_n(\vec{r}-\vec{r}_n)}{|\vec{r}-\vec{r}_n|^3}\right]$$

or

$$\boxed{F = \frac{Q}{4\pi\epsilon}\sum_{i=1}^{n}\frac{Q_i(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3}}$$

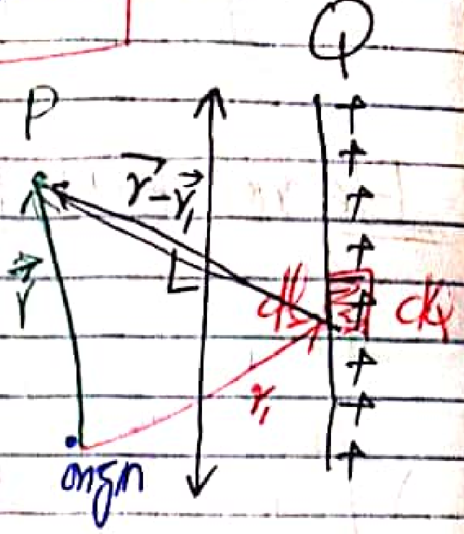
also coulombs law

Electric Field Intensity due to line charge

Line charge density $= \rho_L = \dfrac{Q}{L}$ C/m

$$\boxed{\rho_L = \frac{dQ}{dl} \Rightarrow dQ = \rho_L \, dl}$$

Calculate intensity at P?



$$d\vec{E} = \frac{dQ(\vec{r}-\vec{r}_1)}{4\pi\varepsilon|\vec{r}-\vec{r}_1|^3}$$

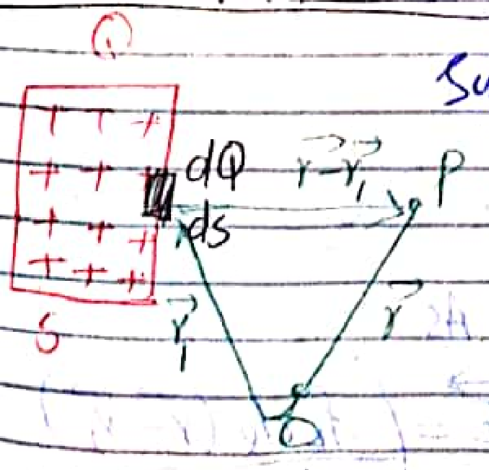
The maximum value of intensity is in free space as its permittivity is least.

$$d\vec{E} = \frac{\rho_L \, dl\,(\vec{r}-\vec{r}_1)}{4\pi\varepsilon|\vec{r}-\vec{r}_1|^3}$$

The integration of above eq results in total intensity

$$\boxed{\vec{E} = \int \frac{\rho_L \, dl\,(\vec{r}-\vec{r}_1)}{4\pi\varepsilon|\vec{r}-\vec{r}_1|^3}}$$

Electric Field Intensity Due to Surface Charge



Surface charge density

$$\rho_s = Q/S \qquad C/m^2$$

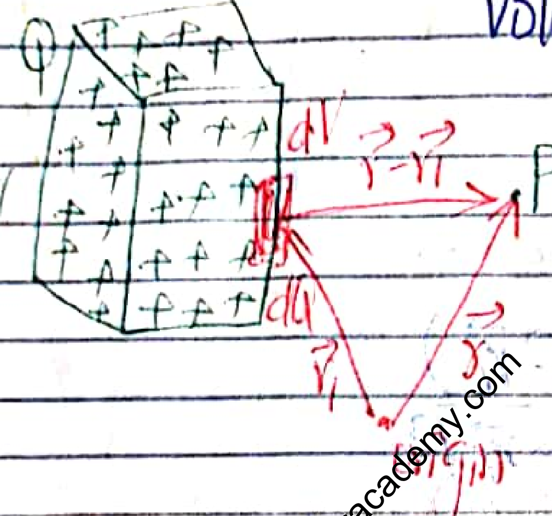
$$\rho_s = \frac{dQ}{ds}$$

$$dQ = \rho_s \, ds$$

$$d\vec{E} = \frac{dQ(\vec{r}-\vec{r}_1)}{4\pi\varepsilon|\vec{r}-\vec{r}_1|^3} = \frac{\rho_s \, ds\,(\vec{r}-\vec{r}_1)}{4\pi\varepsilon|\vec{r}-\vec{r}_1|^3}$$

$$\boxed{\vec{E} = \int \frac{\rho_s\, ds\,(\vec{r}-\vec{r}_1)}{4\pi\varepsilon\,|\vec{r}-\vec{r}_1|^3}}$$ duble integration.

Electric Field Intensity Due to Volume Charge



Volume charge density

$$\rho_v = \frac{Q}{V}$$

$$\rho_v = \frac{dQ}{dV}$$

$$dQ = \rho_v\, dV$$

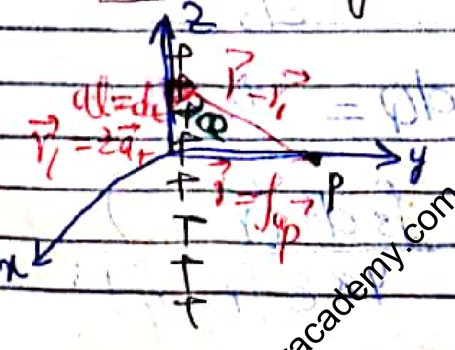
As $d\vec{E} = \dfrac{dQ\,(\vec{r}-\vec{r}_1)}{4\pi\varepsilon\,|\vec{r}-\vec{r}_1|^3} = \dfrac{\rho_v\, dv\,(\vec{r}-\vec{r}_1)}{4\pi\varepsilon\,|\vec{r}-\vec{r}_1|^3}$

triple integration.

$$\boxed{\vec{E} = \int \frac{\rho_v\, dv\,(\vec{r}-\vec{r}_1)}{4\pi\varepsilon\,|r-\vec{r}_1|^3}}$$

Lecture 7 16/10/19

Electric Field Intensity Due to Infinite Line Charge



As

$$\vec{E} = \int \frac{\rho_L\, dl\,(\vec{r}-\vec{r}_1)}{4\pi\varepsilon\,|\vec{r}-\vec{r}_1|^3}$$

$$\vec{r} - \vec{r}_1 = \rho \vec{a}_\rho - z\vec{a}_z$$

$$|\vec{r} - \vec{r}_1| = \sqrt{\rho^2 + z^2} \quad , \quad dl = dz$$

$$\Rightarrow \vec{E} = \int_{-\infty}^{\infty} \frac{\rho_L dz (\rho \vec{a}_\rho - z\vec{a}_z)}{4\pi\varepsilon(\rho^2 + z^2)^{3/2}}$$

z is the only variable in this equation.

↳ very difficult to integrate.

↳ consider a right angled triangle.

base = z perpendicular = ρ

$$\frac{z}{\rho} = \cot\theta \Rightarrow z = \rho\cot\theta$$

Differentiating $\Rightarrow dz = -\rho \operatorname{cosec}^2\theta \, d\theta$

Now find limits for θ.

When $z = -\infty$

$$\frac{\cos\theta}{\sin\theta} = \frac{z}{\rho} = \frac{-\infty}{\rho} = -\infty$$

$$\Rightarrow \frac{\cos\pi}{\sin\pi} = \frac{-1}{0} = -\infty$$

When $z = \infty$

$$\frac{\cos\theta}{\sin\theta} = \frac{\infty}{\rho} = \infty \Rightarrow \frac{\cos 0}{\sin 0} = \frac{1}{0} = \infty$$

$$\Rightarrow \vec{E} = \frac{\rho_L}{4\pi\varepsilon} \int_0^{\pi} \frac{\rho \operatorname{cosec}^2\theta \, d\theta (\rho \vec{a}_\rho - \rho\cot\theta \vec{a}_z)}{(\rho^2 + \rho^2\cot^2\theta)^{3/2}}$$

Simplifying the denominator

$$= \rho^3 (1 + \cot^2\theta)^{3/2} = \rho^3 (\operatorname{cosec}^2\theta)^{3/2}$$

$$= \rho^3 \operatorname{cosec}^3\theta$$

$$\Rightarrow \vec{E} = \frac{\rho_L}{4\pi\varepsilon}\int_0^{\pi} \frac{\rho\,\mathrm{cosec}^2\theta\, d\theta\,(\rho\vec{a}_\rho - \rho\cot\theta\,\vec{a}_z)}{\rho^3\,\mathrm{cosec}^3\theta}$$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon}\int_0^{\pi} \frac{\rho^2\,\mathrm{cosec}^2\theta\, d\theta\,(\vec{a}_\rho - \cot\theta\,\vec{a}_z)}{\rho^3\,\mathrm{cosec}^3\theta}$$

$\rho \rightarrow$ constant $\rightarrow$ radial distance, $\frac{1}{\mathrm{cosec}\,\theta} = \sin\theta$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon\rho}\int_0^{\pi} \sin\theta\, d\theta\,(\vec{a}_\rho - \cot\theta\,\vec{a}_z)$$

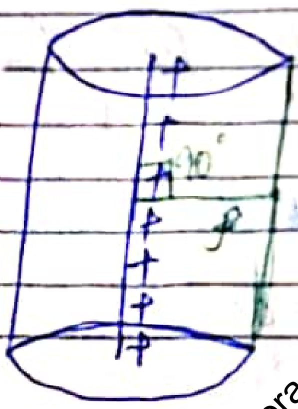
$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon\rho}\left[\int_0^{\pi}\sin\theta\, d\theta\,\vec{a}_\rho - \int_0^{\pi}\frac{\cos\theta}{\cancel{\sin\theta}} \times \cancel{\sin\theta}\, d\theta\,\vec{a}_z\right]$$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon\rho}\left[(-\cos\theta)\Big|_0^{\pi}\vec{a}_\rho - (\sin\theta)\Big|_0^{\pi}\vec{a}_z\right]$$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon\rho} \times 2 \times \vec{a}_\rho \Rightarrow \boxed{\vec{E} = \frac{\rho_L}{2\pi\varepsilon\rho}\vec{a}_\rho}$$

$$\Rightarrow \vec{E} \propto \frac{1}{\rho}$$

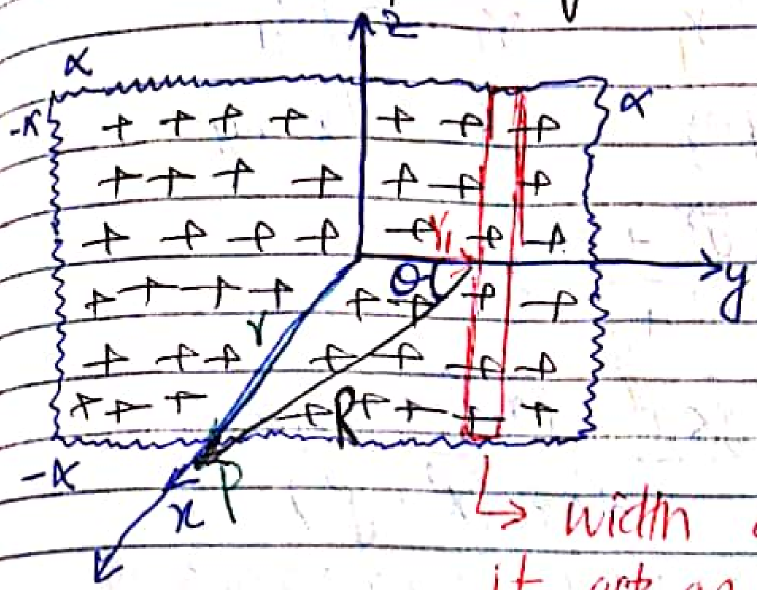
The electrical field along a line parallel to the same will be constant.



let $\rho = R$ $\quad \vec{a}_\rho = \vec{a}_r$

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\varepsilon R}\vec{a}_\rho}$$

Electric Field Intensity Due to Infinite sheet of charge



located in the $x=0$ plane

$$\rho_s = \frac{dQ}{dS}$$

$$\rho_s = \frac{dQ}{dy\,dz} \quad —①$$

↳ width dy → so small that it acts an infinite line charge parallel to z axis

$$\rho_L = \frac{dQ}{dz} \longrightarrow$$ (if line is on or parallel to ± axis)

$$① \Rightarrow \quad \rho_L = \rho_s\,dy \quad \rightarrow ②$$

let the point under observation be on x axis.

As $$\vec{E} = \frac{\rho_L}{2\pi\varepsilon R}\,\vec{a}_R \quad \rightarrow ③$$

$$\vec{R} = \vec{r} - \vec{r}_1 \quad \Rightarrow \vec{R} = x\vec{a}_x - y\vec{a}_y$$

$$R = \sqrt{x^2+y^2} \qquad \vec{a}_R = \frac{\vec{R}}{R} = \frac{x\vec{a}_x - y\vec{a}_y}{\sqrt{x^2+y^2}}$$

$$③ \Rightarrow \quad d\vec{E} = \frac{\rho_s\,dy\,(x\vec{a}_x - y\vec{a}_y)}{2\pi\varepsilon(x^2+y^2)}$$

↳ because of the very small dy.

Considering another strip on the L.H.S of z axis.

$$\vec{R} = \vec{r} - \vec{r}_1 \qquad \vec{r}_1 = -y\vec{a}_y$$

$$\vec{R} = x\vec{a}_x + y\vec{a}_y \qquad R = \sqrt{x^2+y^2}$$

$$\vec{a}_R = \frac{\vec{R}}{R} = \frac{x\vec{a}_x + y\vec{a}_y}{\sqrt{x^2+y^2}}$$

$$d\vec{E} = \frac{\rho_s\, dy\, (x\vec{a}_x + y\vec{a}_y)}{2\pi\varepsilon(x^2+y^2)}$$

B/c of the symmetry, y components will cancel the effect of each other.

$$2d\vec{E} = d\vec{E} + d\vec{E} = \frac{\rho_s\, dy\,[x\vec{a}_x + y\vec{a}_y + x\vec{a}_x - y}{2\pi\varepsilon(x^2+y^2)}$$

$$\Rightarrow \quad d\vec{E} = \int_{-\infty}^{\infty} \frac{\rho_s\, dy\, x\,\vec{a}_x}{2\pi\varepsilon(x^2+y^2)} \quad — \text{(A)}$$

Consider Δ

$$\frac{y}{x} = \cot\theta \quad \Rightarrow \quad y = x\cot\theta$$

$$dy = -x\,\operatorname{cosec}^2\theta\, d\theta$$

when $y = -\infty \Rightarrow \theta = \pi$

when $y = \infty \Rightarrow \theta = 0$

$$\vec{E} = \frac{\rho_s}{2\pi\varepsilon}\int_0^{\pi} \frac{x^2\cos^2\theta\, d\theta}{x^2 + \cot^2\theta}\,\vec{a}_x$$

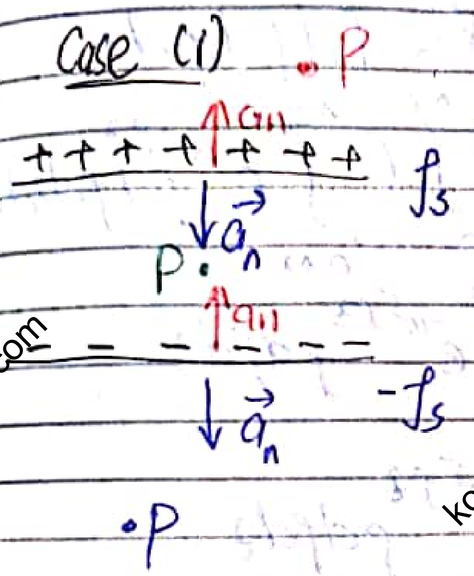
$$\vec{E} = \frac{\rho_s}{2\pi\varepsilon}\int_0^{\pi} \frac{x^2\operatorname{cosec}^2\theta\, d\theta}{x^2\operatorname{cosec}^2\theta}\,\vec{a}_x - \frac{\rho_s}{2\pi\varepsilon}\int_0^{\pi} d$$

$$= \frac{\rho_s}{2\pi\varepsilon}\,(\theta|_0^{\pi}\,\vec{a}_x \Rightarrow \boxed{\vec{E} = \frac{\rho_s}{2\varepsilon}\,\vec{a}_x}$$

this unit vector is normal to the sheet

$$\Rightarrow \vec{E} = \frac{\rho_s}{2\varepsilon}\,\vec{a}_n$$

Case (i)



Superposition Theorem

$$\vec{E}_+ = \frac{\rho_s}{2\varepsilon}\,\vec{a}_n$$

$$\vec{E}_- = -\frac{\rho_s}{2\varepsilon}\,\vec{a}_n$$

$$\vec{E} = \vec{E}_+ + \vec{E}_- = 0$$

Case (ii)

$$\vec{E}_+ = -\frac{\rho_s}{2\varepsilon}\,\vec{a}_n \qquad \vec{E}_- = \frac{\rho_s}{2\varepsilon}\,\vec{a}_n$$

$$\vec{E} = \vec{E}_+ + \vec{E}_- = 0$$

Case (iii)

$$\vec{E}_+ = \frac{\rho_s}{2\varepsilon}\,\vec{a}_n \qquad \vec{E}_- = +\frac{\rho_s}{2\varepsilon}\,\vec{a}_n$$

$$\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{\rho_s}{\varepsilon}\,\vec{a}_n$$

Lecture 8 21/10/19

Example $\rho_v = -5\times10^{-6} \times e^{-10^5 \rho z}$ C/m^3

$\rho = 1cm$

$z = 0.04m$

Electron beam

$z = 0.02m$

$Q = ?$

$$Q = \int \rho_v \, dv$$

$$dv = \rho \, d\rho \, d\phi \, dz$$

$$Q = -5\times10^{-6} \int_0^{0.01}\int_0^{2\pi}\int_{0.02}^{0.04} e^{-10^5 \rho z} \rho \, d\rho \, d\phi \, dz$$

$$Q = -5\times10^{-6} \times \left[\phi\right]_0^{2\pi} \int_0^{0.01}\int_{0.02}^{0.04} e^{-10^5 \rho z} \rho \, d\rho \, dz$$

$$Q = -5\times10^{-6} \times 2\pi \int_0^{0.01}\int_{0.02}^{0.04} e^{-10^5 \rho z} \rho \, d\rho \, dz$$

$$Q = -10^{-5}\pi \int_0^{0.01}\int_{0.02}^{0.04} e^{-10^5 \rho z} \rho \, d\rho \, dz$$

w.r.t z

$$\cancel{Q = -10^{-5}\pi \int_0^{0.01} \left[e^{-4000\rho} - e^{-2000\rho} \right] d\rho}$$

$\cancel{Q = 10}\ 10 \pi$ w.r.t z

$$Q = -10^{-5}\pi \int_0^{0.01} \left. \frac{e^{-10^5 \rho z} \rho}{-10^5 \rho} \right|_{0.02}^{0.04} d\rho$$

$$Q = +10^{(-5-5)} \pi \int_0^{0.01} \left[e^{-4000\rho} - e^{-2000\rho} \right] d\rho$$

$$Q = 10^{(-10)} \pi \left[\int_0^{0.01} e^{-4000\rho} \, d\rho - \int_0^{0.01} e^{-2000\rho} \, d\rho \right]$$

$$Q = 10^{-10}\pi\left[\frac{(e^{-4000\rho})}{-4000}\Big|_0^{0.01} - \frac{e^{-2000\rho}}{-2000}\Big|_0^{0.01}\right]$$

$$Q = -0.025\ PC.$$

Sol 2.1 Start here

$$\vec{F_2} = \frac{Q_1 Q_2}{4\pi\varepsilon_0}\frac{(\vec{r_2}-\vec{r_1})}{|\vec{r_2}-\vec{r_1}|^3}$$

$$\vec{r_2}-\vec{r_1} = -2\vec{a_x} - 2\vec{a_y} + 6\vec{a_z}$$

$$|\vec{r_2}-\vec{r_1}| = 6.63\ m$$

As $\frac{1}{4\pi\varepsilon_0} = 9\times10^9$

$$\vec{F_2} = \frac{9\times10^9 \times 2\times10^{-3}\times -5\times10^{-6}(-2\vec{a_x} - 2\vec{a_y} + 6\vec{a_z})}{(6.63)^3}$$

D 2.1 $Q_1 = 2\times10^{-3}C$, at $P_1(3,-2,-4)$

$Q_2 = -5\times10^{-6}C$ at $P_2(1,-4,2)$

$\vec{F_2}$? F_2 ?

$$\vec{F_2} = 0.617\vec{a_x} + 0.617\vec{a_y} - 1.848\vec{a_z}$$

$$\vec{F_2} = 2.04\ N.$$

2.2 $E = ?$ at $P(3,-4,2)$ caused by

(i) $Q_1 = 2\times10^{-6}\,C$ at $P_1(0,0,0)$

$$\vec{E}_1 = \frac{Q_1(\vec{r}-\vec{r}_1)}{4\pi\varepsilon_0|\vec{r}-\vec{r}_1|^3}$$

$$\vec{r}-\vec{r}_1 = 3\vec{a}_x - 4\vec{a}_y + 2\vec{a}_z$$

$$|\vec{r}-\vec{r}_1| = 5.83\,m$$

So $$\vec{E}_1 = \frac{9\times10^9\times2\times10^{-6}(3\vec{a}_x - 4\vec{a}_y + 2\vec{a}_z)}{(5.83)^3}$$

$$\Rightarrow \vec{E}_1 = 346.7\vec{a}_x - 462\vec{a}_y + 231.2\vec{a}_z \ V/m$$

(ii) $Q_2 = 3\times10^{-6}\,C$ at $P_2(-1,2,3)$

$$\vec{E}_2 = \frac{Q_2(\vec{r}-\vec{r}_2)}{4\pi\varepsilon_0|\vec{r}-\vec{r}_2|^3}$$

$$\vec{r}-\vec{r}_2 = 4\vec{a}_x - 6\vec{a}_y - \vec{a}_z$$

$$|\vec{r}-\vec{r}_2| = 7.28$$

$$\Rightarrow \vec{E}_2 = \frac{9\times10^9\times3\times10^{-6}(4\vec{a}_x - 6\vec{a}_y - \vec{a}_z)}{(7.28)^3}$$

$$\Rightarrow \vec{E}_2 = 280\vec{a}_x - 419.8\vec{a}_y - 69.97\vec{a}_z \ V/m$$

(iii) ? $\vec{E} = ?$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = 626.7\vec{a}_x - 881.8\vec{a}_y + 161.2\vec{a}_z \ V/m$$

2.4 $Q = ?$

1. $\rho_v = 10z^2 e^{-0.1x} \sin \pi y \quad C/m^3$

$-1 \le x \le 2 \quad , \quad 0 \le y \le 1 \quad , \quad 3 \le z \le 3.6$

$$Q = \int \rho_v dv$$

$dv = dx\, dy\, dz$

$\rho_v dv = 10z^2 dz\, e^{-0.1x} dx\, \sin \pi y\, dy$

$$Q = 10 \int_3^{3.6} z^2 dz \int_{-1}^{2} e^{-0.1x} dx \int_0^1 \sin \pi y\, dy$$

$$Q = 10 \left(\frac{z^3}{3} \right|_3^{3.6} \times \left(\frac{e^{-0.1x}}{-0.1} \right|_{-1}^{2} \times \left(\frac{-\cos \pi y}{\pi} \right|_0^1$$

$$\Rightarrow Q = 119.7\, C$$

2. $\rho_v = 4xyz^2$

$0 \le \rho \le 2, \quad 0 \le \phi \le \pi/2, \quad 0 \le z \le 3.$

$Q = \int \rho_v dv$ But first we transform;

$x = \rho \cos \phi \quad , \quad y = \rho \sin \phi .$

$\Rightarrow \rho_v = 4\rho^2 \sin \phi \cos \phi\, z^2 \quad C/m^3$

As $dv = \rho\, d\rho\, d\phi\, dz$

$\rho_v dv = 4\rho^3 d\rho\, \sin \phi \cos \phi\, d\phi\, z^2 dz \quad C$

$$Q = 4\int_0^2 \rho^3 d\rho \int_0^{\pi/2} \sin\phi \cos\phi \, d\phi \int_0^3 z^2 dz$$

$$Q = 4\left(\frac{\rho^4}{4}\Big|_0^2 \times \left(\frac{\sin^2\phi}{2}\Big|_0^{\pi/2} \times \left(\frac{z^3}{3}\Big|_0^3\right.\right.\right.$$

$$\Rightarrow Q = 72 \ C$$

<u>3</u> $\rho_V = \dfrac{3\pi \cos^2\theta \cos^2\phi}{2r^2(r^2+1)}$ C/m³

Universe → we consider the universe to be a sphere

$0 \le r < \infty \ , \ 0 \le \theta \le \pi \ , \ 0 \le \phi \le 2\pi$

In spherical $dV = r^2 \sin\theta \, d\theta \, d\phi$

$$dQ = \rho_V dV = \frac{3\pi}{2} \cdot \frac{dr}{r^2+1} \times \cos^2\theta \sin\theta \, d\theta \times \cos^2\phi$$

$$Q = \frac{3\pi}{2} \int_0^{\infty} \frac{dr}{r^2+1} \int_0^{\pi} \cos^2\theta \sin\theta \, d\theta \int_0^{2\pi} \frac{1+\cos 2\phi}{2} d\phi$$

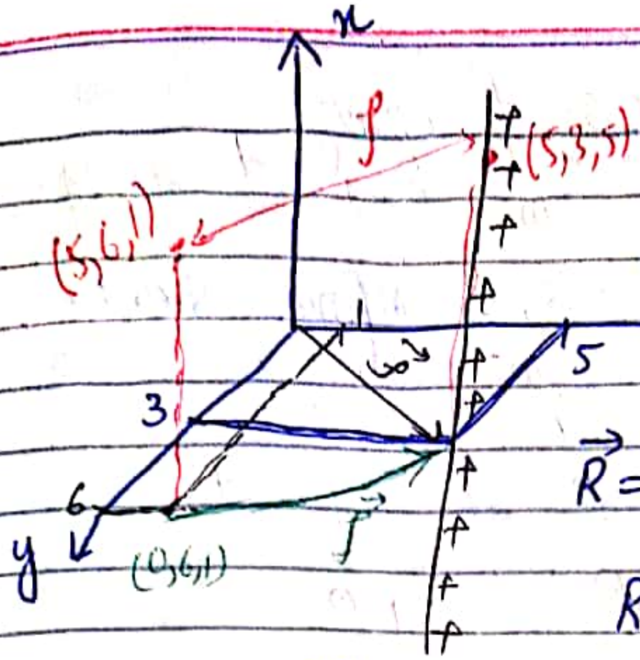
$$Q = \frac{3\pi}{2}\left(\tan^{-1} r\Big|_0^{\infty} \times \left(\frac{-\cos^3\theta}{3}\Big|_0^{\pi} \times \left(\frac{\phi}{2}\Big|_0^{2\pi}\right.\right.\right.$$

$$Q = \frac{3\pi}{2} \times \frac{\pi}{2} \times \frac{2}{3} \times \pi = 15.50 \ C$$

<u>2.5</u> Line charge density, $\rho_L = 30 \, nC/m$

located at $y = 3$, $z = 5$

$E = ?$



(a) $P(0,0,0)$

① $\Rightarrow \vec{E} = \dfrac{\rho_L}{2\pi\varepsilon_0 \rho} \vec{a}_\rho$

$\vec{R} = \vec{\rho} = -3\vec{a}_y - 5\vec{a}_z$

$R = \rho = 5.83 m$

$\vec{a}_R = \vec{a}_\rho = \dfrac{\vec{R}}{R} = -0.514\vec{a}_y - 0.857\vec{a}_z$

① $\Rightarrow \vec{E} = \dfrac{30\times10^{-9}(-0.514\vec{a}_y - 0.857\vec{a}_z)}{2\pi\times 8.85\times10^{-12}\times 5.83}$

$\Rightarrow E = -47.6\vec{a}_y - 68.3\vec{a}_z$ V/m

(b) $P(0,6,1)$

$\vec{R} = \vec{\rho} = 3\vec{a}_y - 4\vec{a}_z$ $\quad R = \rho = 5m$

$\vec{a}_R = \vec{a}_\rho = 0.6\vec{a}_y - 0.8\vec{a}_z$

$\vec{E} = \dfrac{30\times10^{-9}\times(0.6\vec{a}_y - 0.8\vec{a}_z)}{2\pi\times 8.85\times10^{-12}\times 5}$

$\Rightarrow \vec{E} = 64.7\vec{a}_y - 68.3\vec{a}_z$ V/m

(c) $P(5,6,1)$

If line is parallel to source, the intensity on it will be constant.

