NOTES

Subject

Sr. No: **Date** **Topic**

ELECTRICAL MEASURMENT
AND INSTRUMENTATION

Sir Amjid khattak.
Sir Farooq

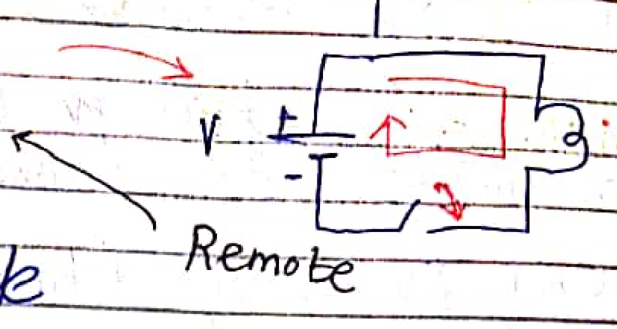
Books:

Accuracy, Precision, Sensitivity,
resolution, significant figures
Error with types
statistical analysis → mean, deviation,
standard deviation, probability of error
system of units, Galvanometer.
Ammeter, multirange ammeter.
egs
DC voltmeter, voltrange eg
Series type ohmmeter eg
shunt " " eg
callibration.
Rectifier type instruments.

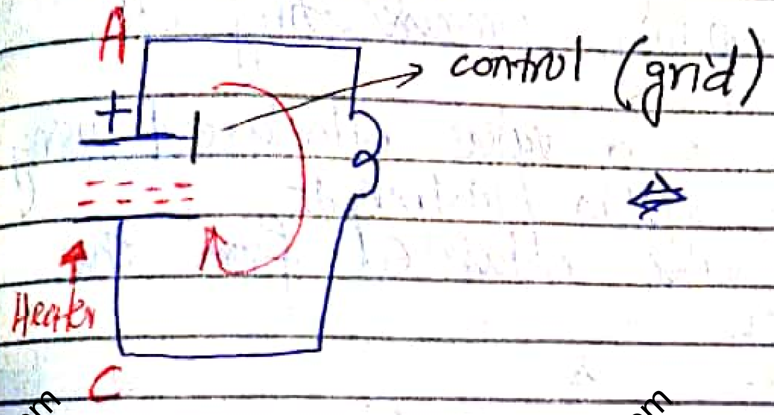
26/9/19

Electronics deal with motion of electronics.

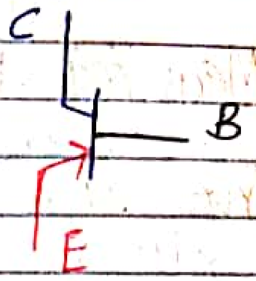
Electrical
Electronics



Vacum tube ⇔ diode



⇔



Thermionic emission ⇔ transistors

Measurment

Man uses his imaginative skills.
↳ to identify a physical phenomena
↳ developed and utilized a means to understand this.

To measure means to determine the magnitude ~~and~~ or extent or degree of the condition of system in terms of some standard.

All measuring systems based on laws of nature.

Meter: Instrument used to indicate or record measured value.

Measurand Variable under measurment.

Meterology science of dealing with precise and acurate measurements.

Instrument A device for determining the value or magnitude of a variable

↳used for sensing, detecting, measuring, recording, controlling, communicating etc.

Measured value is a value calculated using a measurment system / instrument.

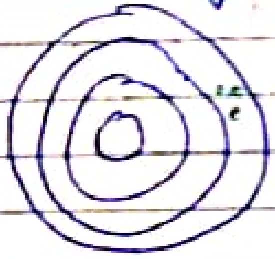
True value is value calculated from the ratted values.

$$R = \frac{\rho L}{A}$$

Accuracy is the degree of closeness of measured value with true value.

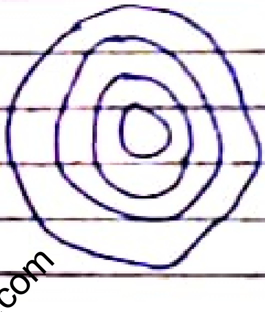
Precision is the degree of closeness of measured values with each other.

eg bird eye target



less accurate more precise

Acarate and precise



Sensitivity of a device is defined as the ratio of change in output signal (responce) to the change in input variable. $S = \frac{\Delta output}{\Delta input}$

Resolution is the smallest change in input that can be detected by device.
↳ depends on the no. of significant figures.