$\Rightarrow P(5,6,1) = P(0,6,1) = 64.7\vec{a}_y - 68.3\vec{a}_z$

$\vec{R} = \vec{\rho} = 3\vec{a}_y - 4\vec{a}_z$

2.6 4 sheets of charge.

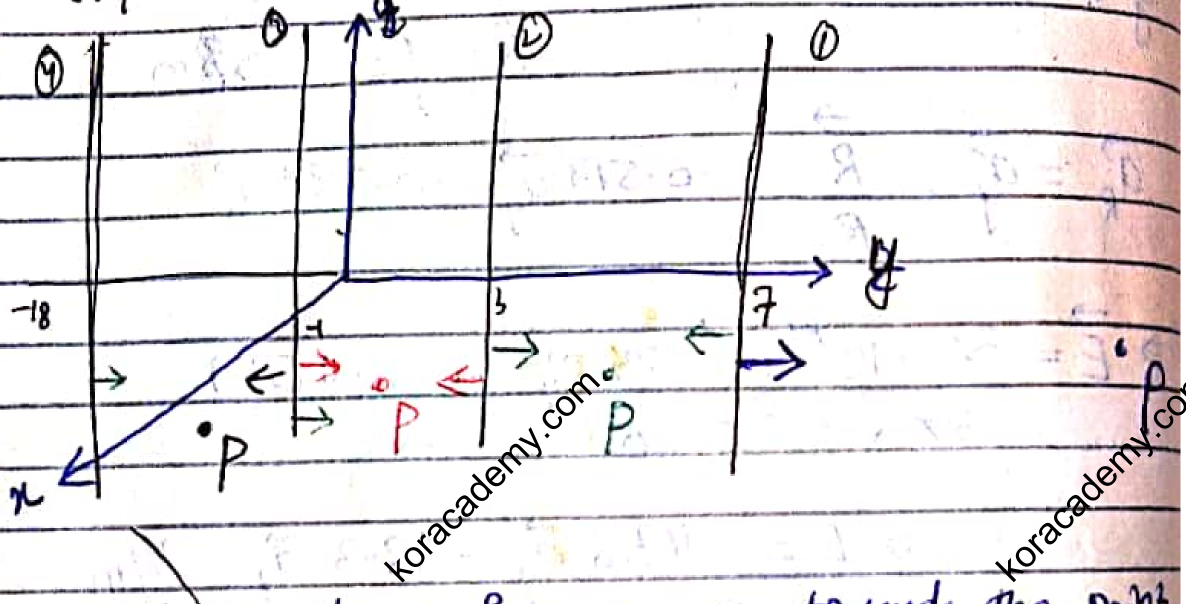
$\rho_{s_1} = 20P\ C/m^2$ located at $y=7$

No other two limits, so infinite sheet.

$\rho_{s_2} = -8\ PC/m^2$ located at $y=3$

$\rho_{s_3} = 6\ PC/m^2$ at $y=-1$

$\rho_{s_4} = -18\ PC/m^2$ at $y=-4$



unit vector → from source towards the point.

1. $P(2,6,-4)$

$$\vec{E}_1 = \frac{-\rho_{s1}}{2\varepsilon_0}\vec{a}_y = \frac{-20\times10^{-12}}{2\times8.85\times10^{-12}}\vec{a}_y = -1.13\vec{a}_y$$

$$\vec{E}_2 = \frac{\rho_{s2}}{2\varepsilon_0}\vec{a}_y = \frac{-8\times10^{-12}}{2\times8.85\times10^{-12}}\vec{a}_y = +0.45\vec{a}_y\ V/m$$

$$\vec{E}_3 = \frac{\rho_{s3}}{2\varepsilon_0}\vec{a}_y = -0.338\,\vec{a}_y\ V/m$$

$$\vec{E}_4 = \frac{\rho_{s1}}{2\varepsilon_0}\vec{a}_y = -1.017\vec{a}_y\ V/m$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 = -2.26\,\vec{a}_y\ V/m$$

(b) $P(0,0,0)$

Only change the direction of E_2 and calculate $\vec{E}$.

(c) $P(-1,-1,5)$

change sign of E_3 in part b.

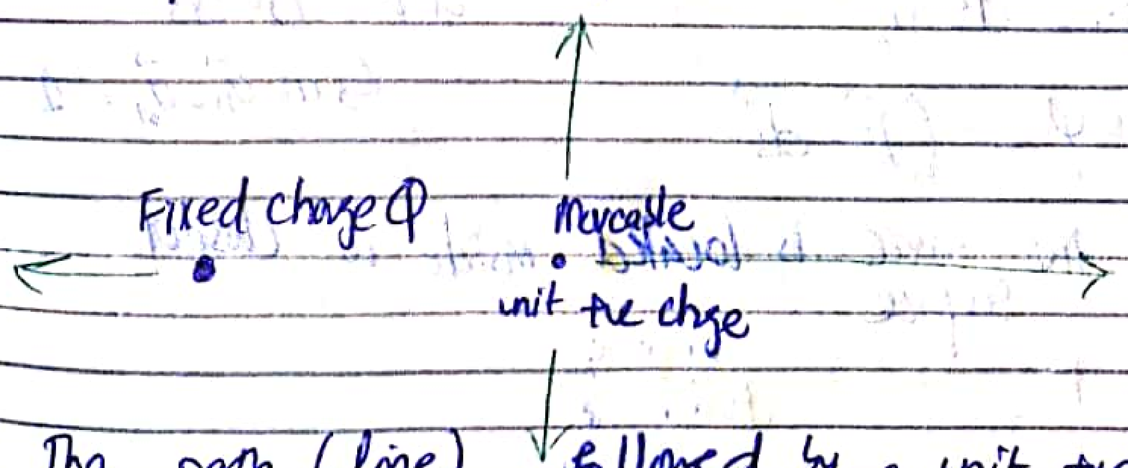
$$\vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3} + \vec{E_4} = -2.03\vec{a_y} \ V/m$$

(d) $P(10^6, 10^6, 10^6)$

change the direction of E_1 in part a.

Chapter 3 Electric Flux Density

Electric flux is
- Scalar quantity. - unit is coulomb (C)
- represented by Ψ (si)



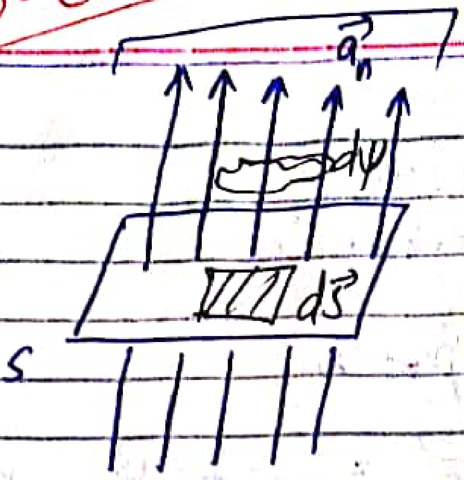
The path (line) followed by a unit +ve charge in an electric field is known as electric flux.

Arrow represents direction of force, electric field intensity and electric flux.

Electric flux density is a vector
↳ symbol $\vec{D}$ and unit C/m^2

$$\boxed{\vec{D} = \varepsilon \vec{E}}$$

Ψ



S = area

$\vec{S} = S\vec{a}_n$

$\vec{D} = D\vec{a}_n$

$$\boxed{D = \frac{\Psi}{S}} \quad C/m^2$$

$$\Rightarrow \boxed{\Psi = DS}$$

$$\Psi = \vec{D} \cdot \vec{S} = DS \qquad \because \vec{a}_n \cdot \vec{a}_n = 1$$

Consider a small portion of the surface represented by $d\vec{s}$, a small portion of flux passes through it represented as $d\Psi$.

$$D = \frac{d\Psi}{ds} \Rightarrow d\Psi = D\,ds$$

$$d\vec{s} = ds\,\vec{a}_n \qquad d\Psi = \vec{D} \cdot d\vec{s}$$

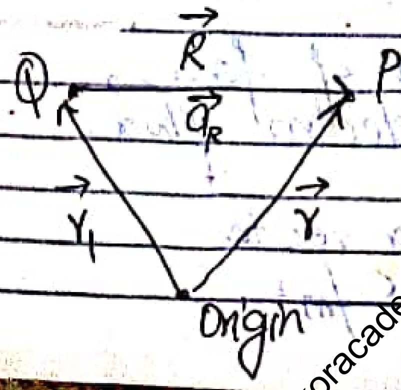
↳ as $\vec{a}_n \cdot \vec{a}_n = 1$

$$\Psi = \int \vec{D} \cdot d\vec{s}$$

If the source is located inside a closed surface

$$\Psi = \oint \vec{D} \cdot d\vec{s}$$

<u>Electric Flux Density Due to Point Charge</u>



$$\vec{E} = \frac{Q}{4\pi\varepsilon R^2}\vec{a}_R$$

$$\vec{D} = \varepsilon\vec{E}$$

$$\boxed{\vec{D} = \frac{Q}{4\pi R^2}\vec{a}_R}$$

$$a_R = \frac{\vec{R}}{R} \rightarrow \boxed{\vec{D} = \frac{Q\vec{R}}{4\pi R^3}}$$

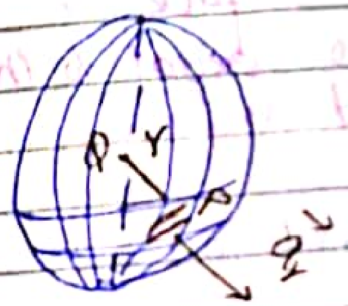
$$\vec{R} = \vec{r} - \vec{r}_1 \qquad R = |\vec{r} - \vec{r}_1|$$

$$\boxed{\vec{D} = \frac{Q(\vec{r} - \vec{r}_1)}{4\pi |\vec{r} - \vec{r}_1|^3}}$$

Electric flux density will be along the straight line joining the source and point.

Lecture 9 28/10/19

Guass's Law

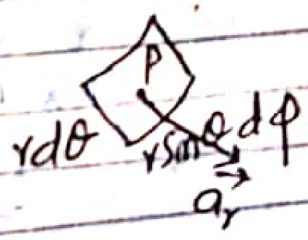


$$d\vec{s} = ds\, \vec{a}_r$$

$$\vec{D} = \frac{Q}{4\pi r^2} \vec{a}_r$$

The small portion of the sphere

Closed surface (sphere of radius r)



$$d\vec{S} = r^2 \sin\theta\, d\theta\, d\phi\, \vec{a}_r$$

$$d\psi = \vec{D} \cdot \vec{ds}$$

$$\psi = \oint \vec{D} \cdot \vec{ds}$$

As $\vec{a}_r \cdot \vec{a}_r = 1$

$$\rightarrow d\psi = \vec{D} \cdot \vec{ds} = \frac{Q}{4\pi r^2} \times r^2 \sin\theta\, d\theta\, d\phi$$

$$\rightarrow d\psi = \frac{Q}{4\pi} \sin\theta\, d\theta\, d\phi$$

$$\text{Integrating} \Rightarrow \psi = \oint \frac{Q}{4\pi} \sin\theta \, d\theta \, d\phi$$

$$\psi = \oint \vec{D} \cdot \vec{ds} = \frac{Q}{4\pi} \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi$$

$$\psi = \oint \vec{D} \cdot \vec{ds} = \frac{Q}{4\pi} \left(-\cos\theta \Big|_0^{\pi} \times (\phi \Big|_0^{2\pi}\right)$$

$$\psi = \oint \vec{D} \cdot \vec{ds} = \frac{Q}{4\pi} \times 2 \times 2\pi$$

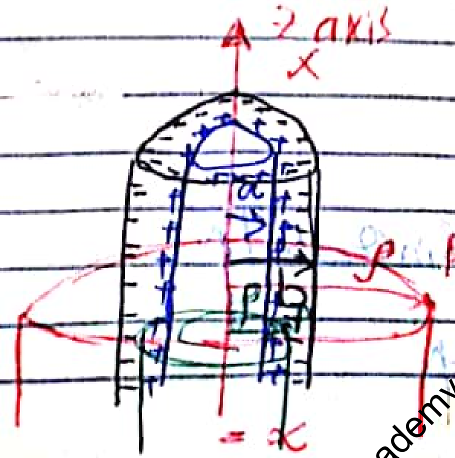
$$\psi = \oint \vec{D} \cdot \vec{ds} = Q_{enclosed}$$

Guass law states that the total electric flux passing through a closed surface in the outward direction is also equal to the charge enclosed by the closed surface.

Consider a very small sphere enclosing a charge Q.
The total flux generated by this source is passing through the sphere.
Hence total flux is equal to charge of the source

Applications of Guass Law

(1) **Coaxial Cable** consists of two conductors.



Inner conductor $\rightarrow$ radius $\rightarrow a$
outer conductor $\rightarrow$ radius $\rightarrow b$
extending from $-x$ to x along the z axis.

There is a dielectric b/w two conductors of the cable.
Assume that charge is uniformly distributed on the outer surface of inner conductor.

electrostatic induction → some amount of -ve charge is induced on the inner surface of the outer conductor.

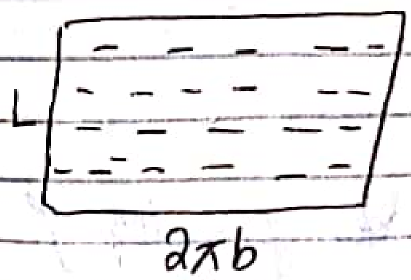
Consider length L.

Assume that charge on this area is Q_{in}.

The surface charge density is $\rho_{s_{in}}$.

$$Q_{in} = \rho_{s_{in}} \times 2\pi a L$$

Consider same length L of outer conductor with total charge Q_{out}.



$$Q_{out} = \rho_{s_{out}} \times 2\pi b L$$

According to electrostatic induction.

$$Q_{out} = -Q_{in}$$

$$\Rightarrow \rho_{s_{out}} \times 2\pi b L = -\rho_{s_{in}} \times 2\pi a L$$

$$\rho_{s_{out}} = \frac{-\rho_{s_{in}} \times a}{b}$$

Similarly for $\rho_{s_{in}}$.

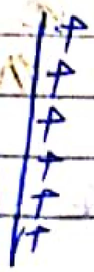
Consider a point P in the dielectric & radial distance $= \rho$

ρ → always perpendicular to z axis.

Calculate $\vec{E}$ at P with Guass law.

guassian surface is in the form of cylinder of radius ρ.

The entire inner cylinder will be inside the guassian cylinder.
Assume that radius of inner conductor is so small that it almost behaves like an infinite line charge



and the electric field intensity due to infinite line charge can be calculated by

$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon\rho}\vec{a}_\rho$$

$\rho_L = ?$

from Q_{in}/L

$$\rho_L = \frac{Q_{in}}{L} = \rho_{s_{in}} \times 2a\pi$$

$$\Rightarrow \vec{E} = \frac{\rho_{s_{in}} \times 2\pi a}{2\pi\varepsilon\rho}\vec{a}_\rho = \boxed{\frac{\rho_{s_{in}} \times a}{\varepsilon\rho}}$$

As $\vec{D} = \varepsilon\vec{E}$ $\Rightarrow \boxed{\vec{D} = \frac{\rho_{s_{in}} \times a}{\rho}\vec{a}_\rho}$ $\quad a<\rho<b$

→ Consider another point located outside the coaxial cable.

$\vec{E}$ with Guass law → guassian cylinder of radius ρ, the entire cable will be located inside the guassian cylinder.

$$\Psi = \oint \vec{D}\cdot d\vec{s} = Q_{enclosed} = Q + (-Q)$$

$$\Psi = 0$$

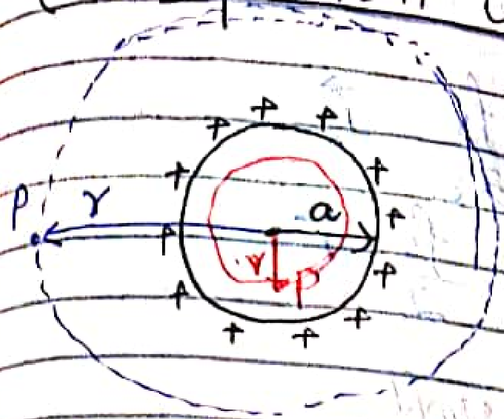
$$\Rightarrow \boxed{\vec{D}\cdot d\vec{s} = 0}$$

$d\vec{s}$ → the differential area of guassian cylinder $\neq 0$

$$\Rightarrow \vec{D} = 0$$

$$\Rightarrow \vec{E} = 0$$

(2) Spherical Charge



Assume that a charge is uniformly distributed on the outer surface.

$S = 4\pi a^2$

$\rho_s \times 4\pi a^2 = Q$

(i) Consider a point outside the source $(r > a)$

Consider a guassian sphere of radius r such that the entire source is located inside it.
Assume that radius of source is so small that it almost behaves as point charge. and $\vec{D}$ due to point charge is

$$\vec{D} = \frac{Q}{4\pi r^2}\vec{a_r}$$

$$\vec{D} = \frac{\rho_s \times 4\pi a^2}{4\pi r^2}\vec{a_r} \qquad \vec{D} = \frac{\rho_s \times a^2}{r^2}\vec{a_r}$$

(ii) Consider a point inside the source $(r < a)$
guass's law → surface → sphere → the source is located outside the guassian surface.

As $\Psi = \oint \vec{D} \cdot \vec{ds} = Q_{enclosed}$

As $Q_{enclosed} = 0 \Rightarrow \oint \vec{D} \cdot \vec{ds} = 0$

$\Rightarrow \vec{D} \cdot \vec{ds} = 0$ As $\vec{ds} \neq 0$

$\Rightarrow \vec{D} = 0$ and $\vec{E} = 0$

(3) $\vec{D}$ due to Infinite line charge

We know that $\vec{E} = \frac{\rho_L}{2\pi\epsilon\rho}\vec{a_\rho}$

$$\vec{D} = \epsilon\vec{E} = \frac{\rho_L}{2\pi\rho}\vec{a_\rho}$$

Infinite sheet of charge

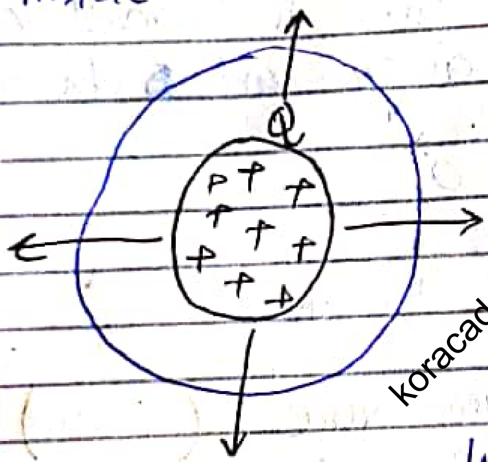
We know that $\vec{E} = \frac{\rho_s}{2\varepsilon} \vec{a}_n$

As $\vec{D} = \varepsilon \vec{E} \Rightarrow \vec{D} = \frac{\rho_s}{2} \vec{a}_n$

MAXWELL'S FIRST EQUATION

Assume that a charge is uniformly distributed inside a volume. → total charge is Q coulomb.

$$Q = \int \rho_v \, dV$$



Enclose the source in a guassian surface.

Electric flux passing through guassian surface?

With the help of Gauss law.

$$\psi = \oint \vec{D} \cdot \vec{ds} = Q_{enclosed}$$

$$\Rightarrow \quad \psi = \oint \vec{D} \cdot \vec{ds} = \int \rho_v \, dV \rightarrow \text{(A)}$$

↳ double integration → difficult → time consuming

Divergence Theorem

$$\oint \vec{D} \cdot \vec{ds} = \int (\nabla \cdot \vec{D}) \, dV$$

Inside bracket ie $(\nabla \cdot \vec{D})$ is called divergence.

How to calculate divergence?

→ Divergence in R.C.S

$$\nabla\cdot\vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

→ Divergence in C.C.S

$$\nabla\cdot\vec{D} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z}$$

→ Divergence in S.C.S

$$\nabla\cdot\vec{D} = \frac{1}{r^2}\left(r^2\frac{\partial D_r}{\partial r}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, d\theta) + \frac{1}{r\sin\theta}\left(\frac{\partial D_\phi}{\partial\phi}\right)$$

(A) → $\int(\nabla\cdot\vec{D})\,dv = \int\rho_v\,dv$

$$\boxed{\nabla\cdot\vec{D} = \rho_v}$$ → first equation of Maxwell.

Numericals

Q3.1 $Q = 25\mu C$ is located at the origin.

$\psi = ?$

(i) Spherical surface, $r = 20cm$, $0 \le \theta \le \pi$

$0 \le \phi \le \pi/2$

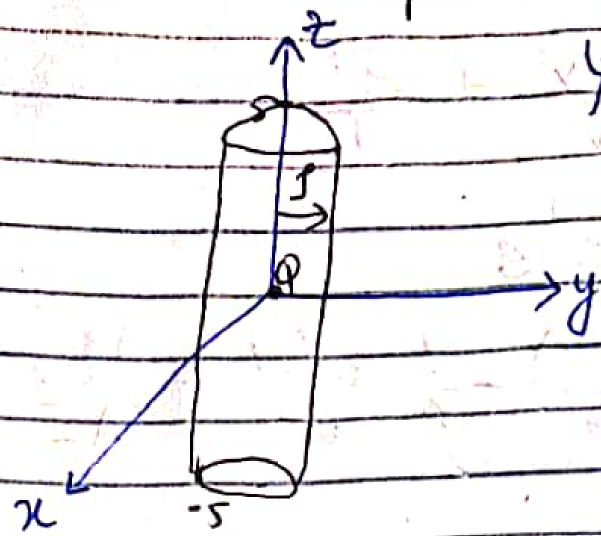
If we have a full sphere of 20cm radius, then the total electric flux passing through this closed surface will be equal to the charge enclosed ie $25\mu C$. $\psi_t = Q_{enc} = 25\mu C$

As we have seen 1/4th portion of the sphere, therefore the flux passing through this portion will be total flux divided by 4.

$$\psi = \frac{\psi_t}{4} = 6.25\mu C$$

(ii) Closed surface $\rho = 0.8m$ $z = \pm 0.5m$

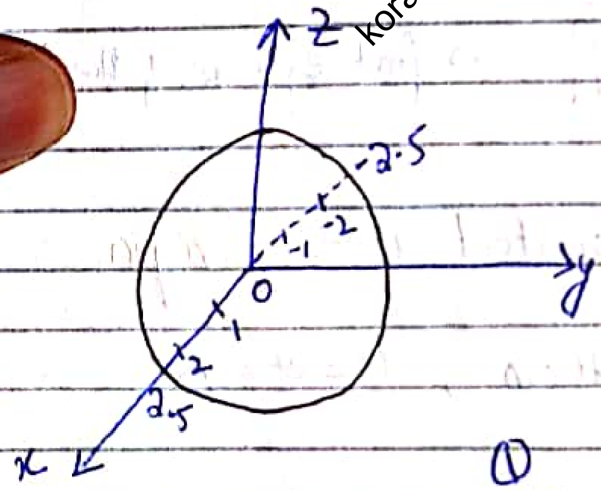
$\Psi = Q_{enclosed} = Q = 25\mu C$



Q3.4 Flux passing through a closed surface of 2.5m radius.

Other two coordinates not given → Universal limits

(i) Q^{-x^2} nC located on the x axis at $x = 0, \pm 1, \pm 2, \pm 3, \pm 4, ------$



$Q_{x=0} = 2^{-0} = 1nC$

$Q_{x=1} = 2^{-1} = 0.5nC$

$Q_{x=-1} = 2^{-1} = 0.5nC$

$Q_{x=2} = 2^{-4} = 0.625nC$

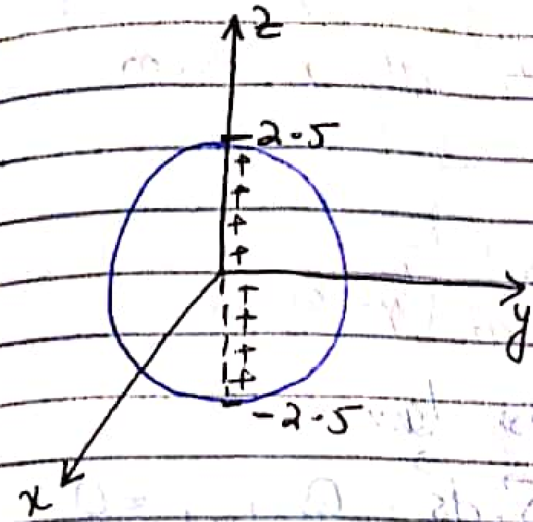
$Q_{x=-2} = 2^{-4} = 0.625nC$

$Q_{enclosed} = (1 + 0.5 + 0.5 + 0.625 + 0.625)nC$

Guass's law $\Psi = Q_{enclosed} = 2.135nC$

(ii) line charge with $\rho_L = \frac{1}{z^2+1}$ nC/m on the z axis ⇒ infinite line of charge

↳ universal limits $-\infty < z < \infty$



$$Q = \int \rho_L \, dl$$

↳ dz

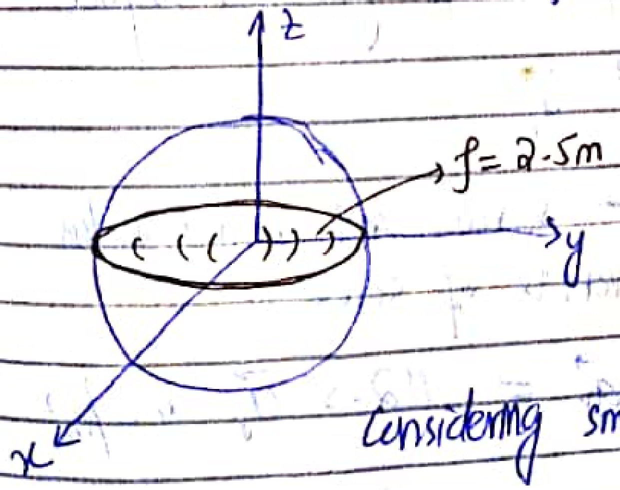
$$Q_{encl} = \int_{-2.5}^{2.5} \frac{dz}{z^2+1} \times 10^{-9}$$

$$Q_{encl} = \left(\tan^{-1} z \Big|_{-2.5}^{2.5}\right) \times 10^{-9}$$

$$\Psi = Q_{enclosed} = 2.38\,nC$$

(iii) Surface charge $\rho_s = \frac{1}{x^2+y^2+4}$ nC/m² on z = 0 plane

↳ universal limits for x and y.

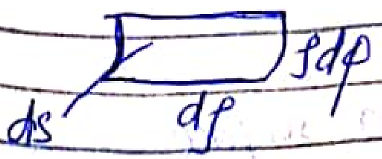


$$Q = \int \rho_s \, ds$$

For circle $x^2+y^2=r^2$

$$\Rightarrow \rho_s = \frac{1}{\rho^2+4}$$

Considering small portion of the area



$$Q = \frac{10^{-9}}{2} \int_0^{2.5} \frac{2\rho \, d\rho}{\rho^2+4} \int_0^{2\pi} d\phi$$

$$= \frac{10^{-9}}{2} \ln(\rho^2+4)\Big|_0^{2.5} \times (\phi\Big|_0^{2\pi} =$$

Lecture 10 30/10/19

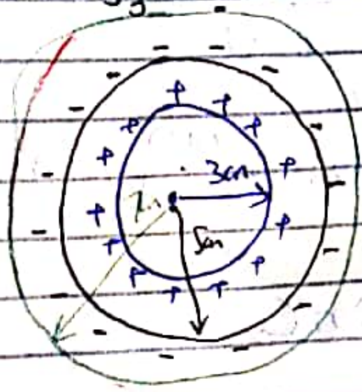
3.5 $\rho_{s1} = 200\,\mu C/m^2$ at $r = 3cm$

↳ universal limits for θ and ϕ.

↳ Charge is uniformly distributed on the surface of sphere.