Significant Figures.

(i) Any non zero digit is significant.
(ii) Any zero b/w two non zeros is significant
(iii) Any zero before non zero is not significant
(iv) Any zero after non zero are not significant if the given number has no decimal part
If a dash on zero it means it is significant
(v) Any zero after non zero is significant if it has decimal.

Number	# of sig F
17	2
132.356	6
101	3
0.5	1
0.00305	3
5000	1
$100\bar{0}$	4
$2\bar{0}00$	2
370.	3
0.003205300	7

$500 = 5 \times 10^2$
$370. = 3.70 \times 10^2$
$5\bar{0}00 = 5.00 \times 10^2$

One trillion, 2 billion, Nine thousand

1 billion = 1000 millions
1 trillion = 1000 billions

Addition And Subtraction.

(i) Math first, sig figures last.
Arithmetic operation [illegible] less and more accurate as results [illegible] less [illegible]

(iv) If both no.s have

150.0
\+ 0.702
150.702 →

0.854
\- 0.0594
0.7946 → 0.795

(ii) If both numbers OR any one does not have a decimal point.

depends on the position of highest ~~lowest~~ significant figures ~~highest less SF~~

4.51
\+ 3
7.51 → 8

→ highest

12000
\- 3500
77500 → 8000 (Answer in thousands)

position

END

500.0 → 100s
\- 90 → 10s
410.0 → 410 ✗

500 → 100s
\- 90 → 10s
410 → 400

Multiplication And Division

Answer has lowest no. of

irrespective of decimal point.

× 815.991
324.6 → 6
264870.6700

Lecture 2 02/oct/19

Error: Difference b/w true and measured value

Accuracy and Precision

↳ degree of closeness of the value of quantity under observation.

Precision → degree of agreement within a group of measurements or instruments.

↳ is composed of two ch.

(i) Conformity

(ii) No. of sig fig. sig fig ↑ P ↑

True value (x_T) $\boxed{\text{Instrument}}$ → measured value (x_m)

Error = $x_m - x_T$

Reducing error by statistical analysis:

→ more than one reading required.

(i) Arithematic Mean.

Readings → $x_1, x_2, x_3 \ldots x_n$

↳ $\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$

↳ most value.

2) Deviation from the mean

$d_1 = x_1 - \bar{x} \qquad d_2 = x_2 - \bar{x} \qquad d_n = x_n - \bar{x}$

Example Readings 2, 4, 3, 3, 2

$\bar{x} = \frac{2+4+3+3+2}{5} = 2.8$

$d_1 = -0.8 \quad d_2 = 1.2 \quad d_3 = 0.2 \quad d_4 = 0.2$
$d_5 = -0.8$

3) Average deviation

$$D = \frac{|d_1| + |d_2| + \cdots + |d_n|}{n}$$

4) Standard Deviation
RMS deviation

$$\delta = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \cdots + d_n^2}{n}}$$

↳ called as population standard deviation

$$\delta = \sqrt{\frac{d_1^2 + d_2^2 + \cdots + d_n^2}{n-1}}$$

↳ sample standard deviation

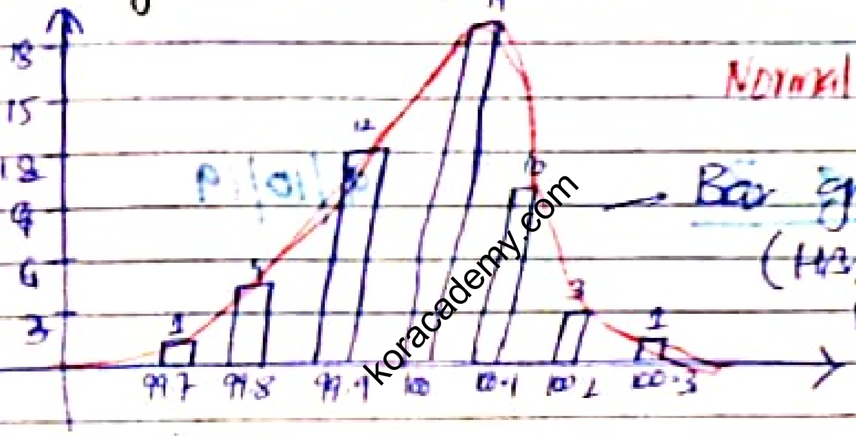
Parameter	Statistic
Each and every individual	Small sample out of the total
mean is parameter	mean is statistic
$= \frac{\sum_{i=1}^{N} x_i}{N}$	$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$

degree of freedom

Probability Of Error.