$\rho_{s2} = -50\,\mu C/m^2$ at $r = 5cm$

$\rho_{s3} = ?$ at $r = 7cm$



(i) $\vec{D}$ at $r = 2cm$

Guass law

$\oint \vec{D}\cdot\vec{ds} = Q_{enclosed} = 0$

$\vec{D}\cdot\vec{ds} = 0$

$\Rightarrow \vec{D} = 0$

(ii) $\vec{D}$ at $r = 4cm$

↳ source 1 comes inside the guassian surface.

$$\vec{D} = \frac{\rho_{s1} \times a^2}{r^2}\,\vec{a_r}$$

~~a is the distance b/w source and point under observation~~. a is radius of source.

$$\vec{D} = \frac{200 \times 10^{-16} \times 9}{16}\,\vec{a_r} = 112.5\,\vec{a_r}\,\mu C/m^2$$

(iii) $\vec{D}$ at $r = 6cm$

↳ Two sources inside the guassian surface then apply superposition theorem.

$$\vec{D_1} = \frac{\rho_{s1} \times a^2}{r^2}\,\vec{a_r} = \frac{200 \times 10^{-6} \times 9}{36}\,\vec{a_r}$$

$= 50\,\vec{a_r}\,\mu C/m^2$

$$\vec{D_2} = \frac{\rho_{s2} \times a^2}{r^2} = \frac{-50 \times 10^{-6} \times 25}{36} \vec{a_r} = -34.7 \vec{a_r} \ \mu C/m^2$$

Now superposition theorem;

$$\vec{D} = \vec{D_1} + \vec{D_2} = 15.28 \vec{a_r} \ \mu C/m^2$$

(iv) $\rho_{s3} = ?$ if $\vec{D} = 0$ at $r = 7.3$ cm

$$\vec{D_1} = \frac{\rho_{s1} \times a^2}{r^2} \vec{a_r} = \frac{200 \times 10^{-6} \times 9}{(7.32)^2} \vec{a_r} = 33.6 \vec{a_r} \ \mu C/m^2$$

$$\vec{D_2} = \frac{\rho_{s2} \times a^2}{r^2} \vec{a_r} = \frac{-50 \times 10^{-6} \times 25}{53.58} \vec{a_r} = -23.33 \vec{a_r} \ \mu C/m^2$$

$$\vec{D_3} = \frac{\rho_{s3} \times a^2}{r^2} \vec{a_r} = \frac{\rho_{s3} \times 49}{53.58} \vec{a_r} = 0.914 \rho_{s3} \vec{a_r} \ /m^2$$

The total density according to superposition theorem;

$\vec{D} = \vec{D_1} + \vec{D_2} + \vec{D_3}$ which is given to be zero.

$$\Rightarrow 33.6 \times 10^{-6} \vec{a_r} - 23.33 \vec{a_r} \times 10^{-6} + 0.914 \rho_{s3} \vec{a_r} = 0$$

$$\Rightarrow \rho_{s3} = -11.23 \ \mu C/m^2$$

3.6 $\vec{D} = y^2 z^3 \vec{a_x} + 2xyz^3 \vec{a_y} + 3xy^2 z^2 \vec{a_z} \ PC/m^2$

$\Psi = ?$ passing through a surface at $x = 3$

$0 \le y \le 2$ $\quad 0 \le z \le 1$

$\vec{ds} = dy\,dz\,\vec{a_x}$ — differential flux passing through the small surface.

$$d\Psi = \vec{D} \cdot \vec{ds} = y^2 z^3 \times 10^{-12} \ C$$

$$\psi = \int \vec{D}\cdot \vec{ds} = \left(\int_0^2 y^2 dy + \int_0^1 z^3 dz\right) 10^{-12}$$

$$\psi = 10^{-12}\left[\left(\frac{y^3}{3}\Big|_0^2 \left(\frac{z^4}{4}\Big|_0^1\right]\right.\right. = 0.667\,PC.$$

(ii) E at $P(3,2,1)$

First we calculate $\vec{D}$ at $\vec{P}$

$$\vec{D} = (4\vec{a_x} + 12\vec{a_y} + 36\vec{a_z})\,10^{-12}\ C/m^2$$

As $\vec{D} = \varepsilon \vec{E} \Rightarrow \vec{E} = \frac{\vec{D}}{\varepsilon}$

$$\vec{E} = \frac{(4\vec{a_x} + 12\vec{a_y} + 36\vec{a_z})\,\cancel{10^{-12}}}{8.85 \times \cancel{10^{-12}}}$$

$$\vec{E} = 0.45\vec{a_x} + 1.355\vec{a_y} + 4.06\vec{a_z}$$

$\Rightarrow E = 4.31\ V/m$

(iii) $Q = ?$ in a sphere $r = 2\mu m$ centered at $P(3,2,1)$

↳ volume charge

As $Q = \int \rho_v\, dV$ $\qquad \rho_v$ is unknown

from Maxwell's first equation

$$\nabla\cdot\vec{D} = \rho_v$$

$$\rho_v = \nabla\cdot\vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\rho_v = (2xz^3 + 6xy^2z)\,10^{-12}\ C/m^3$$

Calculate ρ_v at $P(3,2,1)$

$$\rho_v = 78\ PC/m^3$$

↳ centre of sphere

dV is the differential volume of the sphere.

$$dV = r^2 \sin\theta \, d\theta \, d\phi \, dr$$

From ~~Q = εV~~

$$Q = \int \rho_v \, dv$$

$$Q = 78 \times 10^{-12} \int_0^{2\times10^{-6}} r^2 dr \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi$$

$$= 78 \times 10^{-12} \left[\frac{r^3}{3} \right|_0^{2\times10^{-6}} \times \left[-\cos\theta \right|_0^{\pi} \times \left[\phi \right|_0^{2\pi}$$

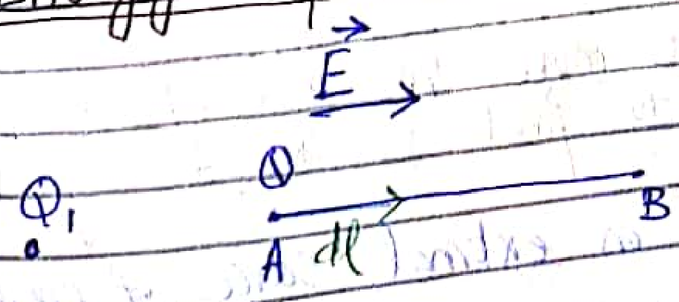
$$\Rightarrow Q = 2.61 \times 10^{-27} \, C$$

<u>3.7</u> divergence from $\vec{D}$.

<u>3.8</u> Calculate ρ_v.

Chapter 4:

Energy Expended In Moving a charge



Q_1 exerts a force of repulsion on Q.
→ tracks from A to B.

dl → a short piece of distance b/w A and B
↓
differential length vector
↓
always considered in direction of $\vec{E}$

$$\vec{F} = F \vec{a_r}$$

$$\vec{E} = E \vec{a_r}$$

$$\vec{dl} = dl \, \vec{a_r}$$

The differential energy provided by the source to move the charge along the differential length is given by

$$dW = F\,dl$$

We can replacing it by

$$dW = \vec{F} \cdot \vec{dl} \qquad \text{as } \vec{a_r} \cdot \vec{a_r} = 1$$

The force experienced by the test charge in presence of the source

$$\vec{F} = Q\vec{E}$$

$$\Rightarrow \boxed{dW = Q\vec{E} \cdot \vec{dl}}$$

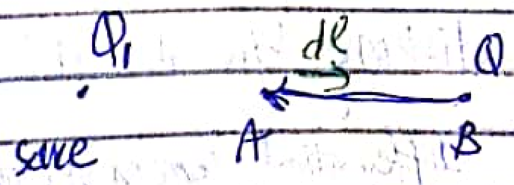
The total energy supplied by the source to move the charge from A to B

$$\boxed{W = Q\int_A^B \vec{E} \cdot \vec{dl}}$$

Reversing.

let we want to move the test charge from initial position B to final position A against the field.

So we need an external source of energy



Considering small segment of the distance b/w A and B.

$\vec{E}$

$$dW = F\,dl \rightarrow \text{external energy}$$

or $$dW = -\vec{F} \cdot \vec{dl}$$

As $\vec{F} = Q\vec{E}$

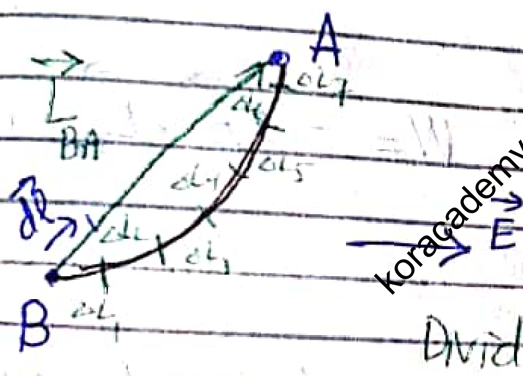
$\Rightarrow dW = -Q\vec{E} \cdot d\vec{l}$ will be required.

So the total energy required → B to A

$$W = -Q\int_B^A \vec{E} \cdot d\vec{l}$$

* Line Integral

Consider two points in a uniform electric field. We want to move a charge of Q coulomb from B to A along the shown path.



So the energy required $W = ?$

Divide the path into a very large no. of very small segments.

$\Delta W_1 = -Q\vec{E} \cdot \Delta\vec{L}_1$ $\qquad \Delta W_2 = -Q\vec{E} \cdot \Delta\vec{L}_2$

$\vdots$

$\Delta W_7 = -Q\vec{E} \cdot \Delta\vec{L}_7$

So the total energy is

$$W = \Delta W_1 + \Delta W_2 + \text{-----} + \Delta W_7$$

$$W = -Q\vec{E} \cdot (\Delta\vec{L}_1 + \Delta\vec{L}_2 + \text{----} + \Delta\vec{L}_7)$$

$$\Delta\vec{L}_1 + \Delta\vec{L}_2 + \dots + \Delta\vec{L}_7 = \vec{L}_{BA}$$

$$\Rightarrow W = -Q\vec{E} \cdot \vec{L}_{BA}$$

Hence

The energy required to move a charge from one point to another against the field is independent of the path followed.

For simplicity, consider the straight path b/w the two points.

Along differential distance $\vec{dl}$

$$dW = -Q\,\vec{E}\cdot\vec{dl}$$

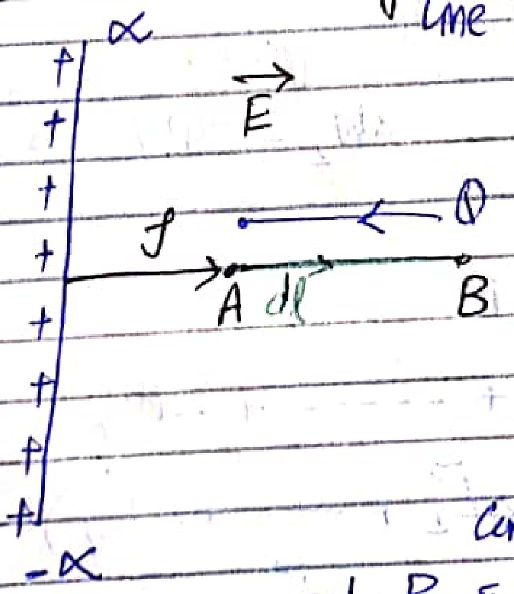
$\Rightarrow$ total energy $\boxed{W = -Q\int_B^A \vec{E}\cdot\vec{dl}}$

Assuming uniform electric field intensity

$$W = -Q\,\vec{E}\cdot\int_B^A \vec{dl}$$

$$\int_B^A \vec{dl} = \vec{L}_{BA} \Rightarrow \boxed{W = -Q\,\vec{E}\cdot\vec{L}_{BA}}$$

Infinite Line Charge:



Line charge density of source is known to be ρ_L.

Point P is located at ρ distance from source.

$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon\rho}\,\vec{a}_\rho$$

Considering two points A and B in field of the source.

How much energy will be required to move this charge from B to A against the field.

Considering small length element

$$\vec{dl} = d\rho\,\vec{a}_\rho$$

The total energy required is

$$W = -Q \int_B^A \vec{E} \cdot \vec{dl}$$

$$\vec{E} \cdot \vec{dl} = \frac{\rho_L}{2\pi\varepsilon} \times \frac{d\rho}{\rho} \qquad \because \vec{a}_\rho \cdot \vec{a}_\rho = 1$$

$$\Rightarrow \quad W = \frac{-Q\rho_L}{2\pi\varepsilon} \int_B^A \frac{d\rho}{\rho} = \frac{-Q\rho_L}{2\pi\varepsilon} \left[\ln\rho\right]_B^A$$

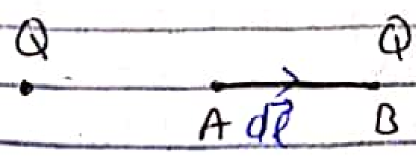
Interchanging the limits.

$$W = \frac{Q\rho_L}{2\pi\varepsilon} \left[\ln\rho\right]_A^B = \frac{Q\rho_L}{2\pi\varepsilon} \left[\ln B - \ln A\right]$$

$$\boxed{W = \frac{Q\rho_L}{2\pi\varepsilon} \ln B/A}$$

Voltage Or Potential Difference

Want to move charge from B to A against field $\rightarrow$ so we will need energy.



$$W = -Q_1 \int_B^A \vec{E} \cdot \vec{dl}$$

Energy is for a charge of Q coulombs $\rightarrow Q_1 \neq 1$

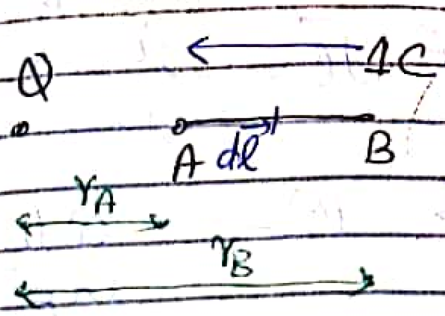
$\hookrightarrow$ energy for 1 C charge $\rightarrow$ potential difference or voltage b/w these two points.

$$\Rightarrow V_{AB} = -1 \int_B^A \vec{E} \cdot \vec{dl} \quad \Rightarrow \quad V_{AB} = \frac{W}{Q_1}$$

Voltage Or Potential Difference due to a point Charge

V b/w A and B due to Q

As $\vec{E} = \dfrac{Q}{4\pi \varepsilon r^2} \vec{a}_r$



$$d\vec{l} = dr\, \vec{a}_r$$

The scalar product

$$\vec{E} \cdot d\vec{l} = \frac{Q}{4\pi \varepsilon} \frac{dr}{r^2} \qquad \because \vec{a}_r \cdot \vec{a}_r = 1$$

$$\Rightarrow \quad V_{AB} = \frac{-Q}{4\pi \varepsilon} \int_{r_B}^{r_A} r^{-2}\, dr = \frac{Q}{4\pi \varepsilon} \left[\frac{1}{r} \right]_{r_B}^{r_A}$$

$$V_{AB} = \frac{Q}{4\pi \varepsilon} \left[\frac{1}{r_A} - \frac{1}{r_B} \right]$$

$$V_{AB} = V_A - V_B$$

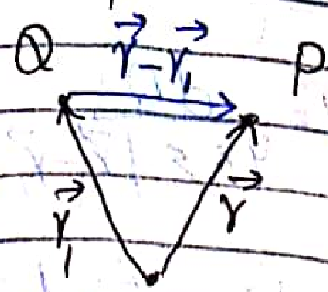
The energy required to bring a unit +ve test charge from infinity to a point is the absolute potential at that point.

$$V_A = \frac{Q}{4\pi \varepsilon r_A} \qquad V_B = \frac{Q}{4\pi \varepsilon r_B}$$

The absolute potential located at a radial distance r generally is

$$V = \frac{Q}{4\pi \varepsilon r}$$

The potential at point P.

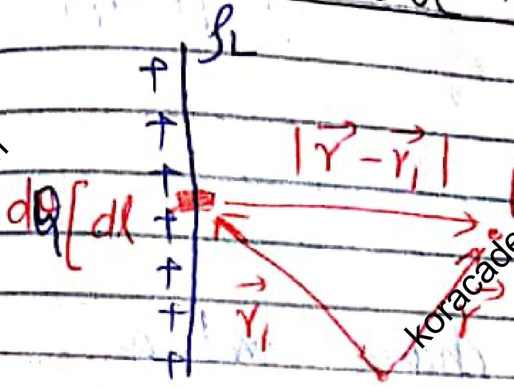


$$\boxed{V = \frac{Q}{4\pi\varepsilon\,|\vec{r}-\vec{r}_1|}}$$

Lecture 11

04/11/19

Potential Due to Line charge



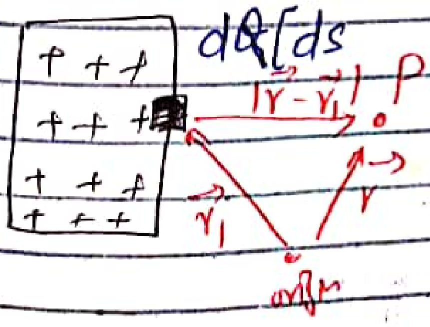
$$dV = \frac{dQ}{4\pi\varepsilon\,|\vec{r}-\vec{r}_1|}$$

$$dQ = \rho_L\, dl$$

$$dV = \frac{\rho_L\, dl}{4\pi\varepsilon\,|\vec{r}-\vec{r}_1|}$$

$$\Rightarrow V = \int \frac{\rho_L\, dl}{4\pi\varepsilon\,|\vec{r}-\vec{r}_1|}$$

Potential due to Surface charge



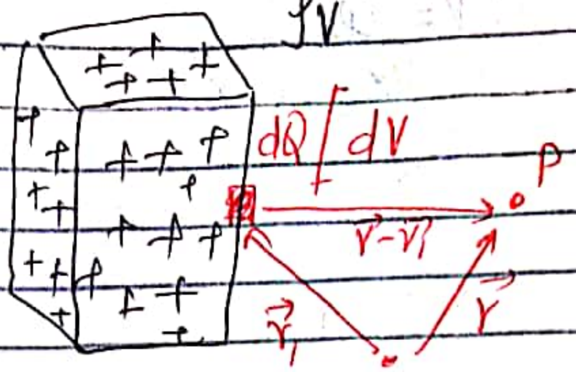
$$dV = \frac{dQ}{4\pi\varepsilon\,|\vec{r}-\vec{r}_1|}$$

$$dQ = \rho_s\, ds$$

$$dV = \frac{\rho_s\, ds}{4\pi\varepsilon\,|\vec{r}-\vec{r}_1|}$$

$$V = \int \frac{\rho_s\, ds}{4\pi\varepsilon\,|\vec{r}-\vec{r}_1|}$$

Potential due to volume charge.



$$dV = \frac{dQ}{4\pi\varepsilon|\vec{r}-\vec{r}_1|}$$

$$dQ = \rho_v \, dv$$

$$dV = \frac{\rho_v \, dv}{4\pi\varepsilon|\vec{r}-\vec{r}_1|}$$

$$V = \int \frac{\rho_v \, dv}{4\pi\varepsilon|\vec{r}-\vec{r}_1|}$$

Potential Gradient

Consider a point charge and a point at a distance r from the source.

$Q \xrightarrow{\quad r \quad} P$

As $\vec{E} = \frac{Q}{4\pi r^2}\vec{a}_r$

↳ This eq is in the spherical coordinate system

Absolute potential,

$$V = \frac{Q}{4\pi\varepsilon r}$$

Gradient of a scalar:

In Rectangular coordinate system

$$\nabla V = \frac{\partial V}{\partial x}\vec{a}_x + \frac{\partial V}{\partial y}\vec{a}_y + \frac{\partial V}{\partial z}\vec{a}_z$$

In cylindrical coordinate system.

$$\nabla V = \frac{\partial V}{\partial \rho}\vec{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \varphi}\vec{a}_\varphi + \frac{\partial V}{\partial z}\vec{a}_z$$

In spherical coordinate system.

$$\nabla V = \frac{\partial V}{\partial r}\vec{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \varphi}\vec{a}_\varphi$$

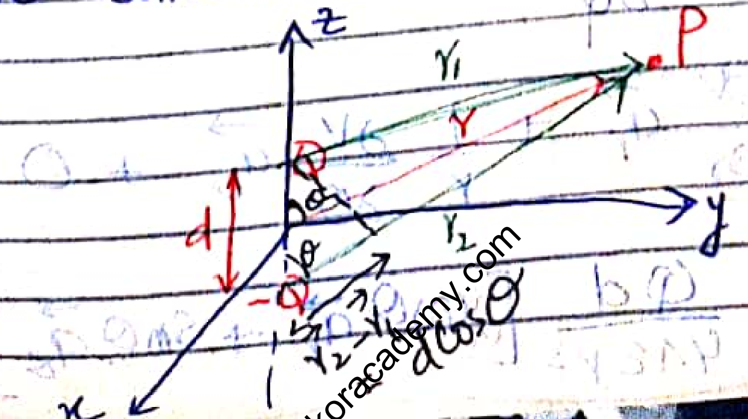
So potential gradient;

$$\boxed{\begin{aligned} \nabla V &= -\frac{Q}{4\pi\varepsilon r^2}\vec{a}_r \\ E &= -\nabla V \end{aligned}}$$

Electric Dipole

If the distance b/w two unlike charges of same magnitude is very small, then this arrangement is called dipole.

Consider a dipole on the z axis whose centre is located at the origin.
Consider a point located away from the dipole somewhere in its field.



$$V_+ = \frac{Q}{4\pi\varepsilon r_1} \qquad V_- = \frac{-Q}{4\pi\varepsilon r_2}$$

Superposition theorem

$$V = V_+ + V_-$$

$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

or $$V = \frac{Q \times (r_2 - r_1)}{4\pi\varepsilon\,(r_1 r_2)}$$

As $r_2 - r_1 = d\cos\theta \Rightarrow V = \dfrac{Q\,d\cos\theta}{4\pi\varepsilon\, r_1 r_2}$

P is very far, d is very small

$\Rightarrow r_1 \approx r_2 \approx r$

$$\Rightarrow \boxed{V = \frac{Q\,d\cos\theta}{4\pi\varepsilon\, r^2}} \rightarrow (A)$$

To calulate E.

$$\vec{E} = -\nabla V$$

As V is not a function of φ so

$$\frac{\partial V}{\partial \varphi} = 0$$

$$\nabla V = \frac{\partial V}{\partial r}\vec{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{a}_\theta + 0$$

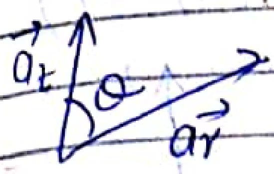
$$\boxed{\vec{E} = -\nabla V = \frac{Q\,d}{4\pi\varepsilon r^3}\left[2\cos\theta\,\vec{a}_r + \sin\theta\,\vec{a}_\theta\right]}$$

P is far away. d is very small.
so r_1, r_2 and r are almost parallel.

Dipole moment ($\vec{p}$)

$+Q$
$\vec{d} = d\vec{a}_z$
$-Q$

$$\vec{p} = Q\vec{d} = Q\,d\,\vec{a}_z$$



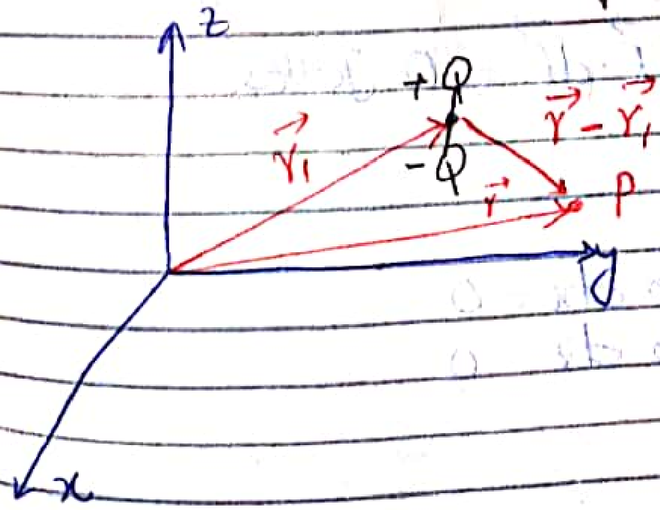
$$\vec{p}\cdot\vec{a}_r = Q\,d\,\vec{a}_z\cdot\vec{a}_r = Q\,d\cos\theta$$

So V in terms of dipole moment

(A) $\Rightarrow$ $$V = \frac{\vec{p}\cdot\vec{a}_r}{4\pi\varepsilon_0 r^2}$$

as $\vec{a}_r = \dfrac{\vec{r}}{|\vec{r}|}$ $\Rightarrow$ $$\boxed{V = \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon r^3}}$$

If the dipole is located somewhere else?
not at the origin?



$$V = \frac{\vec{p}\cdot(\vec{r}-\vec{r}_1)}{4\pi\varepsilon|\vec{r}-\vec{r}_1|^3}$$

Numericals

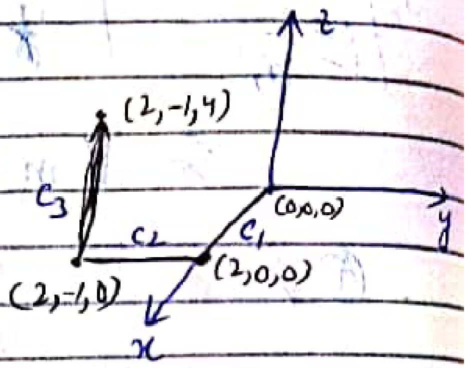
4.2 $Q = 5\mu C$ move from $(0,0,0)$ to $\cancel{(4,2,1)}$ $(2,-1,4)$

$$\vec{E} = 2xyz\,\vec{a_x} + x^2 z\,\vec{a_y} + x^2 y\,\vec{a_z} \quad V/m$$

(i) Straight line segments.

$(0,0,0)$ to $(2,0,0)$ to $(2,-1,0)$ to $(2,-1,4)$

As $W = -Q\int \vec{E}\cdot\vec{dl}$



For C_1

$$\vec{dl} = d_x\vec{a_x} + d_y\vec{a_y} + d_z\vec{a_z}$$

$$y=0 \Rightarrow dy = 0$$
$$z=0 \Rightarrow dz = 0$$

$$\vec{dl} = dx.\vec{a_x}$$

$$\vec{E}\cdot\vec{dl} = 2xyz\,dx = 0 \qquad \because y = z = 0$$

$$W = -Q\int_{C_1} \vec{E}\cdot\vec{dl} = 0 \text{ Joules.}$$

For C_2

$$x=2 \Rightarrow dx = 0$$
$$z=0 \Rightarrow dz = 0$$

$$\vec{dl} = d_y\vec{a_y}$$

$$\vec{E}\cdot\vec{dl} = x^2 z\,dy = 0 \quad \Rightarrow z = 0$$

$$W_2 = -Q\int_{C_2} \vec{E}\cdot\vec{dl} = 0 \text{ Joules}$$

For C_3.

$x = 2 \Rightarrow dx = 0$

$y = -1 \Rightarrow dy = 0$

$\vec{dl} = dz\, \vec{a_z}$

$\vec{E} \cdot \vec{dl} = x^2 y\, dz = (4)(-1) dz = -4 dz$

$$W_3 = -Q \int_{C_3} \vec{E} \cdot \vec{dl}$$

$$W_3 = 5 \times 10^{-6} \times 4 \int^4 dz$$

$$= 20 \times 10^{-6} \times [z|_0^4 = 80 \mu J$$

$= \cancel{66}$

$W = W_1 + W_2 + W_3 = 80 \mu J$.

(ii) Straight line

$x = -2y \Rightarrow y = \frac{-x}{2}$

$z = 2x \Rightarrow x = z/2$

$z = -4y$

$W = -Q \int \vec{E} \cdot \vec{dl}$

$\vec{dl} = dx\, \vec{a_x} + dy\, \vec{a_y} + dz\, \vec{a_z}$

$\vec{dl} = 2x\left(\frac{-x}{2}\right)(2x) dx + 4y^2(-4y) dy + \frac{z^2}{4}\left(\frac{-z}{4}\right) dz$

$\vec{dl} = -2x^3 dx - 16y^3 dy - \frac{1}{16} z^3 dz$

$W = 5 \times 10^{-6} \left[2 \int_0^2 x^3 dx + 16 \int_0^{-1} y^3 dy + \frac{1}{16} \int_0^4 z^3 dz \right]$

$$W = 5\times10^{-4}\left[2\left(\frac{x^4}{4}\right|_0^2 + 16\left(\frac{y^4}{4}\right|_0^{-1} + \frac{1}{16}\left(\frac{z^4}{4}\right|_0^{4}\right]$$

$$\Rightarrow W = 80\,\mu J$$

(iii) Along a curve.

$x = -2y^3 \qquad dx = -6y^2 dy$

$z = 4y^2 \qquad dz = 8y\, dy$

$$\vec{dl} = d_x\vec{a_x} + d_y\vec{a_y} + d_z\vec{a_z}$$

$$\vec{E}\cdot\vec{dl} = 2xyz\,dx + x^2z\,dy + x^2y\,dz$$

In terms of y and dy

$$= 2(-2y^3)y(4y^2)(-6y^2)dy + (4y^6)(4y^2)dy + (4y^6)(y)(8y)\,dy$$

$$= 144y^8\,dy$$

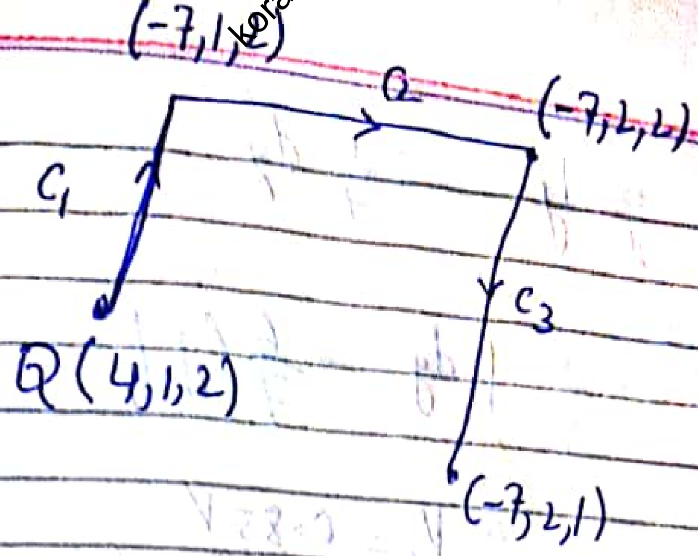
$$\Rightarrow W = -5\times10^{-6}\times144\int_0^{-1} y^8\,dy = -5\times10^{-6}\times144\left(\frac{y^9}{9}\right|_0^{-1}$$

$$\Rightarrow W = 80\,\mu J.$$

Q 4.4 $\quad \vec{E} = \frac{-6y}{x^2}\vec{a_x} + \frac{6}{x}\vec{a_y} + 5\vec{a_z}$ V/m

(a) $V_{PQ} = ?$ $\quad P(-7,2,1) \quad Q(4,1,2)$

V_{PQ} will be energy required to move charge from Q to P



For C1

$$\vec{dl} = dx\,\vec{a_x} + dy\,\vec{a_y} + dz\,\vec{a_z}$$

$$y = 1 \Rightarrow dy = 0$$
$$z = 2 \Rightarrow dz = 0$$

$$\vec{dl} = dx\,\vec{a_x}$$

$$\vec{E}\cdot\vec{dl} = \frac{-6y}{x^2}\,dx$$

$$y = 1 \Rightarrow \vec{E}\cdot\vec{dl} = \frac{-6}{x^2}\,dx$$

$$V_1 = \int_{C1} \vec{E}\cdot\vec{dl} = -6\int_4^{-7} x^{-2}\,dx$$

$$= -6\left(\frac{1}{x}\Big|_4^{-7}\right) \Rightarrow V_1 = 2.35\,V.$$

For C2.

$$\vec{dl} = dx\,\vec{a_x} + dy\,\vec{a_y} + dz\,\vec{a_z}$$

$$x = -7 \Rightarrow dx = 0$$
$$z = 2 \Rightarrow dz = 0$$

$$\vec{dl} = dy\,\vec{a_y}$$

$$\cancel{\vec{E} = \frac{-6y}{x^2}\,a}$$

$$\vec{E}\cdot\vec{dl} = \frac{6}{x}\,dy = -\frac{6}{7}\,dy$$

$$V_2 = -\int_{c_2} \vec{E}\cdot\vec{dl} = \frac{6}{7}\int_1^2 dy = \frac{6}{7}(y)\Big|_1^2$$

$$\Rightarrow V_2 = 0.85\,V$$

<u>For C_3</u>

$$x = -7 \Rightarrow dx = 0$$

$$y = 2 \Rightarrow dy = 0$$

$$\vec{dl} = dz\,\vec{a_z}$$

$$\vec{E}\cdot\vec{dl} = 5\,dz$$

$$V_3 = -\int_{c_3} \vec{E}\cdot\vec{dl} = -5\int_2^1 dz = -5\left[z\right]_2^1$$

$$\Rightarrow V_3 = 5\,V.$$

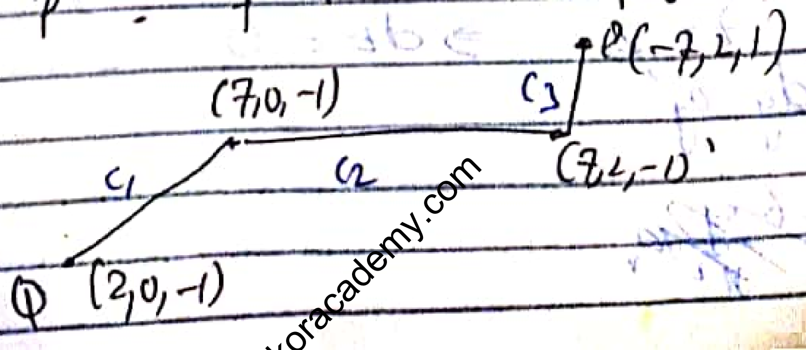
$$V_{PQ} = V_1 + V_2 + V_3 = 8.2\,V.$$

(b) $V_P = ?$ if $V_Q = 0$

$$V_{PQ} = V_P - V_Q$$

$$V_P = V_{PQ} + V_Q \quad \Rightarrow V_P = 8.2\,V$$

(c) $V_P = ?$ if $Q = 0$ at $Q(2,0,-1)$



For C_1 $\vec{dl} = dx\vec{a_x} + dy\vec{a_y} + dz\vec{a_z}$

$y=0 \Rightarrow dy=0$

$z=-1 \Rightarrow dy=0$

$\vec{dl} = dx\,\vec{a_x}$

$\vec{E}\cdot\vec{dl} = -\frac{6y}{x^2}dx$ as $y=0$

$\vec{E}\cdot\vec{dl} = 0$ V

$V_1 = -\int_{c_1} \vec{E}\cdot\vec{dl} = 0$

For C_2 $\vec{dl} = dx\,\vec{a_x} + dy\,\vec{a_y} + dz\,\vec{a_z}$

$x=-7 \Rightarrow dx=0$

$z=-1 \Rightarrow dz=0$

$\vec{dl} = dy\,\vec{a_y}$

$\vec{E}\cdot\vec{dl} = \frac{6}{x}dy = \frac{-6}{7}dy.$

$V_2 = -\int_{c_2} \vec{E}\cdot\vec{dl} = \frac{6}{7}\int_0^1 dy = \frac{6}{7}(y|_0^2 = 1.7$ volts.

For C_3

$x=-7 \Rightarrow dx=0$

$y=2 \Rightarrow dy=0$

$\vec{dl} = dz\,\vec{a_z}$

$\vec{E}\cdot\vec{dl} = 5dz$

$V_3 = \int_{c_3} \vec{E}\cdot\vec{dl} = -5\int_{-1}^{1} dz = -5[z|_{-1}^{1}$

$= 10V$

$V_{PQ} = V_1 + V_2 + V_3 = -8.3V$

$$V_{PQ} = V_P - V_Q$$

$$V_P = V_P = V_{PQ} + \cancel{V_Q}^{\,0}$$

$$V_P = -8.3V$$

Lecture 12:

4.5

$Q = 6\times10^{-9}\,C$ is located at origin.

(i) $V_P = ?$ at $P(-0.2, -0.4, 0.4)$

(ii) $V_P = ?$ at $Q(1,0,0)$ if $V_Q = 0$

(iii) $V_P = ?$ if $V_Q = 20$ at $Q(-0.5, 1, -1)$

$$V_P = \frac{Q}{4\pi\varepsilon_0 r_P} = \frac{9\times10^9\times6\times10^{-9}}{0.6} = 90V$$

$$r_P = \sqrt{(-0.2)^2 + (-0.4)^2 + (0.4)^2} = 0.6$$

(ii) $$V_{PQ} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_P} - \frac{1}{r_Q}\right]$$

$$r_Q = \sqrt{(1)^2 + (0)^2 + (0)^2} = 1$$

$$V_{PQ} = 9\times10^9\times6\times10^{-9}\left[\frac{1}{0.6} - \frac{1}{1}\right] = 36V$$

$$V_{PQ} = V_P - V_Q \Rightarrow V_P = V_{PQ} + \cancel{V_Q}^{\,0}$$

$$V_{PQ} = 36V.$$

(iii) $$V_{PQ} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_P} - \frac{1}{r_Q}\right)$$

$$\vec{E} = -\nabla V$$

$$\vec{E} = \frac{Qd}{4\pi\varepsilon_0 r^3}\left[2\cos\theta\,\vec{a_r} + \sin\theta\,\vec{a_\theta}\right].$$

$$r_\phi = \sqrt{(-0.5)^2 + (1)^2 + (1)^2} = 1.5$$

$$V_{p\phi} = 54\left(\frac{1}{0.6} - \frac{1}{1.5}\right) = 53.94\ V$$

$$V_p = V_{p\phi} + V_\phi = 53.94 + 20 = 73.94\ V.$$

4.6 $V_p = ?$ where $P(0,0,10)$

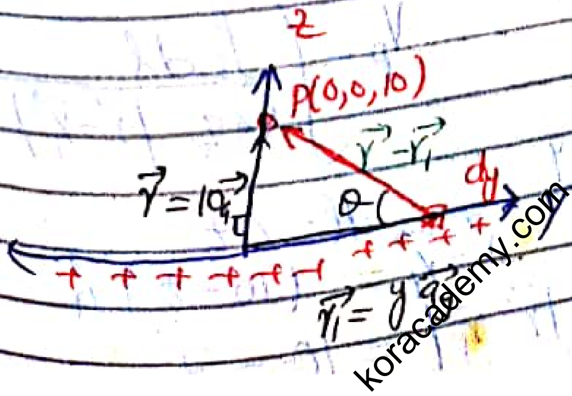
(i) $Q = 20\times10^{-9}\,C$ is located at the origin

$$r_p = 10m$$

$$V_p = \frac{Q}{4\pi\varepsilon_0 r_p} = \frac{20\times10^{-9}\times9\times10^9}{10} = 18V.$$

(ii) Line charge, $\rho_L = 10\,nC/m$ at $x=0$
$z=0$ $\quad -1 \le y \le 1$

$$V_p = \int \frac{\rho_L\,dl}{4\pi\varepsilon_0|\vec{r}-\vec{r_1}|} \qquad dl = dy$$



$$\vec{r}-\vec{r_1} = 10\vec{a_z} - y\vec{a_y}$$

$$|\vec{r}-\vec{r_1}| = y^2+100$$

$$V_p = 9\times10^9 \times 10\times10^{-9}\int_{-1}^{1}\frac{dy}{\sqrt{y^2+100}}$$

This integration is very difficult to perform.
↳ consider the right angled triangle in the figure.

$$\cot\theta = y/10$$

$$y = 10\cot\theta \Rightarrow dy = -10\,\text{cosec}^2\theta\, d\theta$$

$$90\int\frac{-10\,\text{cosec}^2\theta\, d\theta}{\sqrt{100\cot^2\theta+100}} = 90\int\frac{-10\,\text{cosec}^2\theta\, d\theta}{10\,\text{cosec}\,\theta}$$

$$\int -\text{cosec}\,\theta\, d\theta = \ln(\text{cosec}\,\theta+\cot\theta)$$

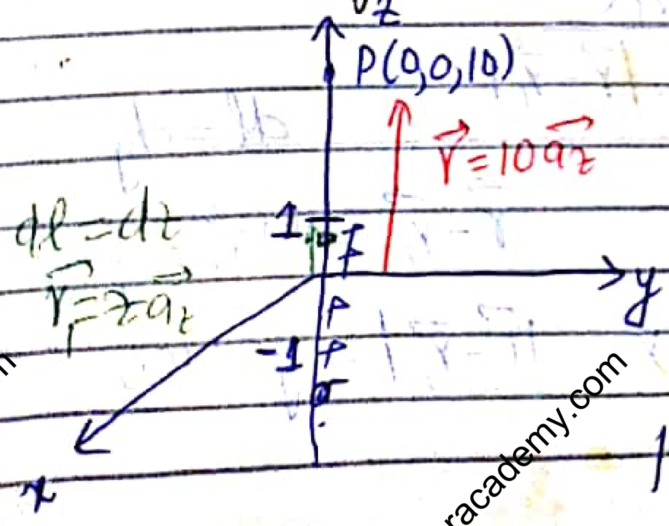
confirm it.

$$V_p = 90\ln\left[\text{cosec}\,\theta+\cot\theta\right] = 90\ln\left[\frac{\sqrt{y^2+100}}{10}+\frac{y}{10}\right]_{-1}^{1}$$

$$\text{cosec}\,\theta = \sqrt{1+\cot^2\theta}$$

$$V_p = 17.95\ V$$

(iii) line charge $\rho_L = 10\,nC/m$ at $x=0, y=0$
$-1 \le z \le 1$



$$V_p = \int\frac{\rho_L\, dl}{4\pi\varepsilon_0|\vec{r}-\vec{r}_1|}$$

$$dl = dz$$

$$\vec{r}-\vec{r}_1 = (10-z)\vec{a}_z$$

$$|\vec{r}-\vec{r}_1| = 10-z$$

$$V_p = -9\times10^9 \times 10\times10^{-9} \int_{-1}^{1} \frac{dz}{10-z}$$

$$V_p = -90\int_{-1}^{1} \frac{dz}{z-10} = -90 \ln(z-10)\Big|_{-1}^{1}$$

$$V_p = 17.95\,V$$

4.8

Potential at a point is $V = \frac{60\sin\theta}{r^2}$ volts

(a) $V_p = ?$ $P(3, 60°, 25°)$

$$V_p = \frac{60\sin 60°}{9} = 5.77\,V$$

(b) $\vec{E}_p$ at $P(3, 60°, 25°)$

$$\vec{E} = -\nabla V = -\left[\frac{\partial V}{\partial r}\vec{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{a}_\theta\right]$$

$$\vec{E}_p = -\left[\frac{-120\sin\theta}{r^3}\vec{a}_r + \frac{60\cos\theta}{r^3}\right]a_\theta$$

$$= \frac{120\sin 60}{27}\vec{a}_r - \frac{60\cos 60}{27}a_\theta$$

$$\Rightarrow \vec{E}_p = 3.85\vec{a}_r - 1.11\vec{a}_\theta \text{ V/m}$$

Ignore remaining parts of this question

4.9 dipole moment $\vec{P} = -4\vec{a_x} + 5\vec{a_y} + 3\vec{a_z}$ nC-m

located at $(1,2,-1)$ in free space.

$V = ?$

(a) At $P(0,0,0)$

$$\vec{V} = \frac{\vec{P}\cdot(\vec{r}-\vec{r_1})}{4\pi\varepsilon_0 |\vec{r}-\vec{r_1}|^3}$$

$$\vec{r}-\vec{r_1} = -\vec{a_x} - 2\vec{a_y} + \vec{a_z}$$

$$|\vec{r}-\vec{r_1}| = 2.45$$

$$\Rightarrow V = \frac{9\times10^{9}\times10^{-9}(-4\vec{a_x} + 5\vec{a_y} + 3\vec{a_z})\cdot(-\vec{a_x} - 2\vec{a_y} + \vec{a_z})}{(2.45)^3}$$

$$\Rightarrow V_p = 1.6835\ V.$$

4.10 Two charge particles of ~~3μC~~

$Q = 3\times10^{-6}$ C at $(0,0,1\,mm)$

$Q = -3\times10^{-6}$ C at $(0,0,-1\,mm)$

Means that it is a dipole.

$$\vec{d} = 2\times10^{-3}\,\vec{a_z}\ m$$

$$\vec{P} = Q\vec{d} = 3\times10^{-6}\times2\times10^{-3}\,\vec{a_z}$$

$$\Rightarrow \vec{P} = 6\,\vec{a_z}\ nC\text{-}m.$$

b) $\vec{E} = ?$ at $P(2, 40°, 50°)$

$$\vec{E} = \frac{Qd}{4\pi\varepsilon_0 r^3}\left[2\cos\theta\, \vec{a_r} + \sin\theta\, \vec{a_\theta}\right]$$

$$= \frac{3\times10^{-6}\times9\times10^{9}}{8}\left[\cos40°\, \vec{a_r} + \sin40°\, \vec{a_\theta}\right].$$

$$= 10.33\, \vec{a_r} + 4.31\, \vec{a_\theta} \quad V/m$$

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EMFT FINAL

Lecture 1.

Magnetic Flux

↳ is a scalar quantity. Symbol ϕ

↳ unit weber

The lines of magnetic force or magnetic field line.

North pole

(N) ⟶ $\vec{F}_M, \vec{B}, \vec{H}$

The path / line which is followed by an isolated north pole on a magnetic field is known as flux.

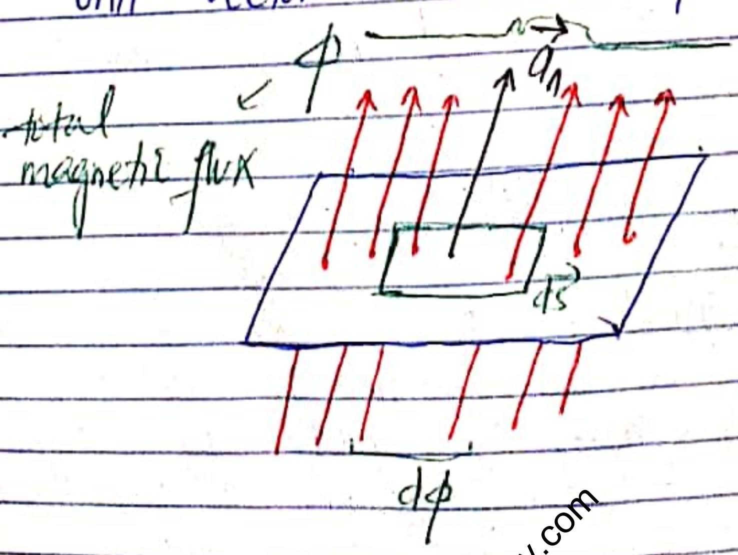
$\vec{B} = \mu \vec{H}$

Magnetic Flux Density

↳ is a vector quantity

→ Represented by $\vec{B}$ Unit is Wb/m^2 or Tesla.

Consider an open surface sheet with area S. To represent it as a vector, we need a unit vector normal to surface of the sheet.



$\vec{S} = S\,\vec{a}_n$

$\vec{B} = B\,\vec{a}_n$

$$\boxed{\begin{aligned} B &= \phi/S \\ \phi &= B \cdot S \end{aligned}}$$

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B and S are vectors and in the same direction.

$$\Rightarrow \quad \phi = \vec{B} \cdot \vec{S}$$

Consider a smaller portion $d\vec{s}$ of the sheet through which a few lines of flux $d\phi$ passes.

$$d\vec{s} = ds\, \vec{a}_n$$

$$B = d\phi / ds$$

$$d\phi = B\, ds$$

Again → B and ds are vectors and in the same direction.

$$d\phi = \vec{B} \cdot \vec{ds} \qquad \because \vec{a}_n \cdot \vec{a}_n = 1$$

The total magnetic flux will be

$$\phi = \int \vec{B} \cdot \vec{ds}$$

If the source is located inside a closed surface

$$\phi = \oint \vec{B} \cdot \vec{ds}$$

Maxwell's third equation

consider a bar magnet inside a closed ~~surfa~~ sphere of radius r.

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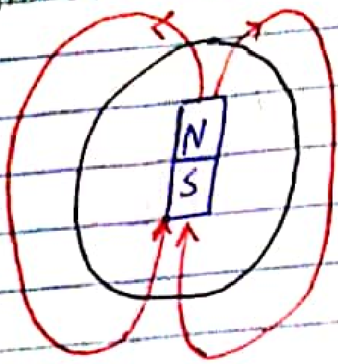
Magnetic flux is continuous ⇒ Hence the no. of lines leaving the surface will be equal to the no. of lines entering the closed surface.
↳ The net magnetic flux in the outward direction will be zero.



Divergence Theorem

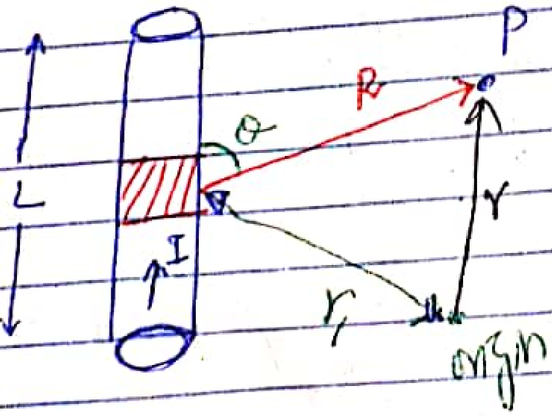
$$\phi = \oint \vec{B}\cdot\vec{ds} = \int(\nabla\cdot\vec{B})\,dV = 0$$

$$\boxed{\nabla\cdot\vec{B} = 0}$$

Biot Savart Law

Consider a current carrying conductor.
↳ is in the upward direction.

Small portion.
We know that current in conductor generates magnetic field.



three parameters to calculate strength of magnetic field
ϕ, B, H.

$$B \propto \frac{IL\sin\theta}{R^2}$$

Constant of proportionality is $\frac{\mu}{4\pi}$.

$\mu \to$ permability of the medium.

$\mu = \mu_0\mu_r$

$\left.\begin{array}{l}\mu_0 = 4\pi\times10^{-7}\\ \mu_r = 1\end{array}\right\}$ free space

$\mu_r > 1$ For any other medium.

↳ So magnetic flux density is greater than in vacum.

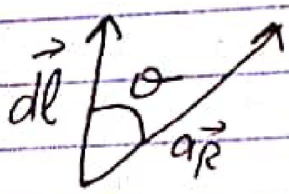
$$\boxed{B = \frac{\mu}{4\pi} \times \frac{IL\sin\theta}{R^2}}$$

Consider a small portion of conductor $\vec{dl}$.

Differential length vector is in the direction of the current.

$$\Rightarrow \boxed{dB = \frac{\mu}{4\pi}\,\frac{I\,dl\,\sin\theta}{R^2}} \rightarrow (A)$$

Direction?



The cross product of these two vectors will give direction of $\vec{B}$

$\cancel{\vec{dl}\times\vec{a_r}}$

$$\vec{dl}\times\vec{a_R} = dl\cdot 1\cdot\sin\theta$$

$$(A) \Rightarrow \boxed{d\vec{B} = \frac{\mu}{4\pi}\,\frac{I\,\vec{dl}\times\vec{a_r}}{R^2}}$$

$$\vec{a_R} = \frac{\vec{R}}{R} \qquad \boxed{\Rightarrow d\vec{B} = \frac{\mu}{4\pi}\,\frac{I\,\vec{dl}\times\vec{R}}{R^3}}$$

The position vector of small portion is represented by $\vec{r_1}$.

Position vector of point P is $\vec{r}$.

Date: / /

$$\vec{R} = \vec{r} - \vec{r}_1 \qquad |\vec{R}| = |\vec{r} - \vec{r}_1|$$

$$d\vec{B} = \frac{\mu}{4\pi} \frac{I\, d\vec{l} \times (\vec{r} - \vec{r}_1)}{|\vec{r} - \vec{r}_1|^3}$$

For total magnetic flux density; integrate

$$\boxed{\vec{B} = \frac{\mu}{4\pi} \int \frac{I\, d\vec{l} \times (\vec{r} - \vec{r}_1)}{|\vec{r} - \vec{r}_1|^3}}$$

$$\vec{B} = \mu \vec{H} \Rightarrow d\vec{H} = \frac{d\vec{B}}{\mu}$$

$$\Rightarrow d\vec{H} = \frac{1}{4\pi} \frac{I\, d\vec{l} \times (\vec{r} - \vec{r}_1)}{|\vec{r} - \vec{r}_1|^3}$$

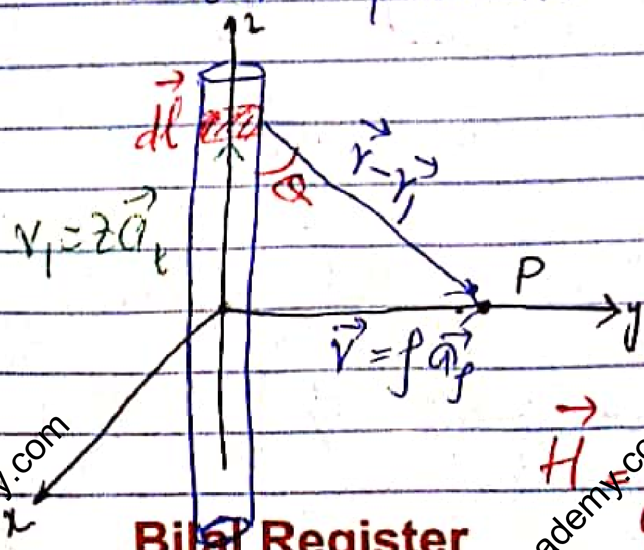
For total magnetic field intensity; integrate

$$\vec{H} = \frac{1}{4\pi} \int \frac{I\, d\vec{l}\, (\vec{r} - \vec{r}_1)}{|\vec{r} - \vec{r}_1|^3}$$

Infinitely Long Current Carrying Conductor

↳ along z axis.

Consider point under observation on y axis.



$$d\vec{l} = dz\, \vec{a}_z$$

$$\vec{r} - \vec{r}_1 = \rho \vec{a}_\rho - z\vec{a}_z$$

$$|\vec{r} - \vec{r}_1| = \sqrt{\rho^2 + z^2}$$

$$\vec{H} = \frac{1}{4\pi} \int \frac{I\, d\vec{l} \times (\vec{r} - \vec{r}_1)}{|\vec{r} - \vec{r}_1|^3}$$

Putting values in this equation

$$\vec{H} = \frac{1}{4\pi}\int_{-\infty}^{\infty} \frac{I\,dz\,\vec{a_z}\,(\rho\vec{a_\rho} - z\vec{a_z})}{(\rho^2+z^2)^{3/2}}$$

We know that $\vec{a_z} \times \vec{a_\rho} = \vec{a_\phi}$

$\vec{a_z} \times \vec{a_z} = 0$

$$\Rightarrow \vec{H} = \frac{1}{4\pi}\int_{-\infty}^{\infty} \frac{I\rho\,dz\,\vec{a_\phi}}{[\rho^2+z^2]^{3/2}}$$

Consider the right angled triangle.

z = base $\quad \rho$ = Perpendicular

$\cot\theta = \frac{z}{\rho} \quad \Rightarrow z = \rho\cot\theta$

$dz = -\rho\,\text{Cosec}^2\theta\,d\theta$

When $z = -\infty$, then $\theta = \pi$

When $z = \infty$, then $\theta = 0$

$$\Rightarrow \vec{H} = \frac{I}{4\pi}\int_{0}^{\pi} \frac{\rho^2\,\text{Cosec}^2\theta\,d\theta\,\vec{a_\phi}}{[\rho^2+\rho^2\cot^2\theta]^{3/2}}$$

Denominator $\rho^3(1+\cot^2\theta)^{3/2}$

$\rho^3(\text{Cosec}^2\theta)^{3/2} = \rho^3\,\text{Cosec}^3\theta$

$$\Rightarrow \vec{H} = \frac{I}{4\pi}\int_{0}^{\pi} \frac{\rho^2\,\text{Cosec}^2\theta\,d\theta\,\vec{a_\phi}}{\rho^3\,\text{Cosec}^3\theta}$$

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$$\vec{H} = \frac{I}{4\pi\rho}\int_0^{\pi} \sin\theta \, d\theta \, \vec{a_\phi} = \frac{I}{4\pi\rho}\left(-\cos\theta\right)\Big|_0^{\pi} \vec{a_\phi}$$

$$\Rightarrow \boxed{\vec{H} = \frac{I}{2\pi\rho}\vec{a_\phi}}$$

(i) DC current will generate constant magnetic field.
AC " " " time varying ".