Normal Distribution of Error;

Voltage Readings	99.7	99.8	99.9	100.0	100.1	100.2	100.3
Number of Readings	1	4	12	19	10	3	1



Normal or Gaussian curve

Bar graph

(Histogram doesn't have spaces)

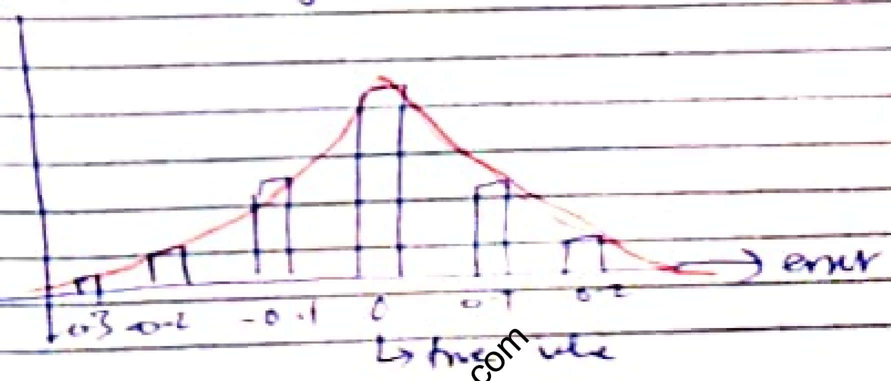
→ All observations include small disturbing effects called random errors.

→ Random error can be +ve or -ve

→ There is an equal probability of +ve and -ve

[illegible] → Small errors are more probable than large errors

→ There is an equal probability of + and - errors so that the probability of a given error will be symmetrical about 0 value



In the standard deviation curve if you move from $-\infty$ to ∞ → you cover 100% of the area.

$-\sigma$ to σ

-2σ to

-3σ to 3σ → 99%

-0.6745σ to 0.6745 → 50%

Probable error $= 0.6745\sigma$

Lecture 3

9/10/19

Chapter 2

1 meter is the length of 1st path travelled by light in vacuum in time interval of $\frac{1}{299,792,453}$ s. was defined in

Kilogram was defined in 1989. 1kg is the mass of international prototype of kilogram (platinum iradium cylinder) kept at International bureau of weights and measures at Paris.

1 second is the duration of 9,192,631,770 periods of radiations corresponding to transition b/w two hyperfind levels of grand state of CS-133 atom. → defined in 1967.

-273.15°C is that temperature at which all the molecular motion in water are ceased and this is equal to 1 kelvin.

0.01°C is called as triple point. At this stage all three states of water exist.

One kelvin is deflection of thermodynamic temperature.

$$\begin{tikzcd} 0.01^\circ C \arrow[r] & 273.10\,K \\ -273.15^\circ C \arrow[r] \arrow[u] & 0^\circ K \arrow[u] \end{tikzcd}$$

One mole is the amount of substance as many elements particles as there are in 0.012 kg of carbon.

→ [illegible]

Lecture 4 16/oct/19

Suspension Galvanometer.

moving coil instruments.

Permanent magnet moving coil (Pmmc)
↳ Dorsonval movement

Electric magnetic moving coil (Emmc)

Galvanometer principle
G A V

①Deflection torque
② opposing torque (spring)

θ → angle b/w Area and $\vec{B}$.
ϕ → deflection of galvanometer.

① Indicating type instruments.
Gives you display.

② Recording type instruments
Display + Recording

③ Integrating type instruments
Display + Recording + Commulative addition

④ Null type instrument
eg wheatstone bridge

$$\vec{F} = \vec{I}L \times \vec{B}$$

1. Deflecting Torque

$T_d \propto I$, $T_d \propto \phi$ $T_d \propto N$

$T_d \propto NI\phi$

$T_d = BANI$ $\qquad\qquad B = \frac{\phi}{A}$

$\Rightarrow T_d \propto I$

deflecting torque is only dependent on current.
In order to control/stop pointer at some point we need controlling torque (T_c).