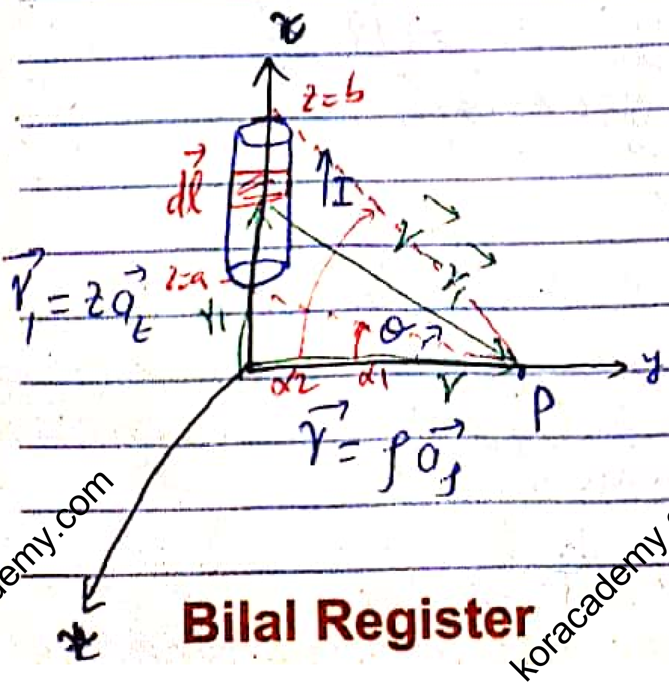
(ii) $\vec{H}$ depends on radial distance b/w source and point under observation.

⇒ If the current carrying conductor is placed in a cylinder → intensity on surface of cylinder will be uniform at distance is constant.

↳ Also if a line is considered parallel to the source, intensity is constant at that line.

<u>Lecture 2</u> 02/12/19

A small conductor



$$\vec{H} = \frac{1}{4\pi}\int \frac{I\vec{dl} \times (\vec{r}-\vec{r_1})}{|\vec{r}-\vec{r_1}|^3}$$

$$\vec{r_1} = z\vec{a_z}$$

$$\vec{r} = \rho\vec{a_\rho}$$

$$\vec{r}-\vec{r_1} = \rho\vec{a_\rho} - z\vec{a_z}$$

$$|\vec{r}-\vec{r_1}| = \sqrt{\rho^2+z^2}$$

$$\vec{dl} = \vec{z}\,\vec{a_z}$$

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$$\vec{H} = \frac{I}{4\pi}\int_a^b \frac{dz\,\vec{a_z} \times (\rho\vec{a_\rho} - z\vec{a_z})}{(\rho^2+z^2)^{3/2}}$$

$$\vec{H} = \frac{I}{4\pi}\int_a^b \frac{\rho\,dz\,\vec{a_\phi}}{(\rho^2+z^2)^{3/2}}$$

$$\frac{z}{\rho} = \operatorname{Tan}\theta \qquad \Rightarrow z = \rho\operatorname{Tan}\theta$$

$$dz = \rho\operatorname{Sec}^2\theta\, d\theta$$

When $z = a$, then $\theta = \alpha_1$

When $z = b$, Then $\theta = \alpha_2$

$$\vec{H} = \frac{I}{4\pi}\int_{\alpha_1}^{\alpha_2} \frac{\rho^2 \sec^2\theta\, d\theta\, \vec{a_\phi}}{(\rho^2 + \rho^2\tan^2\theta)^{3/2}}$$

Denominator $\rho^3(1+\tan^2\theta)^{3/2}$

$= \rho^3(\operatorname{Sec}^2\theta)^{3/2}$

$= \rho^3 \sec^3\theta$

$$\vec{H} = \frac{I}{4\pi}\int_{\alpha_1}^{\alpha_2} \frac{\rho^2 \operatorname{Sec}^2\theta\, d\theta\, \vec{a_\phi}}{\rho^3 \sec^3\theta}$$

$$\vec{H} = \frac{I}{4\pi\rho}\int_{\alpha_1}^{\alpha_2} \cos\theta\, d\theta\, \vec{a_\phi}$$

$$\vec{H} = \frac{I}{4\pi\rho}\left(\sin\theta\right|_{\alpha_1}^{\alpha_2} \vec{a_\phi}$$

$$\Rightarrow \boxed{\vec{H} = \frac{I}{4\pi\rho}\left[\sin\alpha_2 - \sin\alpha_1\right]\vec{a}_\phi}$$

So we conclude that either the conductor is small or large (in length), the magnetic field intensity ($\vec{H}$) will always be in the direction of $\vec{a}_\phi$.

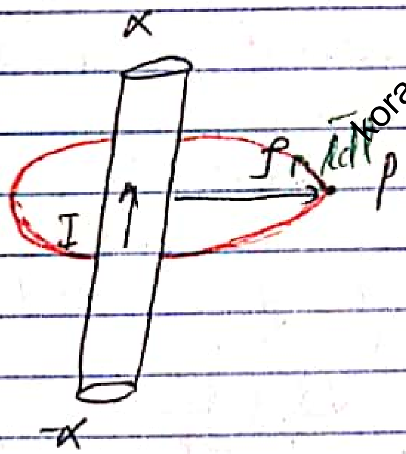
Ampere Circuital Law

Consider a current carrying conductor extending from $-\alpha$ to α.

Consider a point P in the magnetic field of this conductor.

The radial distance b/w conductor and P is given by ρ.

A $\vec{H} = \frac{I}{2\pi\rho}\vec{a}_\phi$



Consider a closed circular path of radius ρ around conductor.

↳ Consider a small portion

↳ small arc ⇒ line

$d\vec{l} = \rho\, d\phi\, \vec{a}_\phi$

$$\vec{H}\cdot d\vec{l} = \frac{I}{2\pi} d\phi \qquad \because a_\phi \cdot a_\phi = 1$$

Integrate along the closed circular path

$$\oint \vec{H}\cdot d\vec{l} = \frac{I}{2\pi}\int_0^{2\pi} d\phi = \frac{I}{2\pi}\left(\phi\right|_0^{2\pi} = I$$

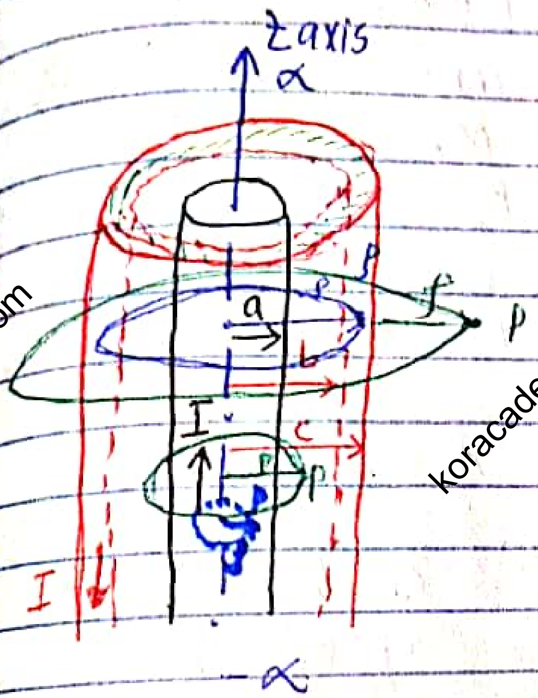
Bilal Register

$$\boxed{\oint \vec{H}\cdot d\vec{l} = I_{enclosed}}$$

This law states that 'The integral of magnetic field intensity around a closed path B always equal to the current enclosed by the closed path.'

Applications of Ampere Circuital Law

1) Coaxial Cable



l_i → thickness of outer conductor

Extending from $-\alpha$ to α.

Say J_i represents the current density of inner conductor.

↳ current per unit area

$$J_i = \frac{I}{\pi a^2}$$

Similarly for outer conductor

$$\frac{I}{\pi(c^2-b^2)} = J_0 = \frac{I}{(\pi c^2 - \pi b^2)}$$

Case 1 $\rho < a$

Point P located inside the inner conductor.

Consider closed circular path of radius ρ.

Top view

$$\vec{H} = H\vec{a}_\phi$$

$$\oint \vec{H} \cdot \vec{dl} = I_{enclosed}$$

L.H.S R.H.S

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Consider L.H.S $\quad \vec{dl} = \rho \, d\phi \, \vec{a_\phi}$

$$\vec{H} \cdot \vec{dl} = H\rho \, d\phi \qquad \because \vec{a_\phi} \cdot \vec{a_\phi} = 1$$

Integrating

$$\oint \vec{H} \cdot \vec{dl} = H\rho \int_0^{2\pi} d\phi = H\rho (\phi) \Big|_0^{2\pi}$$

$\Rightarrow$ L.H.S $\Rightarrow \boxed{\oint \vec{H} \cdot \vec{dl} = 2\pi H\rho}$

R.H.S

current enclosed by path is equal to the current in shaded region.

Current density $\times$ Area = Current

$$I_{enclosed} = J_i \times A_{shaded}$$

$$= \frac{I}{\pi a^2} \times \pi \rho^2$$

$\Rightarrow$ R.H.S $\Rightarrow \boxed{I_{enclosed} = \frac{I\rho^2}{a^2}}$

As L.H.S = R.H.S $\Rightarrow \quad 2\pi H\rho = \frac{I\rho^2}{a^2}$

$$H = \frac{I\rho}{2\pi a^2}$$

$\Rightarrow \boxed{\vec{H} = \frac{I\rho}{2\pi a^2} \vec{a_\phi}}$

Case 2 $\quad$ ~~$\rho < a$~~ $\quad a < \rho < b$

Point is located in the dielectric medium of the coaxial cable.

Bilal Register

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The entire inner conductor will be located inside the closed path.

$$\vec{H} = H\vec{a_\phi}$$

$$\oint \vec{H}\cdot d\vec{l} = I_{enclosed}$$

Shaded area is of inner conductor

L.H.S

$$\vec{dl} = \rho\, d\phi\, \vec{a_\phi}$$

$$\vec{H}\cdot\vec{dl} = H\rho\, d\phi \qquad \because \vec{a_\phi}\cdot\vec{a_\phi} = 1$$

$$L.H.S = \boxed{\oint \vec{H}\cdot\vec{dl} = H\rho\int_0^{2\pi} d\phi = H\rho\, 2\pi}$$

R.H.S $\boxed{I_{enclosed} = I}$

$$\Rightarrow H\rho\, 2\pi = I$$

$$\Rightarrow H = \frac{I}{\rho\, 2\pi}$$

$$\Rightarrow \boxed{\vec{H} = \frac{I}{2\pi\rho}\vec{a_\phi}}$$

Case 3 $\quad b < \rho < c$

The point is in the outer conductor.

So the entire inner conductor and a portion of the outer conductor will be enclosed by the Amper path of radius ρ.

Bilal Register

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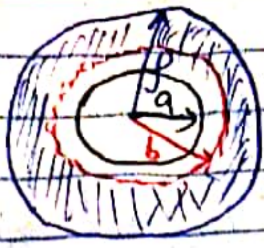


$$\vec{H} = H\vec{a}_\phi$$

$$\oint \vec{H}\cdot\vec{dl} = I_{enclosed}$$

$$\vec{dl} = \rho\, d\phi\, \vec{a}_\phi$$

$$\vec{H}\cdot\vec{dl} = H\rho\, d\phi$$

L.H.S $= \boxed{\oint \vec{H}\cdot\vec{dl} = H\rho \int_0^{2\pi} d\phi = H\rho\, 2\pi}$

R.H.S $\quad I_{enclosed} = I - I_{sh}$

$$I_{sh} = J_0 \times A_{sh} = \frac{I}{\pi(c^2-b^2)} \times \pi(\rho^2-b^2)$$

R.H.S $\quad I_{enclosed} = I - \frac{I(\rho^2-b^2)}{c^2-b^2}$

$$\boxed{= \frac{I(c^2-\rho^2)}{c^2-b^2}}$$

L.H.S = R.H.S $\quad H\rho\, 2\pi = \frac{I(c^2-\rho^2)}{c^2-b^2}$

$$\Rightarrow H = \frac{I}{2\pi\rho}\,\frac{(c^2-\rho^2)}{(c^2-b^2)}$$

$$\Rightarrow \boxed{\vec{H} = \frac{I}{2\pi\rho}\,\frac{(c^2-\rho^2)}{(c^2-b^2)}\,\vec{a}_\phi}$$

Case 4 $\rho > c$

Point is outside the cable.

→ The entire coaxial cable is located inside the closed circular path of radius ρ.

$$\oint \vec{H} \cdot d\vec{l} = I_{enclosed}$$

$$\oint \vec{H} \cdot d\vec{l} = I + (-I) = 0$$

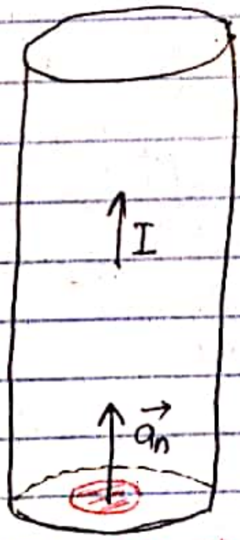
$$\Rightarrow \vec{H} \cdot d\vec{l} = 0$$

$$d\vec{l} \neq 0 \quad \Rightarrow \boxed{\vec{H} = 0}$$

outside the cable.

Maxwell's Second Equation

$$J = \frac{I}{S}$$



$$I = JS$$

$$\vec{J} = J\vec{a_n}$$

$$\vec{S} = S\vec{a_n}$$

$$\boxed{I = \vec{J} \cdot \vec{S}} \quad \because \vec{a_n} \cdot \vec{a_n} = 1$$

Consider a very small portion of area ds.

Area $\vec{S}$ $d\vec{s}, d\vec{I}$

$$d\vec{s} = ds\,\vec{a_n}$$

$$J = \frac{dI}{ds}$$

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$dI = J\,ds = \vec{J}\cdot\vec{ds}$

↳ b/c the vectors are in the same direction. and $\vec{a_n}\cdot\vec{a_n} = 1$

↳ $I = \int \vec{J}\cdot\vec{ds}$

As $\oint \vec{H}\cdot\vec{dl} = I_{enclosed}$

Stokes Theorem: $\oint \vec{H}\cdot\vec{dl} = \int(\nabla\times\vec{H})\cdot\vec{ds}$

If you cannot integrate the L.H.S, then integrate the R.H.S.

The term in bracket on R.H.S is called curl.

Curle of $\vec{H}$ in Rectangular C.S

$$\nabla\times\vec{H} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix}$$

$$\oint \vec{H}\cdot\vec{dl} = \int(\nabla\times\vec{H})\cdot\vec{ds} = \int \vec{J}\cdot\vec{ds}$$

$\Rightarrow$ $\boxed{\nabla\times\vec{H} = \vec{J}}$ incorrect version of Maxwells eq.

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Lecture 3:

04/12/19

8-1 $\Delta \vec{H_2}$ at P_2 caused by $I_1 \Delta \vec{L_1}$ at P_1.

1) $I_1 \Delta \vec{L_1} = 2\pi \times 10^{-6} a_z$ A_m at $P_1(4,0,0)$ and $P_2(0,3,0)$

2) $I_1 \Delta \vec{L_1} = 2\pi \times 10^{-6} a_z$ A_m at $P_1(4,2,-3)$ and $P_2(0,3,0)$

$$\Delta \vec{H_2} = \frac{1}{4\pi} \frac{I_1 \Delta \vec{l_1} \times (\vec{r} - \vec{r_1})}{|\vec{r} - \vec{r_1}|^3}$$

3) $I_1 \Delta L_1 = 2\pi (0.6\vec{a_x} - 0.8\vec{a_y}) 10^{-6}$ A at $P_1(4,2,-3)$
and $P_2(1,3,2)$

$$\Delta \vec{H} = \frac{1}{4\pi} \frac{I_1 \Delta \vec{l_1} \times (\vec{r} - \vec{r_1})}{|\vec{r} - \vec{r_1}|^3} \rightarrow ①$$

$$\vec{r} - \vec{r_1} = -3\vec{a_x} + 5\vec{a_y} - \vec{a_z}$$

$$|\vec{r} - \vec{r_1}| = 5.92 \text{ m}$$

$$① \Rightarrow \Delta \vec{H_2} = \frac{1}{4\pi} \frac{\times 2\pi (0.6\vec{a_x} - 0.8\vec{a_y})(3\vec{a_x} + 5\vec{a_y} - \vec{a_z})}{5.92}$$

$$\Delta \vec{H_2} = 1.93\vec{a_x} + 1.44\vec{a_y} + 1.44\vec{a_z} \text{ nA/m}$$

8.2 $\vec{H}$ at $P(1.5, 2, 3)$ in R.C.S caused by a current of 24 A in $\vec{a_z}$ direction.

Using $\vec{H} = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\vec{a_\phi}$

i) Conductor extends from $z=0$ to $z=6$.

ii) " " " $z=6$ to $z=\infty$

iii) " " " $z=-\infty$ to $z=\infty$

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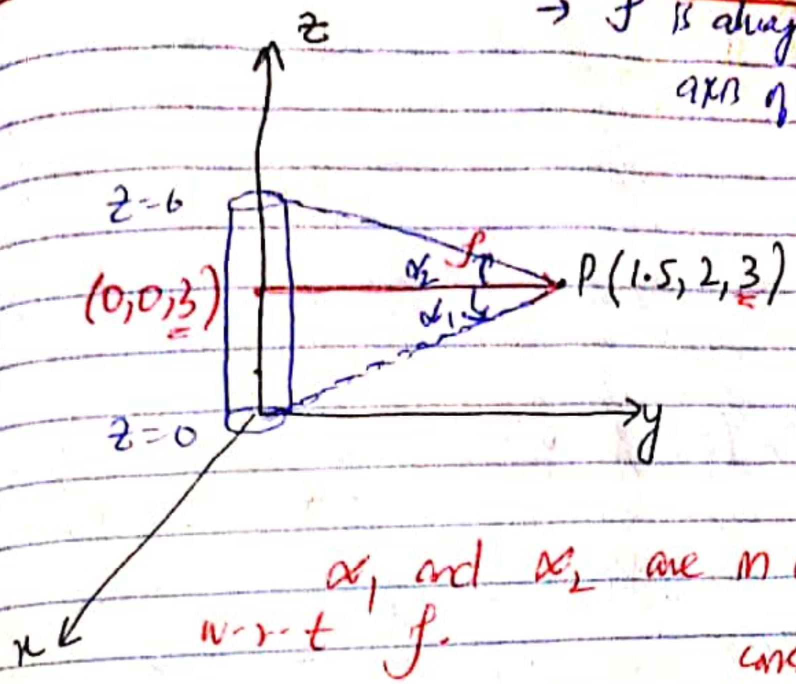
→ ρ is the radial distance b/w axis of conductor and point under observation.

→ ρ is always perpendicular to axis of rotation.



α_1 and α_2 are in clockwise direction w.r.t ρ. $\underbrace{\text{clockwise}}_{\text{considered +ve}}$

If α_1 and α_2 are in anticlockwise direction w.r.t ρ, they are negative.

$$\rho = \sqrt{(1.5-0)^2 + (2-0)^2 + (3-3)^2}$$

$$\Rightarrow \boxed{\rho = 2.5\,m}$$

For α_1, connect the first end of conductor with P.

$$\alpha_1 = -\tan^{-1}\left(\frac{3}{2.5}\right) \Rightarrow \alpha_1 = -50.19^\circ$$

as $\tan\alpha_1 = \frac{Perp = 3}{Base = \rho = 2.5}$

For α_2, connect second end of conductor with P.

$$\alpha_2 = \tan^{-1}\left(\frac{3}{2.5}\right) \Rightarrow \alpha_2 = 50.19^\circ$$

$$\Rightarrow \vec{H} = \frac{24}{4\pi \times 2.5}\left(\sin 50.19^\circ + \sin 50.19^\circ\right)\vec{a_\phi}$$

$$\vec{H} = 1.17\,\vec{a_\phi}\ A/m$$

Bilal Register

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For transformation; first calculate ϕ.

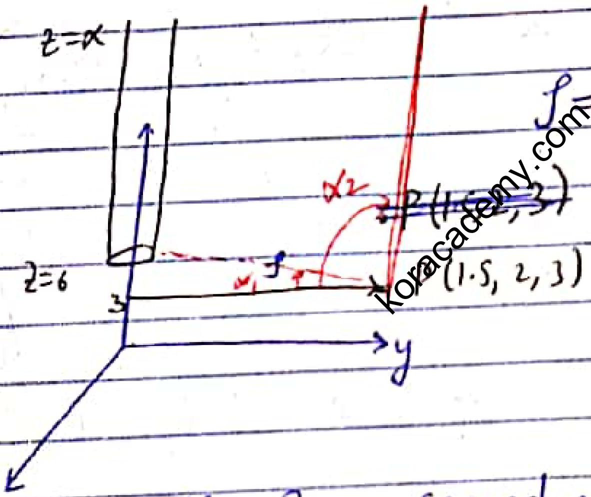
$$\phi = \tan^{-1}\left(\frac{2}{1.5}\right) \Rightarrow \phi = 53.1^\circ$$

as $P(1.5, 2, 3)$

Now
$$\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} \cos 53.1^\circ & -\sin 53.1^\circ & 0 \\ \sin 53.1^\circ & \cos 53.1^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho = 0 \\ A_\phi = 1.17 \\ A_z = 0 \end{bmatrix}$$

$$\Rightarrow \vec{H} = -0.939\vec{a_x} + 0.7025\vec{a_y} \ A/m$$

ii)



$$\rho = \sqrt{(1.5-0)^2 + (2-0)^2 + (3-3)^2}$$

$$\Rightarrow \rho = 2.5m$$

$$\alpha_1 = \tan^{-1}\frac{3}{2.5} = 50.19^\circ$$

$\alpha_2 = ?$ second end $= ?$

In such case draw a line parallel to the conductor i.e a perpendicular at the point P.

Angle w.r.t ρ is α_2.

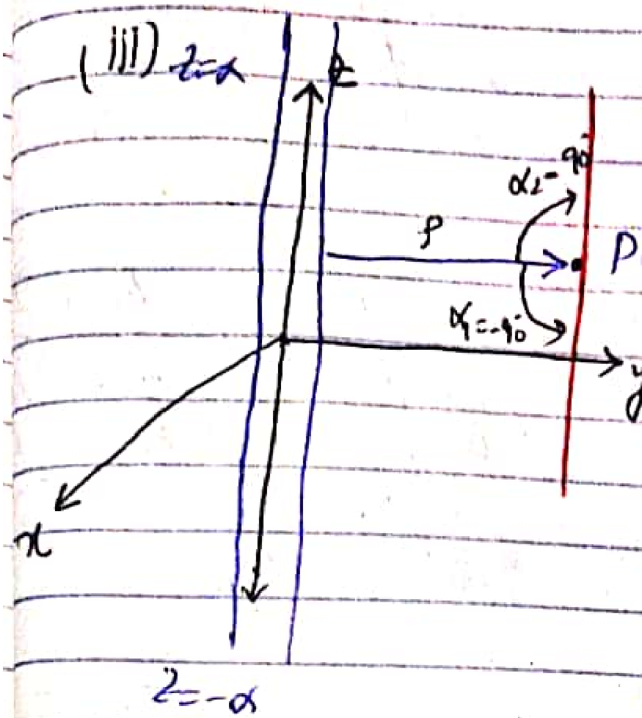
$$\alpha_2 = 90^\circ$$

$$\Rightarrow \vec{H} = \frac{24}{4\pi \times 2.5}(\sin 90^\circ - \sin 50.19) = 0.177\vec{a_\phi} \ A/m.$$

$$\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} \cos 53.1 & -\sin 53.1 & 0 \\ \sin 53.1 & \cos 53.1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho = 0 \\ A_\phi = 0.177 \\ A_z = 0 \end{bmatrix}$$

$$\vec{H} = 0.141\,\vec{a_x} + 0.1602\,\vec{a_y} \text{ Am}.$$

(iii)



$$\vec{H} = \frac{I}{4\pi\rho}(\sin 90^\circ + \sin 90^\circ)\,\vec{a_\phi}$$

$$\Rightarrow \vec{H} = \frac{I}{2\pi\rho}\,\vec{a_\phi}$$

$$= \frac{24}{2 \times 2.5 \times \pi}$$

$$\Rightarrow \vec{H} = 1.527\,\vec{a_\phi} \text{ A/m}$$

$$\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} \cos 53.1 & -\sin 53.1 & 0 \\ \sin 53.1 & \cos 53.1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho = 0 \\ A_\phi = 1.527 \\ A_z = 0 \end{bmatrix}$$

$$\Rightarrow \vec{H} = -1.22\,\vec{a_x} + 0.917\,\vec{a_y} \text{ A/m}$$

If a conductor extends along z axis → the magnetic field does not have z component. Similarly for x and y axis.

8.3 $\vec{H}$ at $P(0.01, 0, 0)$ in R.C.S

(a) Two conductors.

→ $I = 0.08$ A in the $\vec{a_z}$ direction in the conductor on the z axis

→ $I = 0.08$ A in the $-\vec{a_z}$ direction in conductor located at $x = 0.015$ and $y = 0$

(b) Coaxial cable $a = 3$mm $b = 9$mm $c = 12$mm. There is a current of 0.8A in the central conductor in the $\vec{a_z}$ direction.

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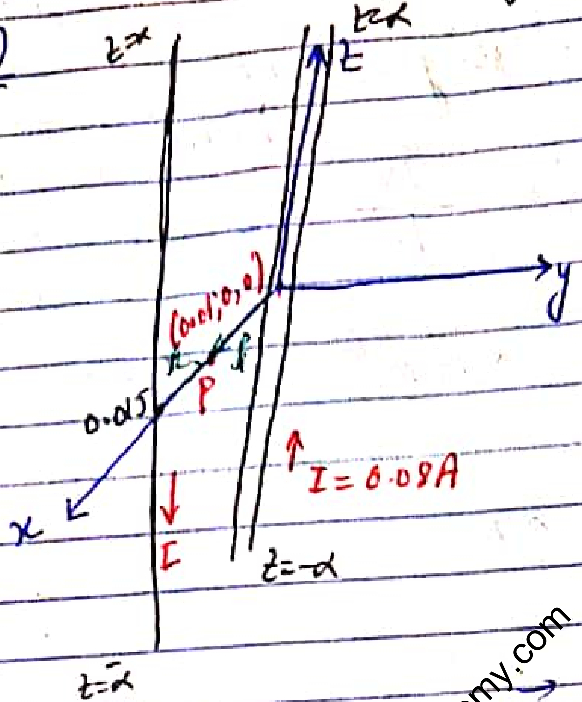
(c) Three sheets $1.5\pi \vec{a_y}$ A/m at $x = -6$mm.
$-3\pi \vec{a_y}$ A/m at $x = 9$mm
$1.5\pi \vec{a_y}$ A/m at $x = 12$mm

(i)



$$\vec{H_1} = \frac{I}{2\pi \rho_1} \vec{a_\phi}$$

$$\vec{H_1} = \frac{0.08}{2\pi \times 0.01} \vec{a_\phi} = a_y$$

$$\vec{H_2} = \frac{I}{2\pi \rho_2} \vec{a_\phi}$$

$$\rho_2 = 0.015 - 0.01 = 0.005$$

$$\vec{H_2} = \frac{0.08}{2\pi \times 0.05} \vec{a_\phi} \text{ A/m} = \vec{a_y}$$

Superposition

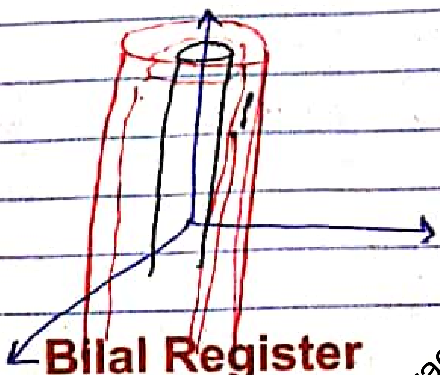
$$\Rightarrow \vec{H} = \vec{H_1} + \vec{H_2} = 3.81 \vec{a_\phi} \text{ A/m}$$
$= a_y$

Right hand rule → thumb = current, fingers curl 90°
↳ if point is in a plane. & finger tips touch P.

Transformation matrix may work in this question

(ii) Point is in the outer conductor.

Eq derived was as $\vec{H} = \frac{I}{2\pi\rho} \frac{(c^2 - \rho^2)}{(c^2 - b^2)} \vec{a_\phi}$



$$\rho = 0.01$$

$$\vec{H} = \frac{0.8}{2\pi \times 0.01} \left(\frac{144 - 100}{144 - 81}\right) \vec{a_\phi}$$

$$\Rightarrow \vec{H} = 8.89 \vec{a_\phi} = 8.89 \vec{a_y}$$

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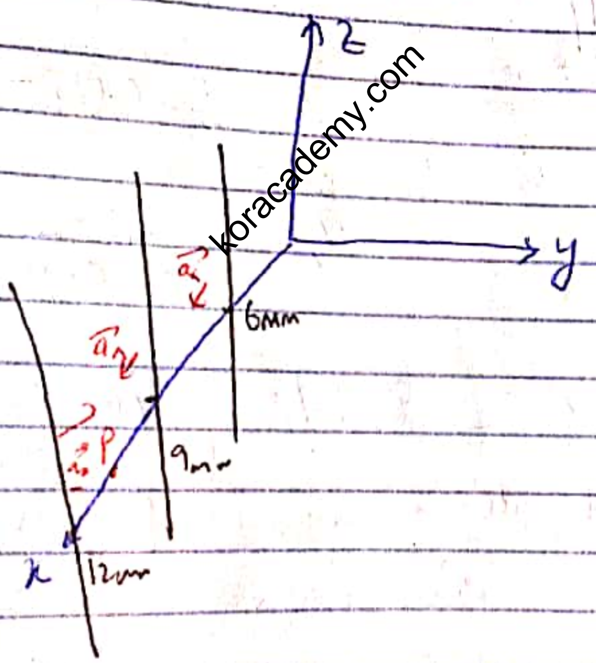
(III) Magnetic field intensity due to infinite sheet of current is $\vec{H} = \frac{1}{2} \vec{K} \times \vec{a_n}$

Surface current density = $\frac{\text{current}}{\text{width}}$

$\Rightarrow K = \frac{I}{b}$ A/m

$\vec{a_n} \rightarrow$ normal unit vector



$\vec{H_1} = \frac{1}{2} \times 1.5\pi \, \vec{a_y} \times \vec{a_x} = -2.35 \vec{a_z}$ A/m

$\vec{H_2} = \frac{1}{2} \times -3\pi \, \vec{a_y} \times \vec{a_x} = 4.7 \vec{a_z}$ A/m

$\vec{H_3} = \frac{1}{2} \times 1.5 \vec{a_y} \times -\vec{a_x} = 2.35 \vec{a_z}$ A/m

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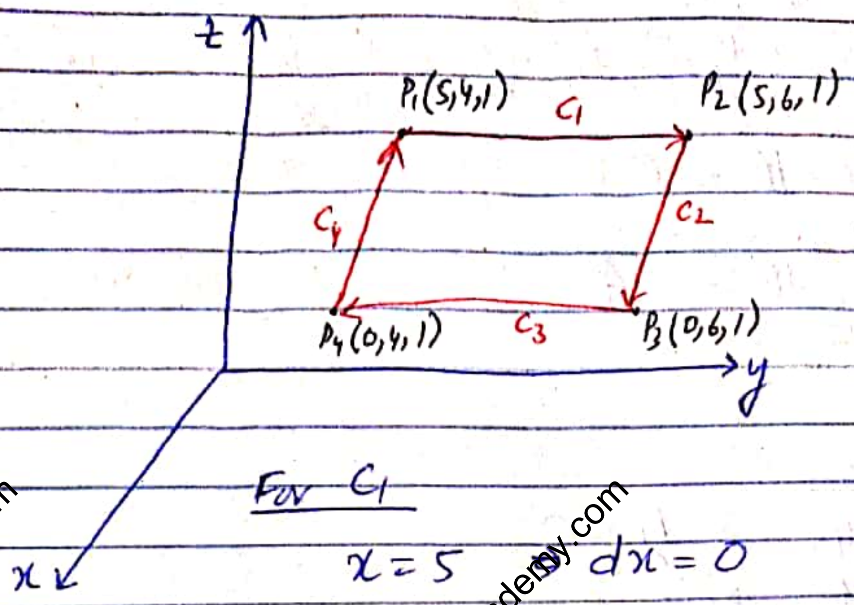
8.4 $\vec{H} = 0.1y^3\vec{a_x} + 0.4x\vec{a_z}$ A/m in a region.

Closed path starting from $P_1(5,4,1)$ to $P_2(5,6,1)$ to $P_3(0,6,1)$ to $P_4(0,4,1)$ to $P_1(5,4,1)$



$$\oint \vec{H}\cdot\vec{dl} = ?$$

For C_1

$$x = 5 \Rightarrow dx = 0$$
$$z = 1 \Rightarrow dz = 0$$
$$4 \le y \le 6$$

$$\vec{dl} = dx\,\vec{a_x} + dy\,\vec{a_y} + dz\,\vec{a_z} = dy\,\vec{a_y}$$

$$\vec{H}\cdot\vec{dl} = 0$$

$$\Rightarrow \int_{C_1} \vec{H}\cdot\vec{dl} = 0 \text{ Ampere.}$$

For C_2

$$y = 6 \Rightarrow dy = 0$$
$$z = 1 \Rightarrow dz = 0$$
$$0 \le x \le 5 \qquad \Rightarrow \vec{dl} = dx\,\vec{a_x}$$

$$\vec{H}\cdot\vec{dl} = 0.1y^3\,dx = 21.6\,dx$$

$$\int_{C_2} \vec{H}\cdot\vec{dl} = \int_5^0 21.6\,dx = -108 \text{ A}$$

Bilal Register

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For C_3

$x=0 \Rightarrow dx=0$

$z=1 \Rightarrow dz=0$

$4 \le y \le 6$

$$\vec{dl} = dx\,\vec{a_x} + dy\,\vec{a_y} + dz\,\vec{a_z} = dy\,\vec{a_y}$$

$$\vec{H}\cdot\vec{dl} = 0$$

$$\int_{C_3} \vec{H}\cdot\vec{dl} = 0A$$

For C_4

$y=4 \Rightarrow dy=0$

$z=1 \Rightarrow dz=0$

$0 \le x \le 5$

$$\vec{dl} = dx\,\vec{a_x} + dy\,\vec{a_y} + dz\,\vec{a_z} = dx\,\vec{a_x}$$

$$\vec{H}\cdot\vec{dl} = 0.1y^3\,dx = 0.1\times 64\,dx = 6.4\,dx$$

$$\int_{C_4} \vec{H}\cdot\vec{dl} = 6.4\int_0^5 dx = 32A$$

$$\Rightarrow \oint \vec{H}\cdot\vec{dl} = \int_{C_1} + \int_{C_2} + \int_{C_3} + \int_{C_4}$$

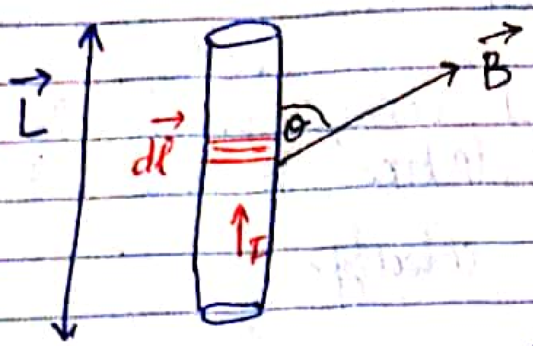
$$= 0 - 108 + 0 + 32$$

$$\Rightarrow \oint \vec{H}\cdot\vec{dl} = -76A$$

Ans

Lecture 4 09/10/19

Force On a Current Carrying Conductor



$F \propto ILB \sin\theta$

$k = 1$

$F = ILB \sin\theta$

direction of $\vec{F}$?

$\vec{L} \times \vec{B} = LB \sin\theta$

$$\boxed{\vec{F} = I(\vec{L} \times \vec{B})}$$

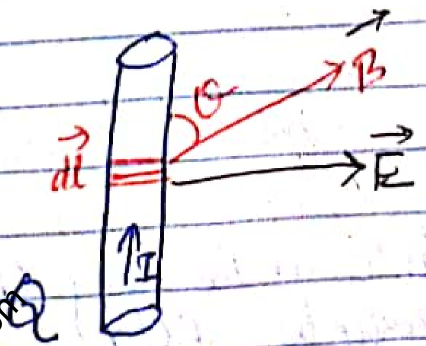
For small portion of conductor in given magnetic field

$$d\vec{F} = I\, \vec{dl} \times \vec{B}$$

For total force; integrate

$$\boxed{\vec{F} = \int I\, \vec{dl} \times \vec{B}}$$

Force On A Moving Charge In A Magnetic field



The current in this conductor is due to free charge in it (Q).

so $I = \frac{Q}{t} = \frac{dQ}{dt}$

Multiply both sides by dl

$$I\,\vec{dl} = \frac{dQ}{dt}\,\vec{dl}$$

$$I\,\vec{dl} = dQ \cdot \frac{\vec{dl}}{dt}$$

Say the differential charge $\to dQ$ inside the conductor covers $\vec{dl}$ distance in time dt
so $\frac{\vec{dl}}{dt}$ gives the velocity.

And we assume that the charge is moving with a uniform velocity.

$$\Rightarrow I\,\vec{dl} = dQ \times \vec{V}$$

As $\vec{F} = \int I\,\vec{dl} \times \vec{B}$

$$\Rightarrow \vec{F} = \int dQ\,(\vec{V} \times \vec{B})$$

As $\vec{V}$ and $\vec{B}$ are assumed to be uniform.

$$\int dQ = Q_{\text{total free charge}}$$

$$\Rightarrow \boxed{\vec{F} = Q(\vec{V} \times \vec{B})}$$

Now assume that this conductor is located in an electric field as well
↳ so two sources of force on the moving charge.
(i) electric field (ii) magnetic field

Apply superposition Theorem;

$\vec{E}=0 \Rightarrow \vec{F_m} = Q(\vec{V} \times \vec{B})$

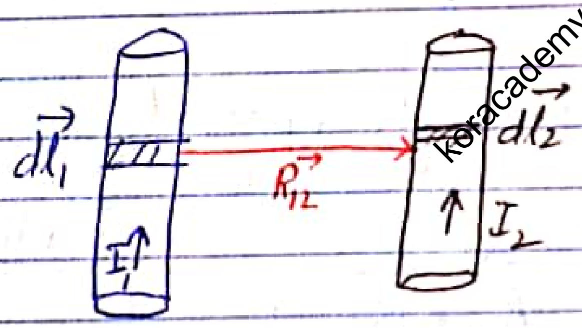
$\vec{B}=0 \Rightarrow \vec{F_E} = Q\vec{E}$

Total force; $\vec{F} = \vec{F_m} + \vec{F_E}$

$$\Rightarrow \boxed{\vec{F} = Q[\vec{E} + (\vec{V} \times \vec{B})]}$$

Lorentz force equation.

Force B/w Two Differential Current Elements



They both will produce a magnetic field

we assume that both of them are present in the magnetic field of each other

↳ force.

Attraction if both currents are parallel.
Repulsion if both currents are in opposite direction

Biot Savart law;

$$d\vec{B_1} = \frac{\mu_0}{4\pi} \frac{I_1 \vec{dl_1} \times \vec{R_{12}}}{R_{12}^3} \rightarrow (1)$$

$$d(d\vec{F_2}) = I_2 \vec{dl_2} \times d\vec{B_1}$$

Bilal Register

Similarly

$$\vec{dB_2} = \frac{\mu}{4\pi} \frac{I_2 \, \vec{dl_2} \times \vec{R_{21}}}{R_{21}^3} \rightarrow (2)$$

Force experienced by small portion of first conductor;

$$d(d\vec{F_1}) = I_1 \, \vec{dl_1} \times d\vec{B_2}$$

Total Magnetic flux density generated by current in first conductor;

Integrating (1) ⇒

$$\vec{B_1} = \frac{\mu}{4\pi} \int \frac{I_1 \, \vec{dl_1} \times \vec{R_{12}}}{R_{12}^3}$$

Differential force experienced by small portion of second conductor due to the entire first conductor

$$d\vec{F_2} = I_2 \, \vec{dl_2} \times \vec{B_1}$$

Putting value of $\vec{B_1}$.

$$d\vec{F_2} = I_2 \, \vec{dl_2} \times \frac{\mu}{4\pi} \int \frac{I_1 \, \vec{dl_1} \times \vec{R_{12}}}{R_{12}^3}$$

For total force on second conductor; integrate;

$$\vec{F_2} = \int I_2 \, \vec{dl_2} \times \frac{\mu}{4\pi} \int \frac{I_1 \, \vec{dl_1} \times \vec{R_{12}}}{R_{12}^3}$$

Total magnetic flux density produced by the current in the entire second conductor

Integrate ⑥ ; $\vec{B_2} = \frac{\mu}{4\pi} \int \frac{I_2 \vec{dl_2} \times \vec{R_{21}}}{R_{21}^3}$

Diff force experienced by small portion of first conductor due to mag field of entire second conductor.

$$d\vec{F_1} = I_1 \vec{dl_1} \times \vec{B_2}$$

Putting value of B_2

$$d\vec{F_1} = I_1 \vec{dl_1} \times \frac{\mu}{4\pi} \int \frac{I_1 \vec{dl_1} \times \vec{R_{21}}}{R_{21}^3}$$

For total force on first conductor; integrate;

$$\vec{F_1} = \int I_1 \vec{dl_1} \times \frac{\mu}{4\pi} \int I_1 \frac{\vec{dl_1} \times \vec{R_{21}}}{R_{21}^3}$$

Right hand rule → Thumb = current, fingers = ~~ed~~ magnetic flux
↓
curled

Region b/w the two conductors is the region of lower magnetic potential
↳ opposite direction → cancel effect of ~~effect~~ each other.

Outside the conductors → higher magnetic potential

Everything moves from high to low potential so they are attracted towards each other.

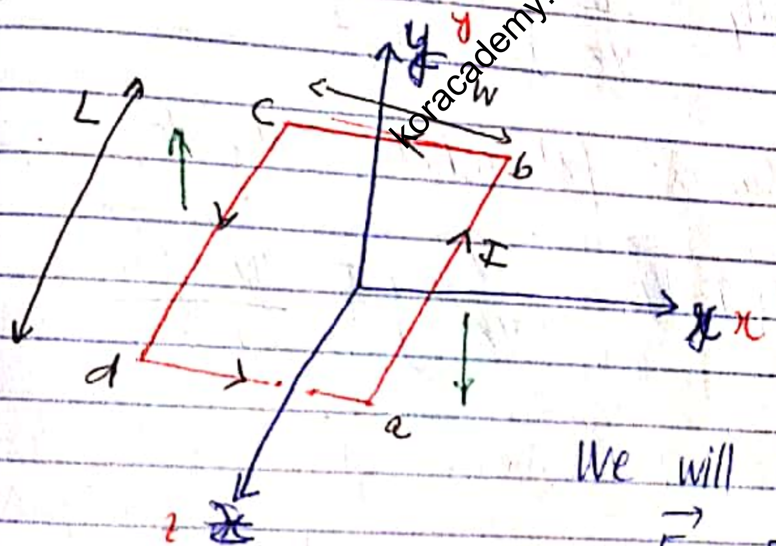
If the currents are in opposite direction

↓ magnetic field reinforce each other b/w the two conductors.

So Repel each other.

Force On A Current Carrying Loop

Current carrying loop is located in the $y=0$ plane.
↳ it is free to rotate around z axis.
↳ is axis of the loop



$\vec{B} = B\vec{a}_x$

length $= L$
Width $= W$

We will use this equation
$$\vec{F} = I\vec{L}\times\vec{B}$$

Consider length vector in the direction of current.

Side ab

$$\vec{F}_{ab} = I\vec{L}\times\vec{B}$$

$$\vec{F}_{ab} = I \times -L\vec{a}_z \times B\vec{a}_x$$

$$\vec{F}_{ab} = -BIL\,\vec{a}_y$$

Side BC

$$\vec{F}_{BC} = I\vec{L}\times\vec{B}$$

$$= I \times -W\vec{a_x} \times B\vec{a_x} \Rightarrow \vec{F_{BC}} = 0$$

As $\vec{a_x}\times\vec{a_x} = 0$

Side CD

$$\vec{F}_{CD} = I\vec{L}\times\vec{B}$$

$$= I \times L\vec{a_z} \times B\vec{a_x}$$

$$\Rightarrow \vec{F}_{CD} = BIL\,\vec{a_y}$$

Side DA

$$\vec{F}_{dA} = I\vec{L}\times\vec{B} = IW\vec{a_x}\times\vec{a_x}$$

$$\Rightarrow \vec{F}_{da} = 0\,N \qquad \text{As } \vec{a_x}\times\vec{a_x} = 0$$

So in the presence of these two forces, the current carrying loop will rotate in the clockwise direction about the z axis. with a uniform angular velocity.

$\Rightarrow$ it will be in a state of equilibrium

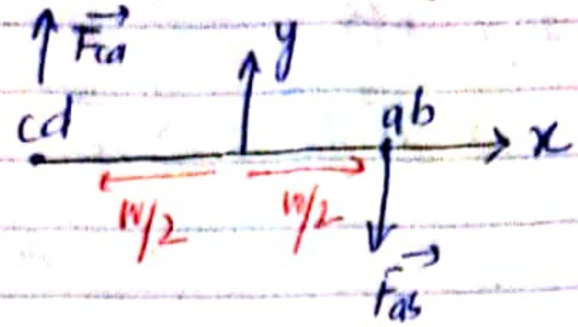
Equilibrium $\Rightarrow$ Net force on object $= 0$

Total force on loop, $\vec{F_T} = \vec{F_{ab}} + \vec{F_{bc}} + \vec{F_{cd}} + \vec{F_{da}}$

$$= -BIL\vec{a_y} + 0 + BIL\vec{a_y} + 0$$

$$\Rightarrow F_T = 0$$

Consider the front view of the loop.



Cross product of moment arm and force is torque.

$$\vec{\tau}_{ab} = \frac{W}{2}\vec{a}_x \times \vec{F}_{ab} = \frac{1}{2}W\vec{a}_x \times -BIL\vec{a}_y$$

$$\vec{\tau}_{ab} = -\frac{1}{2}BILW\vec{a}_z$$

$$\vec{\tau}_{bc} = \text{Moment arm} \times \underbrace{F_{bc}}_{0} \Rightarrow \vec{\tau}_{bc} = 0$$

$$\vec{\tau}_{cd} = -\frac{W}{2}\vec{a}_x \times \vec{F}_{cd} = -\frac{1}{2}W\vec{a}_x \times BIL\vec{a}_y$$

$$\Rightarrow \vec{\tau}_{cd} = -\frac{1}{2}BILW\vec{a}_z$$

$$\vec{\tau}_{da} = \text{Moment arm} \times \underbrace{F_{da}}_{0} \Rightarrow \vec{\tau}_{da} = 0$$

Total torque on the loop;

$$\vec{\tau} = \vec{\tau}_{ab} + \vec{\tau}_{bc} + \vec{\tau}_{cd} + \vec{\tau}_{da}$$

$$\Rightarrow \boxed{\vec{\tau} = -BILW\vec{a}_z \ N.m}$$

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We can also calculate torque with this equation;

$$\boxed{\vec{T} = \vec{m} \times \vec{B}}$$

$\vec{m} \rightarrow$ dipole moment $\qquad \boxed{\vec{m} = I\vec{S}}$

current . area

$\vec{m} = ILW \vec{a_y}$

$\vec{T} = ILW \vec{a_y} \times B\vec{a_x} = -BILW\vec{a_z}$ N.m

Lecture 5: 11/12/19

9-1 $\quad Q = -40 \times 10^{-9}$ C $\qquad V = 6 \times 10^{6}$ m/s

$\vec{a_V} = -0.48\vec{a_x} - 0.6\vec{a_y} + 0.64\vec{a_z}$

$\vec{B} = 2\vec{a_x} - 3\vec{a_y} + 5\vec{a_z}$ mT

i) $\vec{F_m} = ?$ $\qquad \vec{E} = 2\vec{a_x} - 3\vec{a_y} + 5\vec{a_z}$ kV/m

ii) $\vec{F_E} = ?$ $\qquad$ iii) $\vec{F_T} = ?$ due to $\vec{E}$ and $\vec{H}$.

$\vec{V} = V\vec{a_V} = (-2.88\vec{a_x} - 3.6\vec{a_y} + 3.84\vec{a_z}) \times 10^{6}$ m/s

i) $\vec{F_M} = Q(\vec{V} \times \vec{B}) = (-40 \times 10^{-9} \times 10^{6} \times 10^{-3}) \times$

$$\begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ -2.88 & -3.6 & 3.84 \\ 2 & -3 & 5 \end{vmatrix}$$

$\vec{F_m} = 259.2\vec{a_x} + 883.7\vec{a_y} - 633.6\vec{a_z} \times 10^{-6}$ N

$F_m = 1117.44$ μN

ii) $\vec{F_E} = Q\vec{E} = -40 \times 10^{-9} \times 10^{3} (2\vec{a_x} - 3\vec{a_y} + 5\vec{a_z})$

$\Rightarrow \vec{F} = (-80\vec{a_x} + 120\vec{a_y} - 200\vec{a_z}) \times 10^{-6}$ N

Bilal Register

$F_E = 246.57\ \mu N.$

iii) $\vec{F}_T = \vec{F}_E + \vec{F}_m = 179.2\vec{a}_x - 763.7\vec{a}_y - 833.6\vec{a}_z\ \mu N.$

$\Rightarrow F_T = 1444\ \mu N.$

9.4 Two current carrying conductors.

$I_1\Delta\vec{L}_1 = 10^{-5}\vec{a}_z$ Am at $P_1(1,0,0)$

$I_2\Delta\vec{L}_2 = (0.6\vec{a}_x - 2\vec{a}_y + 3\vec{a}_z)10^{-5}$ Am at $P_2(-1,0,0)$

$\Delta(\Delta F_2) = ? \qquad \Delta(\Delta F_1) = ?$

$$\Delta\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{I_1\Delta\vec{L}_1 \times \vec{R}_{12}}{R_{12}^3}$$

$\mu_0 = 4\pi \times 10^{-7}$

$\vec{R}_{12} = \vec{r}_2 - \vec{r}_1 = -2\vec{a}_x \Rightarrow |\vec{R}_{12}| = 2m$

$$\Rightarrow \Delta\vec{B}_1 = \frac{4\pi\times 10^{-7}}{4\pi}\frac{(10^{-5}\vec{a}_z \times -2\vec{a}_x)10^{-7}}{8}$$

$\Rightarrow \Delta\vec{B}_1 = -2.5\times 10^{-13}\vec{a}_y\ T$

$\Delta(\Delta\vec{F}_2) = I_2\Delta\vec{L}_2 \times \Delta\vec{B}_1$

$= (0.6\vec{a}_x - 2\vec{a}_y + 3\vec{a}_z)10^{-5} \times (-2.5\times 10^{-13}\vec{a}_y)$

$\Rightarrow \Delta(\Delta\vec{F}_2) = (7.5\vec{a}_x - 1.5\vec{a}_z)10^{-18}$ N.

For $\Delta(\Delta F_1)$ calculate ΔB_2 first;

$$\Delta \vec{B_2} = \frac{\mu_0}{4\pi} \frac{I_2 \Delta \vec{L_2} \times \vec{R_{21}}}{R_{21}^3}$$

$\mu_0 = 4\pi \times 10^{-7}$ $\quad \vec{R_{21}} = \vec{r_1} - \vec{r_2} = 2\vec{a_x}$ $\quad R_{21} = 2m$

$$\Rightarrow \Delta B_2 = \frac{4\pi \times 10^{-7}}{4\pi} \frac{(0.6\vec{a_x} - 2\vec{a_y} + 3\vec{a_z})10^{-5} \times 2\vec{a_x}}{(2)^3}$$

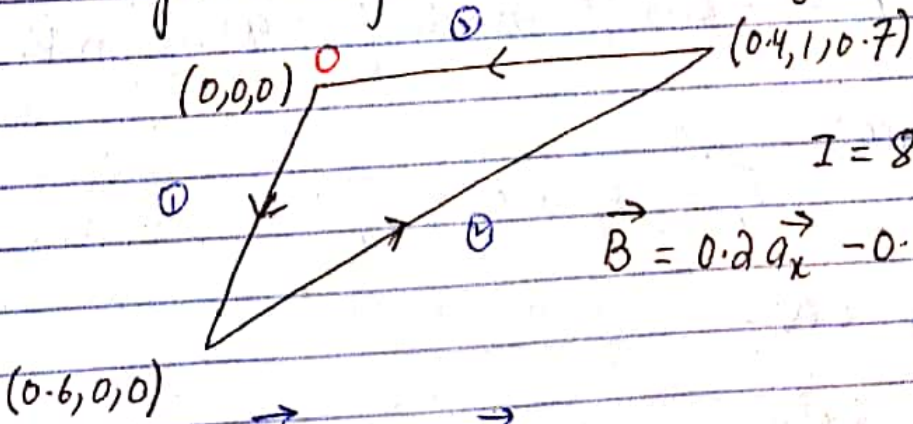
$$\Rightarrow \Delta B_2 = (7.5\vec{a_y} + 5\vec{a_z})10^{-13} \ Wb/m^2 \text{ or } T.$$

$$\Delta(\Delta \vec{F_1}) = I_1 \Delta \vec{L_1} \times \Delta B_2$$

$$= 10^{-5}\vec{a_z} \times (7.5\vec{a_y} + 5\vec{a_z})10^{-13}$$

$$\Rightarrow \Delta(\Delta \vec{F_1}) = -7.5 \times 10^{-18} \vec{a_x} \ N.$$

9.5 Current carrying loop located in a constant magnetic field. (loop → triangular.)



$I = 8mA$

$\vec{B} = 0.2\vec{a_x} - 0.1\vec{a_y} + 0.2\vec{a_z}$ T

$$\vec{L_1} = 0.6\vec{a_x}$$

$$\vec{F_1} = I\vec{L_1} \times \vec{B} = 8 \times 10^{-3} \times 0.6\vec{a_x} \times (0.2\vec{a_x} - 0.1\vec{a_y} + 0.2\vec{a_z})$$

$$\Rightarrow \vec{F_1} = -0.96\vec{a_y} - 0.48\vec{a_z} \ mN.$$

Length of second line segment in the direction of the current.

$$\vec{L_2} = -0.2\vec{a_x} + \vec{a_y} + 0.7\vec{a_z}$$

$$\vec{F_2} = I\vec{L_2} \times \vec{B_x}$$

$$= 8\times10^{-3} \times (-0.2\vec{a_x} + \vec{a_y} + 0.7\vec{a_z}) \times (0.2\vec{a_x} - 0.1\vec{a_y} + 0.2\vec{a_z})$$

$$\Rightarrow \vec{F_2} = 2.16\,\vec{a_x} + 1.44\vec{a_y} - 1.44\vec{a_z} \text{ mN.}$$

Similarly $\vec{L_3} = -0.4\vec{a_x} - \vec{a_y} - 0.7\vec{a_z}$

$$\vec{F_3} = I\vec{L_3} \times \vec{B}$$

$$= 8\times10^{-3}(-0.4\vec{a_x} - \vec{a_y} - 0.7\vec{a_z}) \times (0.2\vec{a_x} - 0.1\vec{a_y} + 0.2\vec{a_z})$$

$$\Rightarrow \vec{F_3} = -2.16\vec{a_x} - 0.48\vec{a_y} + 1.92\vec{a_z} \text{ mN.}$$

Total force; $\vec{F_T} = F_1 + F_2 + F_3$

$$\Rightarrow \vec{F_T} = 0N.$$

Torque on the loop.

$$\vec{T} = \vec{m} \times \vec{B}$$

As $\vec{m} = I\vec{S}$

$$\vec{S} = \frac{1}{2}(\vec{L_1} \times \vec{L_3})$$

L_1 and L_3 in reference to an origin.

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$$\vec{S} = \frac{1}{2}\begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 0.6 & 0 & 0 \\ 0.4 & 1 & 0.7 \end{vmatrix} = 0\vec{a_x} - 0.21\vec{a_y} + 0.3\vec{a_z}$$

$$\vec{m} = I\vec{S} = (-0.21\vec{a_y} + 0.3\vec{a_z})\, 8\times10^{-3}$$

$$\vec{T} = \vec{m}\times\vec{B} = 8\times10^{-3}\begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 0 & -0.21 & 0.3 \\ 0.2 & -0.1 & 0.2 \end{vmatrix}$$

$$\Rightarrow \vec{T} = -0.096\vec{a_x} + 0.048\vec{a_y} + 0.336\vec{a_z} \text{ Nm.}$$

Example $I_1 \vec{dl_1} = -3\vec{a_y}$ Am at $P_1(5,2,1)$

$I_2 \vec{dl_2} = -4\vec{a_z}$ Am at $P_2(1,8,5)$.

$d(dF_2) = ?$

$$d(d\vec{F_2}) = I_2 \vec{dl_2} \times d\vec{B_1} \rightarrow ①$$

$$d\vec{B_1} = \frac{\mu_0}{4\pi}\,\frac{I_1\vec{dl_1}\times\vec{R_{12}}}{R_{12}^3}$$

$$\vec{R_{12}} = \vec{r_2} - \vec{r_1} = -4\vec{a_x} + 6\vec{a_y} + 4\vec{a_z}$$

$$|\vec{R_{12}}| = 8.24 \text{ m}$$

$$\Rightarrow \frac{4\pi\times10^{-7}}{4\pi} \times \frac{-3\vec{a_y} \times (-4\vec{a_x} + 6\vec{a_y} + 4\vec{a_z})}{(8.24)^3}$$

$$d\vec{B_1} = (-2.13\vec{a_x} - 2.13\vec{a_z})\times10^{-10} \text{ T}$$

$$① \Rightarrow d(dF_2) = -4\vec{a_z} \times (-2.13\vec{a_x} - 2.13\vec{a_z})\times10^{-10}$$

$$\Rightarrow d(dF_2) = 8.54\times10^{-9}\,\vec{a_y} \text{ N.}$$

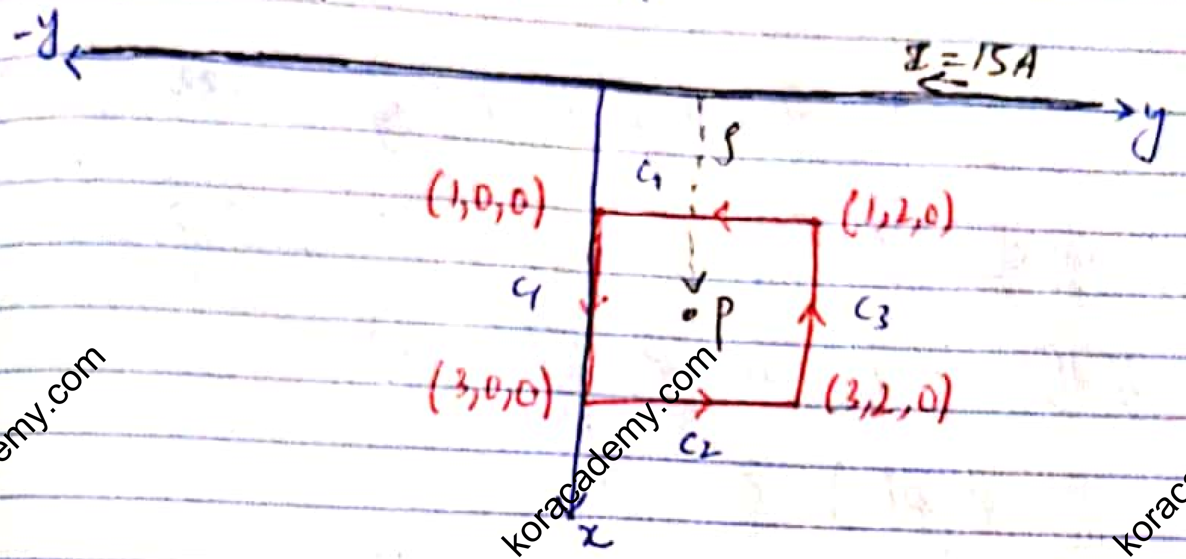
Example

Current carrying conductor along y axis of 15A. moving in the direction of $-\vec{a_y}$.

Conductor extending from $-\infty$ to ∞ on y axis.

Current in conductor will create a magnetic field.

In the magnetic field, we have a current carrying loop having $I_1 = 2mA$



As $\vec{H} = \dfrac{I}{2\pi\rho}\vec{a_\phi}$

Consider point P at distance ρ.

To rectangular system;

$$\rho = x \qquad \vec{a_\phi} \Leftrightarrow \vec{a_z}$$

$$\Rightarrow \vec{H} = \frac{I}{2\pi x}\vec{a_z}$$

$$\vec{B} = \mu_0\vec{H} = \frac{4\pi\times10^{-7}\times15}{2\pi x}\vec{a_z} = \frac{3\times10^{-6}}{x}\vec{a_z}\ T.$$

As x is changing so we consider different segments.

Bilal Register

C_1 $\quad \vec{dl_1} = dx\, \vec{a_x} \quad ; \quad y = 0$

$$\vec{dF_1} = I_1 \vec{dl_1} \times \vec{B} \Rightarrow \vec{F_1} = \int I_1 \vec{dl_1} \times \vec{B}$$

$$= 2\times10^{-3} \times 3\times10^{-6} \int_1^3 \frac{dx\, \vec{a_x} \times \vec{a_z}}{x}$$

$$\Rightarrow \vec{F_1} = -6\times10^{-9} \ln(x|_1^3\, \vec{a_y}\ N.$$

C_2 $\quad \vec{dl_2} = dy\, \vec{a_y} \quad ; \quad x = 3$

$$\vec{F_2} = \int I_1 \vec{dl_2} \times \vec{B} = 2\times10^{-3} \times \frac{3\times10^{-6}}{3} \int_0^2 dy\, \vec{a_y} \times \vec{a_z}$$

$$\Rightarrow \vec{F_2} = 4\times10^{-9}\, \vec{a_x}\ N.$$

C_3

$$\vec{dl_3} = dx\, \vec{a_x}$$

$$\vec{dF_3} = I \vec{dl_3} \times \vec{B} \qquad \vec{F_3} = \int I_1 \vec{dl_3} \times \vec{B}$$

$$= 2\times10^{-3} \times 3\times10^{-6} \int_3^1 \frac{dx}{x}\, \vec{a_x} \times \vec{a_z}$$

$$\Rightarrow \vec{F_3} = -\vec{F_1} = \_\_\_\_$$

C_4 $\quad \vec{dl_4} = dy\, \vec{a_y} \quad ; \quad x = 1$

$$\vec{F_4} = \int \vec{dl_4} \times \vec{B} = 2\times10^{-3} \times \frac{3\times10^{-6}}{1} \int_2^0 dy\, \vec{a_y} \times \vec{a_z}$$

$$\Rightarrow \vec{F_4} = -12\times10^{-9}\, \vec{a_x}\ N.$$

$$\vec{F_T} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} = -8\, \vec{a_x}\ nN.$$

The loop is attracted by conductor carrying current.

Bilal Register

Lecture 6

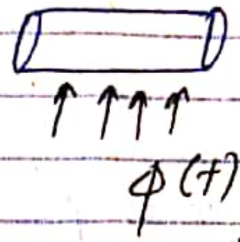
16/12/19

Chapter 10

Maxwell's 4th Equation

Consider a conductor in time varying magnetic field → so voltage will be induced as according to ampere's law

We know that



$$\phi = \int \vec{B} \cdot \vec{ds}$$

$$V = -\frac{d\phi}{dt}$$

Differentiate w.r.t time

$$\frac{d\phi}{dt} = \int \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$$

↳ partial integration is used b/c $\vec{B}$ might be a function of other parameters such as x, y, z or r, θ, ρ, r, θ, t etc.

$$V = -\frac{d\phi}{dt} = \int -\frac{\partial \vec{B}}{\partial t} \cdot \vec{ds} \rightarrow (A)$$

Also $V = \oint \vec{E} \cdot \vec{dl} \rightarrow (B)$

comparing (A) and (B)

$$\oint \vec{E} \cdot \vec{dl} = \oint -\frac{d\vec{B}}{dt} \cdot \vec{ds} \rightarrow (1)$$

Apply Stokes' theorem on L.H.S of (1)

$$\hookrightarrow \oint \vec{E} \cdot \vec{dl} = \int (\nabla \times \vec{E}) \cdot \vec{ds}$$

$(\nabla \times \vec{E})$ is curl of electric field intensity.

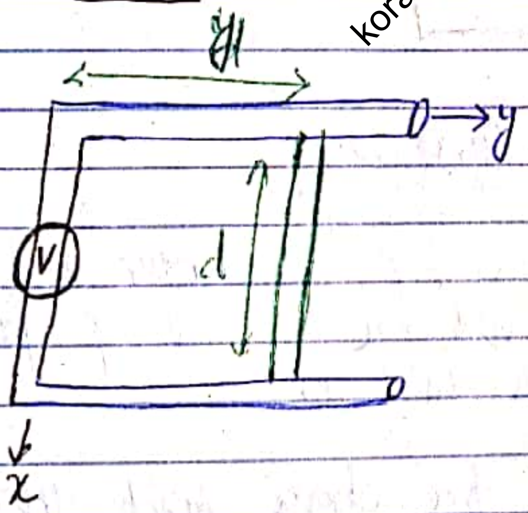
Curl in cylindrical system

$$\nabla \times \vec{E} = \frac{1}{\rho} \begin{vmatrix} \vec{a}_\rho & \rho\vec{a}_\phi & \vec{a}_z \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ E_\rho & \rho E_\phi & E_z \end{vmatrix}$$

① ⇒ $$\int (\nabla \times \vec{E}) \cdot d\vec{s} = \int -\frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

↳ A time varying magnetic field can create a time varying electric field.



Consider a conductor along y axis.
Another along x axis
voltmeter is also placed
Another conductor parallel to y axis.
Movable conductor

$y \rightarrow$ initial position of movable conductor.
$d \rightarrow$ separation b/w two conductors.

<u>Rail</u> → this whole arrangement is called rail.
↳ this arrangement is present in a constant magnetic field.
Move with uniform velocity V. along y axis.

Date: / /

Acurding to Faraday's law

$$V = -\frac{d\phi}{dt}$$

The magnetic flux linking the area $(S) = yd$ is given as;

$$\phi = B \cdot S = Byd$$

$$\Rightarrow V = -\frac{d\phi}{dt} = -B\frac{dy}{dt}d$$

$\frac{dy}{dt}$ represents velocity of this conductor.

$$\boxed{V = -BVd}$$

Equation Of Continuity

Consider a piece of conductor and we know that there will be a lot of free electrons inside this conductor.

Say the total free charge inside this conductor is Q coulomb.

Say somehow we remove the free electrons from this conductor.

↳ so definitely the magnitude of charge inside the conductor will decrease with respect to time.

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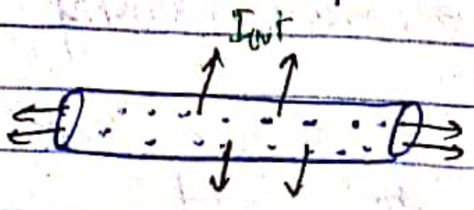
The law of conservation of charge states "that charge can neither be created nor be destroyed. It can be moved from one place to another place."



$$\frac{-dQ}{dt} = I_{out} \rightarrow ①$$

$$I_{out} = \oint \vec{J} \cdot \vec{ds}$$

$$Q = \int \rho_v \, dv$$

Partial integration b/c ρ_v might be a function of other parameters like $x, y, t, \rho, \theta, \phi$, etc

$$-\frac{dQ}{dt} = \int -\frac{\partial \rho_v}{\partial t} \, dv$$

$$① \Rightarrow \oint \vec{J} \cdot \vec{ds} = \int -\frac{\partial \rho_v}{\partial v} \, dV$$

Apply Divergence Theorem on the L.H.S

$$\hookrightarrow \oint \vec{J} \cdot \vec{ds} = \int (\nabla \cdot \vec{J}) \, dV$$

$$\rightarrow \int (\nabla \cdot \vec{J}) \, dV = \int -\frac{\partial \rho_v}{\partial t} \, dV$$

$$\Rightarrow \boxed{\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}}$$ eq. of continuity

closed surface integration → divergence theorem
closed path integration → stoke's theorem.

Maxwell's 3rd Equation

(2nd eq in book)

$$\nabla \times \vec{H} = \vec{J} \rightarrow ①$$

↳ current density

↳ incorrect version

↳ A time varying current can create a time varying magnetic field.

We will correct it with the help of eq. of continuity.

Consider a current carrying conductor extending from $-\infty$ to ∞.

The magnetic field intensity generated by the current in this conductor is

$$\vec{H} = \frac{I}{2\pi\rho}\,\vec{a_\phi}$$

Magnitude of $\vec{H}$ in general form H_ϕ

$$\vec{H} = H_\phi\,\vec{a_\phi}$$

Calculating curl of this $\vec{H}$.

$$\nabla \times \vec{H} = \frac{1}{\rho}\begin{vmatrix} \vec{a_\rho} & \rho\vec{a_\phi} & \vec{a_z} \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ H_\rho & \rho H_\phi & H_z \end{vmatrix}$$

$$\nabla \times \vec{H} = \frac{1}{\rho}\begin{vmatrix} \vec{a_\rho} & \rho\vec{a_\phi} & \vec{a_z} \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ 0 & \frac{I}{2\pi\rho} & 0 \end{vmatrix}$$

?

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Expanding; will result in a vector, say $\vec{A}$

$$\vec{A} = \nabla \times \vec{H} = \frac{-I}{2\pi\rho^2}\ \vec{a_z}$$

or $$\vec{A} = A_z\ \vec{a_z}$$

Calulating divergence of this new vector;

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \vec{A}$$

$$= \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}(\rho A_\rho)\right] + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} = 0$$

Divergence of curl of magnetic field intensity is 0.

$$\nabla \cdot (\nabla \times \vec{H}) = 0$$

→ Divergence of curl of any vector is zero.

① ⇒ If L.H.S is zero
R.H.S must also be zero

ie $\nabla \cdot \vec{J} = 0$

↳ which is violating the eq of continuity.

⇒ There is something wrong with the third equation of Maxwell.

Consider the eq. of continuity and 1st eq of Maxwell.

$$\nabla \cdot \vec{J} = \frac{-\partial \rho_v}{\partial t} \qquad \nabla \cdot \vec{D} = \rho_v$$

Bilal Register

Partially differentiate both sides of Max 1st eq w.r.t time:

$$\frac{\partial}{\partial t}(\nabla\cdot\vec{D}) = \frac{\partial \rho_v}{\partial t}$$

Can interchange order of these two.
↳ derivative and divergence

$$-\nabla\cdot\frac{d\vec{D}}{dt} = -\frac{\partial \rho_v}{\partial t}$$

Putting this value in eq. of continuity

$$\Delta\cdot\vec{J} = -\nabla\cdot\frac{\partial\vec{D}}{\partial t}$$

$$\Rightarrow \nabla\cdot\vec{J} + \nabla\cdot\frac{\partial\vec{D}}{\partial t} = 0$$

$$\nabla\cdot\left(\vec{J} + \underline{\underline{\frac{\partial\vec{D}}{\partial t}}}\right) = 0$$

↳ This term is missing in eq.

So the corrected Maxwell's eq is as

$$\boxed{\nabla\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}}$$

$\frac{\partial\vec{D}}{\partial t}$ is called displacement current density

$\vec{J}$ is called conduction current density.

$\int$ = Jenclosed → Ampere circuital law)
= Qenclosed → Gauss law

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Review of Maxwell's Equations In Instantaneous Form

$$\boxed{\begin{aligned} &1-\quad \nabla\cdot\vec{D} = \rho_v \\ &2-\quad \nabla\cdot\vec{B} = 0 \\ &3.\quad \nabla\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t} \\ &4.\quad \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \end{aligned}}$$

chp 3
chap 8
corrected chp 10

1- $\vec{D}$ is the instantaneous value of magnetic flux density which must be in terms of t.

β $\vec{H}$, $\vec{J}$, $\vec{E}$, $\vec{B}$ also instantaneous values in terms of time.

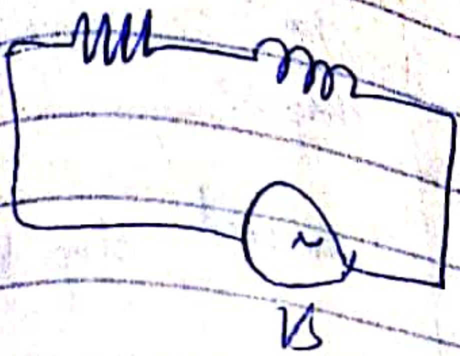
All the values (of fields) in these equation are in terms of time.

Maxwell's Equations In Integral Form

From Gauss's law

$$\boxed{\begin{aligned} &1.\quad \oint\vec{D}\cdot\vec{ds} = \int\rho_v\,dv \\ &2.\quad \oint\vec{B}\cdot\vec{ds} = 0 \qquad \text{magnet in a sphere.} \\ &3.\quad \oint\vec{H}\cdot\vec{dl} = \int\vec{J}\cdot\vec{ds} + \int\frac{\partial\vec{D}}{\partial t}\cdot\vec{ds} \\ &4.\quad \oint\vec{E}\cdot\vec{dl} = \int-\frac{\partial\vec{B}}{\partial t}\cdot\vec{ds} \end{aligned}}$$

Maxwell's Equation In Phasor Form



In instntnus form

$$V_s = iR + L\frac{di}{dt}$$

In phasor form

$$V_s = IR + jI\omega L$$

Steps

(i) instntnus values to phasor values.

(ii) $\frac{d}{dt} \sim \frac{\partial}{\partial t} \to j\omega$.

$$\vec{E} \sim \overset{2}{\vec{E}} \qquad \vec{D} \sim \overset{2}{\vec{D}} \qquad \vec{B} \sim \overset{2}{\vec{B}}$$

$$\vec{J} \sim \overset{2}{\vec{J}} \qquad \vec{H} \sim \overset{2}{\vec{H}}$$

1. $\nabla \cdot \overset{2}{\vec{D}} = \rho_v$
2. $\nabla \cdot \overset{2}{\vec{B}} = 0$
3. $\nabla \times \overset{2}{\vec{H}} = \overset{2}{\vec{J}} + j\omega \overset{2}{\vec{D}}$
4. $\nabla \times \overset{2}{\vec{E}} = -j\omega \overset{2}{\vec{B}}$

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10.1 $\vec{B} = 6 \cos 10^6 t \sin 0.01x \ \vec{a_z}$ mT

a) $\phi = ?$ passing through the surface $z = 0$, $0 \le x \le 20$; $0 \le y \le 3$

at $t = 1 \times 10^{-6}$ sec.

$$\phi = \int \vec{B} \cdot \vec{ds} \quad \text{and} \quad \vec{ds} = dx\, dy\, \vec{a_z}$$

$$\Rightarrow \phi = 6 \times 10^{-3} \cos 10^6 t \int_0^{20} \sin 0.01x \, dx \int_0^3 dy$$

$$\Rightarrow \phi = 6 \times 10^{-3} \cos 10^6 t \left(\frac{-\cos 0.01x}{0.01} \Big|_0^{20} \right) \times (y) \Big|_0^3$$

$$\Rightarrow \phi = 35.82 \times 10^{-3} \cos 10^6 t \text{ weber} \rightarrow (1)$$

Put $t = 1\mu s$

$$\Rightarrow \phi = 35.82 \times 10^{-3} \cos (10^6)(10^{-6})$$

$$\Rightarrow \phi = 19.34 \text{ mWb}$$

b) $\oint \vec{E} \cdot \vec{dl} = ?$ at $t = 1\mu s$

$$\oint \vec{E} \cdot \vec{dl} = V \quad \text{or} \quad V = \frac{-d\phi}{dt}$$

$$\Rightarrow \oint \vec{E} \cdot \vec{dl} = \frac{-d\phi}{dt}$$

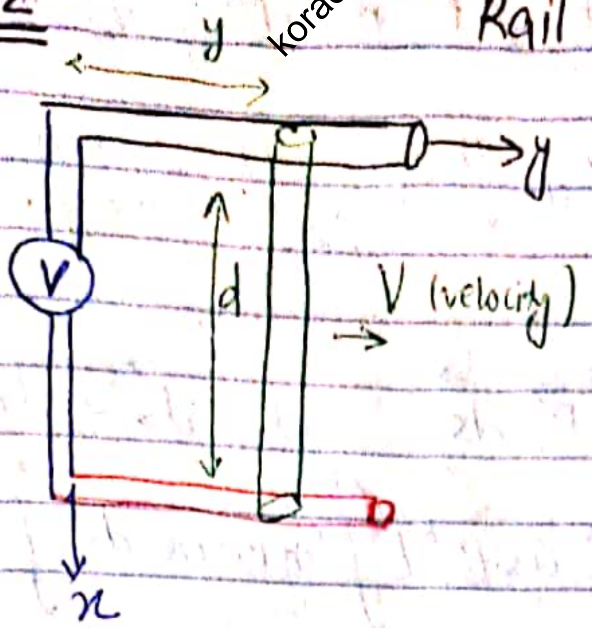
$$(1) \Rightarrow \frac{-d\phi}{dt} = 35.82 \times 10^{-3} \times 10^6 \times \sin 10^6 t$$

Put $t = 1\mu s$

$$\oint \vec{E} \cdot \vec{dl} = \frac{-d\phi}{dt} = 35.82 \times 10^3 \times \sin(10^6 \times 10^{-6})$$

$$\oint \vec{E} \cdot \vec{dl} = 30.14 \text{ kV}.$$

10.2



$d = 0.05m \quad \vec{B} = 0.25 \ Wb/m^2 \text{ or } T \ \vec{a_z}$

$\vec{V} = 20\sqrt{y} \ m/s \ \vec{a_y}$

Let $y = 0.04m$ at $t = 0sec.$

At $t = 0.06 sec.$

Find (i) y (ii) V (iii) V (iv) I_V

if $R_V = 200k\Omega$

Ideally,

Resistance of voltmeter is infinite.

Resistance of ammeter is zero.

$V = 20\, y^{1/2} \Rightarrow \frac{dy}{dt} = 20y^{1/2}$

$\int y^{-1/2}\, dy = \int 20\, dt$

$2\sqrt{y} = 20t + K$

$2\sqrt{0.04} = 20(0) + K$

Initial condition

$K = 0.4$

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$\Rightarrow 2\sqrt{y} = 20t + 0.4$

$\sqrt{y} = 10t + 0.2$

$\Rightarrow y = (10t + 0.2)^2$

At ~~t = 0.64m~~ $t = 0.06s$

$\Rightarrow y = 0.64m$

$V = 20\sqrt{0.64} = 16 \, m/s$

Also $V = -BVd = -0.2V$

$I_V = \frac{V}{R_V} = \frac{-0.2 \times 10^{-3}}{200} = -1 \mu A$

10.3 Amplitude of $\frac{\partial \vec{D}}{\partial t} = ?$

a) $\vec{E} = 80 \cos(6.277 \times 10^8 t - 2.092y)\, \vec{a_z}$ in air.

Calculate the amplitude of displacement current density.

$\vec{D} = \varepsilon_0 \vec{E}$

$\vec{D} = 8.85 \times 10^{-12} \times 80 \cos(6.277 \times 10^8 t - 2.092y)\, \vec{a_z}$

$\frac{\partial D}{\partial t} = -0.44 \sin(6.277 \times 10^8 t - 2.092y)\, \vec{a_z} \; A/m^2$

$\Rightarrow$ Amplitude $= 0.445 \; A/m^2$

b) Magnetic field intensity in air space of large Power transformer.

$\vec{H} = 10^6 \cos(377t + 1.256 \times 10^{-6} z)\, \vec{a_y}$

Air space $\rightarrow$ insulator (dielectric medium).

We assume it ideal insulator.

Bilal Register

Ideal $\sigma = 0$ conductivity = 0

Current density and electric is assumed zero.

$$\vec{J} = \sigma\vec{E} = 0$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

Also $\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

$$\frac{\partial \vec{D}}{\partial t} = \begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 10^6 \cos_2 (377t + 1.25\times10^{-6} z) & 0 \end{vmatrix}$$

$$= 1.25 \sin(377t + 1.25\times10^{-6} y)\, \vec{a_x} \ A/m^2$$

Amplitude = $1.256 \ A/m^2$

(c) $\varepsilon_r = 600$ (permitivity)

insulator b/w two plates of a capacitor.

$$\vec{D} = 3\times10^{-6} \sin(6\times10^6 t - 0.3464x)\, \vec{a_x} \ C/m^2$$

Calculate the displacement current density and its amplitude.

$$\frac{\partial \vec{D}}{\partial t} = 18 \cos(6\times10^6 t - 0.364x)\, \vec{a_x} \ A/m^2$$

Amplitude = $18 \ A/m^2$

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(d) conductor $\epsilon_r = 1$ conductivity $\sigma = 5\times10^7$

$\vec{J} = 10^7 \sin(6.283t - 444z)\,\vec{a_x}$ A/m^2

conductivity of ideal insulator is zero.

$\vec{J} = \sigma\vec{E}$

$\vec{E} = \dfrac{\vec{J}}{\sigma} = 0.2\sin(6.283t - 444z)\,\vec{a_x}$ A/m^2

$\vec{D} = \epsilon\vec{E} = \dfrac{\vec{J}}{\sigma} = \underbrace{8.85\times10^{-12}\times 0.2}\sin(6.283t - 444z)\,\vec{a_x}$ C/m^2

$\vec{D} = 1.77\times10^{-12}$

$\dfrac{\partial\vec{D}}{\partial t} = 11.13\times10^{-9}\cos(6.283t - 444z)\,\vec{a_x}$ A/m^2

$\Rightarrow$ Amplitude $= 11.13\times10^{-9}$ A/m^2

10.4 Calculate unknown constant K for all three parts. If conductivity $\sigma = 0$ $\rho_v = 0$

Note:

All fields satisfy maxwell's equations

a) $\vec{E} = (Kx - 100t)\,\vec{a_y}$ V/m

$\vec{H} = (x + 20t)\,\vec{a_z}$ A/m

$\vec{B} = 4\vec{H} = 0.25(x + 20t)\,\vec{a_z}$ A/m.

$\mu = 0.25$ $\quad \epsilon = 0.01$

$\nabla\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t}$

$\nabla\times\vec{E} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & Kx-100t & 0 \end{vmatrix} = K\vec{a_z}$

Bilal Register

$$-\frac{\partial \vec{B}}{\partial t} = -5\vec{a_x}$$

$$K\vec{a_z} = -5\vec{a_z} \Rightarrow \boxed{K = -5}$$

(b) Electric flux density

$$\vec{D} = (5x\vec{a_x} + 2y\vec{a_y} + kz\vec{a_z})10^{-6}\ C/m^2$$

$\vec{B} = 2 \times 10^{-3}\vec{a_y}\ T$, $\mu = \mu_0$, $\varepsilon = \varepsilon_0$ (Free space)

By Maxwell's first equation.

$$\nabla \cdot \vec{D} = \rho_v = 0$$

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = 0$$

$$(5 + 2 + k)10^{-6} = 0 \Rightarrow \underline{k = -3}$$

(c) $\vec{E} = 60 \sin 10^6 t \sin 0.01z\ \vec{a_x}$

$\vec{B} = \mu\vec{H} = 0.6k\ 10^6 t \cos 0.01z\ \vec{a_y}\ wb/m^2$

$$\nabla \times \vec{E} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \partial/dx & \partial/dy & \partial/dz \\ 60\sin 10^6 t \ \sin 0.01z & 0 & 0 \end{vmatrix}$$

$$\nabla \times \vec{E} = 0.6 \sin 10^6 t \cos 0.01z\ \vec{a_y} \rightarrow (A)$$

$$-\frac{\partial \vec{B}}{\partial t} = 0.6 \times 10^6 \times k \sin 10^6 t \cos 0.01z\ \vec{a_y} \rightarrow (B)$$

(A) and (B) $\Rightarrow$

$$0.6 \times 10^6 \times k = 0.6$$

$$\Rightarrow \underline{k = 10^{-6}}\ \text{Ans.}$$

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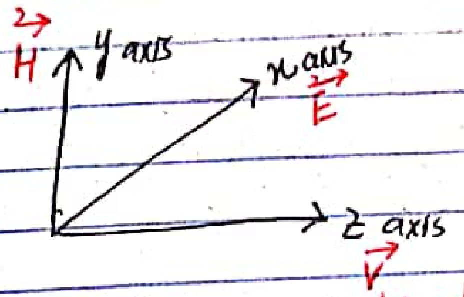
Chapter 11 :

The Propagation of TEM Wave In a Lossy Dielectric Medium

TEM : Transverse Electromagnetic Waves / Uniform plane wave.

↳ $\vec{E}$, $\vec{H}$ and $\vec{V}$ are perpendicular to each other.



↳ direction of propagation.

The electric field intensity ($\vec{E}$) and magnetic field intensity ($\vec{H}$) are located in a rectangular plane.

Electromagnetic wave oscillates with a very high frequency ranging from 3KHz - 300GHz.

Light is also considered an EM wave with frequency range 1.5×10^{14} to 15×10^{14} Hz.

Air is a practical medium.

↳ attenuation takes place.

↓

Energy of EM wave will decrease w.r.t the distance travelled by the wave.

For practical medium;
conductivity, $\sigma \neq 0$

If $\sigma = 0 \rightarrow$ lossless medium.

Consider Maxwell's equation in phasor form;

$$\nabla \times \vec{E} = -j\omega \vec{B}$$

And also $\vec{B} = \mu \vec{H}$

The maximum speed of any wave will occur in free space. 3×10^8 m/s

$$\nabla \times \vec{E} = -j\omega\mu \vec{H}$$

Calculating curl on both sides.

$$\nabla \times (\nabla \times \vec{E}) = -j\omega\mu (\nabla \times \vec{H}) \rightarrow (1)$$

Another Maxwell's equation

$$\nabla \times \vec{H} = \vec{J} + j\omega \vec{D}$$

$$\vec{J} = \sigma \vec{E}$$

$$\vec{D} = \varepsilon \vec{E}$$

$$\Rightarrow \nabla \times \vec{H} = \sigma \vec{E} + j\omega\varepsilon \vec{E}$$

$$(1) \Rightarrow \nabla \times (\nabla \times \vec{E}) = -j\omega\mu (\sigma \vec{E} + j\omega\varepsilon \vec{E})$$

We simplify this equation

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$$\nabla \times (\nabla \times \vec{E}) = \omega^2 \mu \varepsilon \vec{E} - j\omega\mu\sigma\vec{E}$$

$$\nabla \times (\nabla \times \vec{E}) = -\nabla^2 \vec{E}$$ Proof is H.W

$\nabla^2 \vec{E}$ in rectangular cordinate system.

$$\nabla^2 \vec{E} = \frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2}$$

$$\Rightarrow \nabla^2 \vec{E} = -\omega^2 \mu \varepsilon \vec{E} + j\omega\mu\sigma\vec{E}$$

Rearrange

$$\nabla^2 \vec{E} + \omega^2 \mu \varepsilon \vec{E} - j\omega\mu\sigma\vec{E} = 0$$

$$\nabla^2 \vec{E} + \omega^2 \mu \varepsilon \underbrace{\left(1 - j\frac{\sigma}{\omega\varepsilon}\right)} \vec{E} = 0$$

The permitivity of a lossy medium is a complex quantity. Whereas for lossless it is real ie ε.

$$\text{let} \quad \hat{\varepsilon} = \varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right)$$

$$\boxed{\nabla^2 \vec{E} + \omega^2 \mu \hat{\varepsilon} \vec{E} = 0}$$ Wave Equation

$$\text{Power} \propto E$$

$$E = f(z)$$

$\vec{E}$ does not w.r.t x and y ; so

Bilal Register

$$\nabla^2 \vec{E} = \frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2}$$

$$\Rightarrow \nabla^2 \vec{E} = \frac{\partial^2 \vec{E}}{\partial z^2}$$

$$\Rightarrow \frac{\partial^2}{\partial z^2} \vec{E} + \omega^2 \mu \hat{\epsilon} \vec{E} = 0$$

This is a second order homogenous differential equation.

↳ solution?

Auxilary / Characteristic Equation.

let $\frac{\partial}{\partial z} = m$

$$\Rightarrow m^2 \vec{E} + \omega^2 \mu \hat{\epsilon} \vec{E} = 0$$

$$\vec{E} (m^2 + \omega^2 \mu \hat{\epsilon}) = 0$$

As $\vec{E} \neq 0$
b/c EM wave.

$$\Rightarrow m^2 + \omega^2 \mu \hat{\epsilon} = 0$$

↳ auxilary equation.

Roots

$$m = \pm j\omega \sqrt{\mu \hat{\epsilon}} = \pm \hat{\gamma} = \pm (\alpha + j\beta)$$

γ = propagation constant.

α = Attenuation constant.

β = Phase constant.

$\boxed{\text{For lossless medium; } \alpha = 0 \quad (\delta = 0)}$

The two roots are;

$$m_1 = -\hat{\gamma} = -\alpha - j\beta$$

$$m_2 = \hat{\gamma} = \alpha + j\beta$$

$$\Rightarrow \vec{E} = \left(E_o e^{m_1 z} + E_b e^{m_2 z}\right)\vec{a}_x$$

$$\vec{E} = \Big(\underbrace{E_o e^{-\hat{\gamma} z}}_{+ve\ \vec{z}} + \underbrace{E_b e^{\hat{\gamma} z}}_{\overleftarrow{-ve\ z}}\Big)\vec{a}_x \quad \text{components}$$

When there is change in characteristics of medium, reflection of wave will take place.
↳ undesirable

T → R ←

↳ Forward travelling wave. →
Backward travelling wave. ←
↳ Not considering it.

$$\Rightarrow \boxed{\vec{E} = E_o e^{-\hat{\gamma} z}\,\vec{a}_x} \quad \text{In phasor form}$$

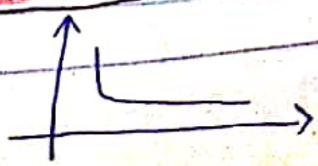
Put value of γ

$$\vec{E} = E_o e^{-\alpha z - j\beta z}\,\vec{a}_x$$

indicates wave moving along the z axis

Writing in instantaneous form.

$$\boxed{\vec{E} = \underbrace{E_o e^{-\alpha z}}_{\text{amplitude}} \sin(\omega t - \beta z)\,\vec{a}_x}$$



For Magnetic Field intensity, consider the equation

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$

$$\vec{H} = \frac{1}{-j\omega\mu}(\nabla \times \vec{E})$$

$$\vec{H} = \frac{1}{-j\omega\mu}\begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_0 e^{-\hat{\gamma} z} & 0 & 0 \end{vmatrix}$$

$$\vec{H} = \frac{\hat{\gamma}E_0}{j\omega\mu} e^{-\hat{\gamma}z}\vec{a_y}$$

Putting value of $\hat{\gamma} = j\omega\sqrt{\mu\hat{\varepsilon}}$

$$\vec{H} = \frac{\cancel{j\omega}\sqrt{\mu\hat{\varepsilon}}}{\cancel{j\omega}\mu} E_0 e^{-\hat{\gamma}z}\vec{a_y}$$

$$\vec{H} = \sqrt{\frac{\hat{\varepsilon}}{\mu}} E_0 e^{-\hat{\gamma}z}\vec{a_y}$$

$$\vec{H} = \frac{E_0}{\sqrt{\mu/\hat{\varepsilon}}} e^{-\hat{\gamma}z}\vec{a_y}$$

↳ amplitude

$\hat{\eta} = \mu/\hat{\varepsilon}$ is called the intrinsic/characteristic impedance of the medium.

$$\hat{\eta} = \frac{\mu}{\hat{\varepsilon}} = \eta e^{j\theta} = \eta\angle\theta$$

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$$\vec{H} = \frac{E_o}{\hat{\eta}} e^{-\hat{\gamma} z} \vec{a_y}$$

$$\vec{H} = \frac{E_o}{\eta e^{j\theta}} e^{-\alpha z} \times e^{-j\beta z} \times \vec{a_y}$$

$$\boxed{\vec{H} = \frac{E_o}{\eta} e^{-\alpha z} \times e^{-j(\beta z + \theta)} \vec{a_y}}$$

↳ Phasor valve

Writing in instantanous form,

$$\boxed{\vec{H} = \frac{E_o}{\eta} e^{-\alpha z} \sin(\omega t - \beta z - \theta) \vec{a_y}}$$

→ The average power density can be calculated by;

$$\boxed{\vec{S} = \frac{1}{2} \text{Real} [\vec{E} \times \vec{H}]}$$

calculated with the help of Pointing therem.

Lecture 9 30/12/19

Propagation of TEM Wave In a Lossless dielectric medium

Ideal mediam.
No attenuation.
Conductivity $\sigma = 0$

For a lossy mediam

$$\vec{E} = E_0 e^{-\alpha z} \sin(\omega t - \beta z)\,\vec{a_x}$$

$$\vec{H} = \frac{E_0}{\eta} e^{-\alpha z} \sin(\omega t - \beta z - \theta)\,\vec{a_y}$$

$$\hat{\varepsilon} = \varepsilon\left(1 - \frac{j\sigma}{\omega\varepsilon}\right)$$

$$m = \pm j\omega\sqrt{\mu\hat{\varepsilon}}$$

$$m = \pm(\alpha + j\beta)$$

<u>For lossless mediam</u>

$$\sigma = 0 \quad , \quad \varepsilon = \varepsilon_0$$

$$m = \pm j\omega\sqrt{\mu\varepsilon} = \pm j\beta$$

$$\Rightarrow \beta = \omega\sqrt{\mu\varepsilon} \qquad \alpha = 0$$

$$\vec{E} = E_0 \sin(\omega t - \beta z)\,\vec{a_x}$$

As $\hat{\eta} = \sqrt{\frac{\mu}{\hat{\varepsilon}}} = \eta e^{j\theta}$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \qquad \theta = 0$$

$$\vec{H} = \frac{E_0}{\eta} \sin(\omega t - \beta z)\,\vec{a_y}$$

In phasor form; $\vec{E} = E_0 e^{-j\beta z} \vec{a_x}$

$$\vec{H} = \frac{E_0}{\eta} e^{-j\beta z} \vec{a_y}$$

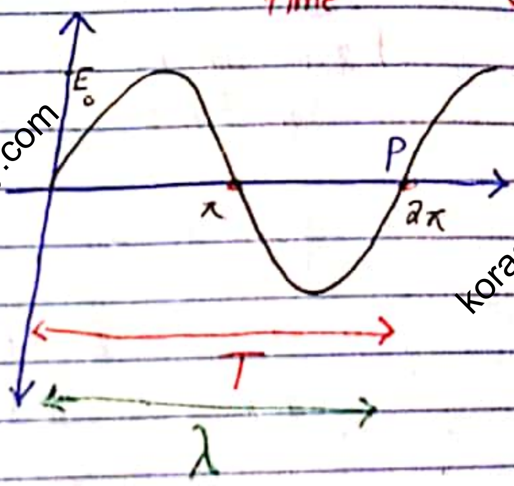
$$\vec{S}_{av} = \frac{1}{2} \text{Real}[\vec{E} \times \vec{H}]$$

$\underline{\underline{W = 2\pi f}}$?

$$E = E_0 \sin(\omega t - \beta z)$$

let $\beta z = 0$ → we see variation only w.r.t time

$$E = E_0 \sin \omega t$$



At P

$t = T$

$\omega t = \omega T = 2\pi$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$\underline{\underline{\beta = 2\pi / \lambda}}$?

As $E = E_0 \sin(\omega t - \beta z)$

Here we see variation w.r.t distance.
Let time be any constant value.
Say $\omega t = \pi$

$$E = E_0 \sin \beta z$$

Distance travelled by wave in one complete cycle is wavelength of the wave.

At P,

$$z = \lambda$$

$$\beta z = \beta\lambda = 2\pi$$

$$\beta = \frac{2\pi}{\lambda}$$

Velocity of wave

$$E = E_0 \sin(\omega t - \beta z)$$

$$\text{Phase} = \omega t - \beta z$$

Phase of a wave is always equal to a constant quantity

→ Differentiate

$$\omega \frac{dt}{dt} - \beta \frac{dz}{dt} = 0$$

$$\omega - \beta \frac{dz}{dt} = 0$$

$$V = \frac{dz}{dt} = \frac{\omega}{\beta} \rightarrow (A)$$

$$\boxed{\vec{V} = \frac{\omega}{\beta} \vec{a_z}}$$

Wave is moving along z axis

Put $\beta = \omega\sqrt{\mu\epsilon}$ in (A)

$$V = \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{\mu\epsilon}}$$

$$\boxed{V = \frac{1}{\sqrt{\mu \varepsilon}}}$$ → These two parameters control speed of the wave.

~~Propagation of TEM In Free Space~~

Propagation of TEM In Free Space

→ Special lossless mediam.

→ $\mu = \mu_0$, $\varepsilon = \varepsilon_0$, $\sigma = 0$, $\alpha = 0$

$$\vec{E} = E_0 \sin(\omega t - \beta_0 z)\, \vec{a_x}$$

where $\beta_0 = \omega \sqrt{\mu_0 \varepsilon_0}$

$$\beta_0 = 2\pi / \lambda_0$$

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$$\vec{H} = \frac{E_0}{\eta_0} \sin(\omega t - \beta_0 z)\, \vec{a_y}$$

Where $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}}$

$$\eta_0 \approx 120\pi \approx 377\ \Omega$$

$$V = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^{8}\ m/s = c$$

$$f = \frac{c}{\lambda_0} = \text{constant.}$$

Reflection of TEM Wave:

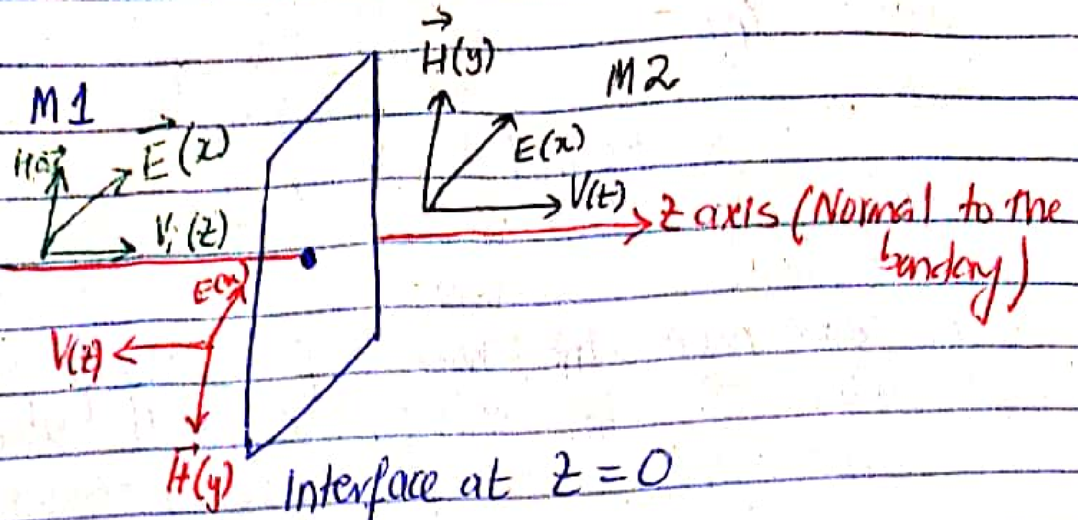
Dielectric Dielectric Interface

We assume that both media 1 and 2 are lossless.

Mediam 1 with parameters $\mu_1, \varepsilon_1, \eta_1$

Mediam 2 with $\mu_2, \varepsilon_2, \eta_2$.

Incident wave



Interface at $z=0$

At the boundary a fraction of wave will be reflected.

↳ change direction of either $\vec{E}$ or $\vec{H}$.

→ Consider the incident wave in median 1.

$$\vec{E}_i = E_o e^{-j\beta_1 z} \vec{a}_x$$

$$\vec{H}_i = \frac{E_o}{\eta_1} e^{-j\beta_1 z} \vec{a}_y$$

Power density, $\vec{S}_{i\,avg} = \frac{1}{2} \text{Real} \left[\vec{E}_i \times \vec{H}_i \right]$

→ Consider the reflected wave;

Power of a wave is directly proportional to the square of amplitude of electric field intensity.

$$\vec{E}_r = \Gamma E_o e^{j\beta_1 z} \vec{a}_x$$

Γ → reflection coefficient.

$$\vec{H_r} = \frac{-\rho E_0}{\eta_1} e^{j\beta_1 z} \vec{a_y}$$

$$\vec{S_r}_{avg} = \frac{1}{2} \text{Real} \left[\vec{E_r} \times \vec{H_r} \right]$$

As we have two waves of same frequency in medium 1 so interference will take place.

$$\vec{E_1} = \vec{E_i} + \vec{E_r}$$

$$\vec{E_1} = \left(E_0 e^{-j\beta_1 z} + \rho E_0 e^{j\beta_1 z} \right) \vec{a_x}$$

Similarly

$$\vec{H_1} = \vec{H_i} + \vec{H_r}$$

$$\vec{H_1} = \left(\frac{E_0}{\eta_1} e^{-j\beta_1 z} - \frac{\rho E_0}{\eta_1} e^{j\beta_1 z} \right) \vec{a_y}$$

→ Consider the transmitted wave in medium 2.

$$\vec{E_2} = \tau E_0 e^{-j\beta_2 z} \vec{a_x}$$

$\tau \rightarrow$ coefficient of transmission.

$$\vec{H_2} = \frac{\tau E_0}{\eta_2} e^{-j\beta_2 z} \vec{a_y}$$

$$\vec{S_2}_{avg} = \frac{1}{2} \text{Real} \left[\vec{E_2} \times \vec{H_2} \right]$$

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x and y axis are parallel (tangential) to the interface whereas z axis is normal to it.

The boundary condition states that whenever a wave travels from one medium to another medium, the tangential component of the wave does not change; however the normal component (if any) will change.

According to boundary condition;

$$\vec{E_1} = \vec{E_2}$$

Let putting $z=0$

$$\Rightarrow \quad 1+\rho = T$$

Similarly for $\vec{H}$ $\quad \vec{H_1} = \vec{H_2}$

let putting $z=0$

$$\Rightarrow \frac{1}{\eta_1} - \frac{\rho}{\eta_1} = \frac{T}{\eta_2}$$

$$\Rightarrow \frac{1}{\eta_1} - \frac{\rho}{\eta_1} = \frac{1}{\eta_2} + \frac{\rho}{\eta_2}$$

$$\rho = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

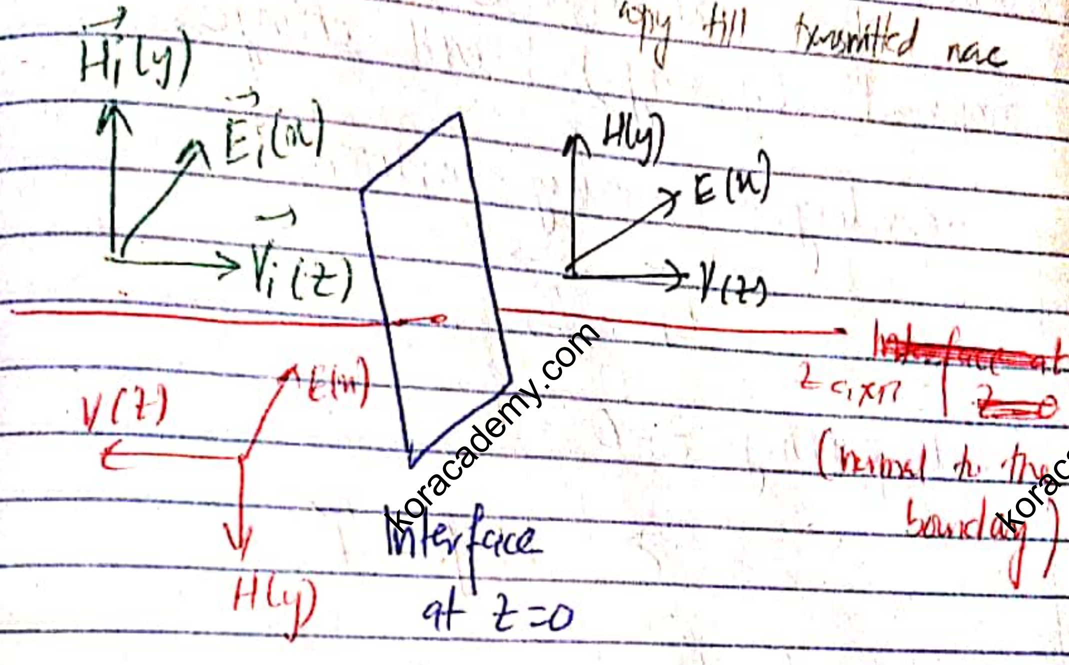
Reflection of TEM Wave

DiElectric Conductor Interface:

Assuming mediam 1 lossless and mediam 2 to be a pure conductor.

diagram same.

copy till transmitted wave



Consider the incident wave in medium 1.

$$\vec{E}_1 = E_0 \, e^{-j\beta_1 z} \, \vec{a_x}$$

$$\vec{H}_1 = \frac{E_0}{\eta} \, e^{-j\beta_1 z} \, \vec{a_y}$$

$$\vec{S}_{avg} = \frac{1}{2} \text{Real} \left[\vec{E}_1 \times \vec{H}_1^* \right]$$

Consider the reflected wave.

$$\vec{E}_r = \rho E_0 \, e^{j\beta_1 z} \, \vec{a_x}$$

$\rho \rightarrow$ reflection coefficient

$$\vec{H_r} = -\frac{\rho E_0}{\eta} e^{j\beta_1 z} \vec{a_y}$$

$$\vec{S}_{r\,avg} = \frac{1}{2} \text{Real} \left[\vec{E_r} \times \vec{H_r} \right]$$

As we have two waves of same frequency in medium 1, so interferance will take place.

$$\vec{E_1} = \vec{E_i} + \vec{E_r}$$

$$\vec{E_1} = \left(E_0 e^{-j\beta_1 z} + \rho E_0 e^{j\beta_1 z} \right) \vec{a_x}$$

Similarly

$$\vec{H_1} = \vec{H_i} + \vec{H_r}$$

$$\vec{H_1} = \left(\frac{E_0}{\eta_1} e^{-j\beta_1 z} - \rho \frac{E_0}{\eta_1} e^{j\beta_1 z} \right) \vec{a_y}$$

Consider the transmitted wave in medium 2.

$$\vec{E_2} = \tau E_0 e^{-\hat{\gamma}_2 z} \vec{a_x}$$

$$\vec{H_2} = \tau \frac{E_0}{\eta_2} e^{-\hat{\gamma}_2 z} \vec{a_y}$$

$$\vec{S}_{2\,avg} = \frac{1}{2} \text{Real} \left[\vec{E_2} \times \vec{H_2} \right]$$

x and y are parallel (tangential) to the interface and z is normal to it.

Bondary condition;

$$\vec{E_1} = \vec{E_2}$$

put $z=0$

$$1+\rho = \tau$$

Similarly $\vec{H_1} = \vec{H_2}$

Put $z=0$

$$\frac{1}{\eta_1} - \frac{\rho}{\eta_1} = \frac{\tau}{\eta_2}$$

$$\Rightarrow \frac{1}{\eta_1} - \frac{\rho}{\eta_1} = \frac{1}{\eta_2} + \frac{\rho}{\eta_2}$$

$$\rho = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

- For pure conductor $\eta_2 = 0$

$\Rightarrow \rho = -1$

↳ All of the signal has been reflected.

$\Rightarrow \tau = 0$

↳ No transmission

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Numericals :

1- Amplitude of magnetic field, $H_0 = 20$ A/m in the $\vec{a_y}$ direction in free space.
Angular velocity, $w = 2 \times 10^9$ rad/sec

$$B_0 = \frac{w}{c} = w\sqrt{\mu_0 \varepsilon_0} = \frac{2 \times 10^9}{3 \times 10^8}$$

$$\Rightarrow B_0 = 6.67$$

$$\vec{H} = 20 \sin(2 \times 10^9 t - 6.67z)\vec{a_y} \ A/m$$

$$\lambda_0 = \frac{2\pi}{B_0} = 0.942 \ m$$

$$f = \frac{w}{2\pi} = 318 \ MHz$$

$$T = \frac{1}{f} = 3.14 \ ms$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377$$

As $H_0 = \frac{E_0}{\eta_0} \Rightarrow E_0 = H_0 \eta_0$

$$\Rightarrow E_0 = 7.537 \times 10^3 \ V/m.$$

$$\vec{E} = 7.537 \times 10^3 \sin(2 \times 10^9 t - 6.67z)\vec{a_x}$$

$$\vec{S}_{avg} = \frac{1}{2} Real \left[7.537 \times 10^3 \times e^{-j6.67} \vec{a_x} \times 20 \times e^{j6.67} \vec{a_y} \right].$$

Neglect imaginary part (if any)

2) $\vec{E} = 377 \sin(10^9 t - 5y)\vec{a_z}$

$\mu = \mu_0$

$\varepsilon_r = ?$ $\rightarrow$ $v = ?$ (velocity) $\rightarrow$ $\eta = ?$ $\rightarrow$

$\lambda = ?$ $\rightarrow$ $\vec{H} = ?$ $\rightarrow$ $S_{avg} = ?$

$w = 10^9$ rad/sec $\qquad \beta = 5$

As $v = \frac{w}{\beta} = \frac{10^9}{5} = 2 \times 10^8$ m/s

$$v = \frac{1}{\sqrt{\mu \varepsilon}} \qquad v^2 = \frac{1}{\mu_0 \mu_r \varepsilon_r \varepsilon_0}$$

$$\varepsilon_r = \frac{1}{\mu_r \mu_0 \varepsilon_0 v^2}$$

$$\Rightarrow c^2 = \frac{1}{\mu_0 \varepsilon_0}$$

$$\varepsilon_r = \frac{c^2}{v^2} = \frac{(3 \times 10^8)^2}{(2 \times 10^8)^2}$$

$$\varepsilon_r = 2.25$$

Also $\lambda = \frac{2\pi}{\beta} = 1.257$ m

Also $\eta = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}} = 251.33\ \Omega$

Also $\vec{H} = \frac{E_0}{\eta} \sin(wt - \beta y)\,\vec{a_x}$

$$\vec{H} = \frac{377}{251.33} \sin(10^7 t - 5y)\, \vec{a_x}$$

Also, $\vec{S_{av}} = \frac{E_0^2}{2\eta} \vec{a_y}$ lossless medium

$\rightarrow$ By simplifying $\frac{1}{2} Real[\vec{E} \times \vec{H}]$.

$$\vec{S_{avg}} = 282.5\, \vec{a_y} \;\; W/m^2$$

(3) $\vec{E} = 94.25 \sin(wt + 6z)\, \vec{a_x}$ V/m $\rightarrow$ vector direction ($\vec{E}$)

wave moving along $-\vec{a_z}$

in free space.

$c = 3 \times 10^8$ m/s $\quad \eta_0 = 377\ \Omega$

$$c = \frac{\omega}{\beta_0} \Rightarrow \omega = c\beta_0$$

$$\Rightarrow \omega = 1.8 \times 10^9 \text{ rad/sec}$$

$$H_0 = \frac{E_0}{\eta_0} = \frac{94.25}{377} = 0.25 \text{ A/m}$$

$$\vec{H} = -0.25 \sin(1.8 \times 10^9 t + 6z)\, \vec{a_y} \text{ A/m}$$

$\hookrightarrow \vec{a_x} \times -\vec{a_y} = -\vec{a_z}$

$\hookrightarrow$ came mathematically to verify direction of wave

$$\lambda_0 = \frac{2\pi}{6} = 1.047 \text{ nm.}$$

$$\vec{S_{avg}} = \frac{-E_0^2}{2\eta_0} \vec{a_z} = -11.78\, \vec{a_z} \;\; Wb/m^2$$

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4. $f = 1.8\,GHz$ $\quad \epsilon_r = 2.5 \quad \mu_r = 1.6$
$\sigma = 2.5$ (lossy)

$$\vec{E} = 0.1\, e^{-\alpha t} \sin(\omega t - \beta t)\, \vec{a_x} \; V/m$$

$$\hat{\gamma} = \alpha + j\beta = j\omega\sqrt{\mu \hat{\epsilon}}$$

$$\hat{\gamma} = j\omega\sqrt{\mu_0 \mu_r \epsilon_r \epsilon_0 \left(1 - \frac{j\sigma}{\omega \epsilon_0 \epsilon_r}\right)}$$

$$\hat{\epsilon} = \epsilon\left(1 - \frac{j\sigma}{\omega \epsilon}\right)$$

$$\hat{\gamma} = \alpha + j\beta = 108.51 + j\,261.97$$

$$\Rightarrow \alpha = 108.51 \qquad \beta = 261.97$$

Put values of α and β in $\vec{E}$.

Also $\hat{\eta} = \sqrt{\frac{\mu}{\hat{\epsilon}}} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r \left(1 - \frac{j\sigma}{\omega \epsilon_0 \epsilon_r}\right)}}$

$$\hat{\eta} = 80.2 \angle 22.5^\circ \; \Omega$$

Also $V = \frac{\omega}{\beta} = 4.37 \times 10^7 \; m/s$

Also $\vec{H} = \frac{E_0}{\eta} e^{-\alpha t} \sin(\omega t - \beta z - \theta)\, \vec{a_y}$

$$\Rightarrow \vec{H} = \frac{0.1}{80.2} e^{-108.51 t} \sin(1.13 \times 10^{10} t - 261.97 t - 22.5)\, \vec{a_y}$$

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5. Repeat Q4 for $f = 1.8\,\text{KHz}$

Answer $\alpha = \beta = 0.1686$ $\qquad \frac{\sigma}{\omega\varepsilon} >>> 10$

intrinsic impedance

$\hat{\eta} = 0.0954 \angle 45^\circ$

$v = 6.7 \times 10^{4}\ m/s$

Amplitude $\quad M_\delta = 1.048$