2- Controlling torque

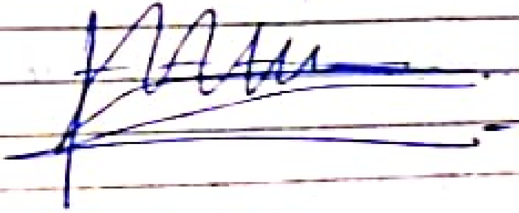
we use spiral spring $\qquad T = K\theta$

when $T_d = T_c \rightarrow$ pointer stops.

$BANI = K\theta \Rightarrow \theta = \frac{BAN}{K} I$

$\theta = cI \Rightarrow \theta \propto I$

Due to inertia, initially it doesn't stop.



oscillations → underdamped
ideally → critically damped
↳ we need third torque

3_ Damping torque

(a) Mechanical damping: we used special bearings, but we have losses in it as we introduce friction.

(b) Electromagnetic damping: eddy current damping.

- aluminium disc - vibration changes magnetic flux - produces eddy current - direction opposite to its cause (lenz's law)

(c) Critical damping ressistances.

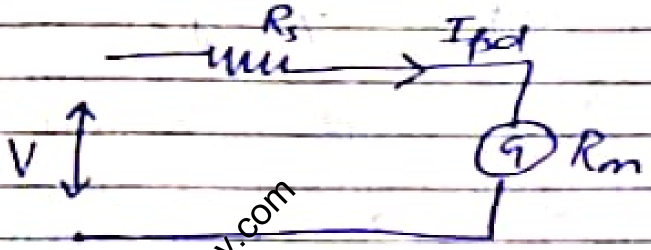
Temperature also effects galvanometers

Lecture 5 22/10/19

DC voltmeter (G) 0 to I_{fsd} ↓
(V) 0 to V

G has very small resistance

After R_s



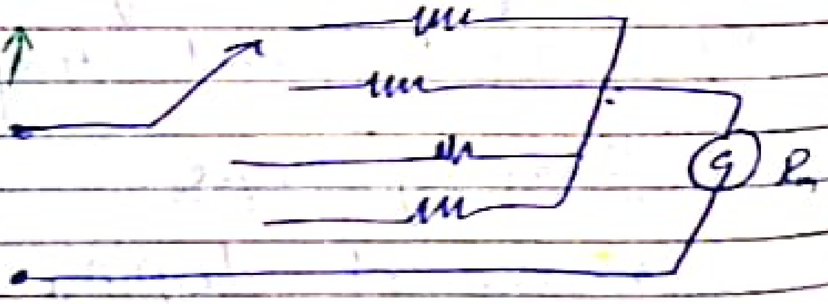
R_s is also known as multiplier

KVL ⇒ $V = I_{fsd} (R_m + R_s)$

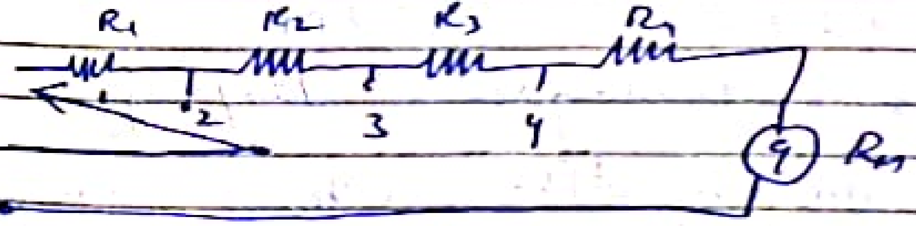
$$\frac{V}{I_{fsd}} - R_m = R_s$$

multirange voltmeters have more than 1 series resistance.

R_s ↑ Range of voltmeter ↑



more practical



In first case we need all 4 resistors of high quality while in second case we need only.

→ R_4 should be of high quality

Ex 4.3

voltmeter sensitivity - ohms per unit rating of the voltmeter

$$S = \frac{R_T}{V_{fsd}}$$

R_T → total resistance of voltmeter. (internal + series)

As $V_{fsd} = I_{fsd}(R_s + R_m)$

$\Rightarrow V_{fsd} = I_{fsd}(R_T)$

$$S = \frac{R_T}{V_{fsd}} = \frac{1}{I_{fsd}}$$

As it depends on full scale deflection of galvanometer.

$S \times V = R_s + R_m$

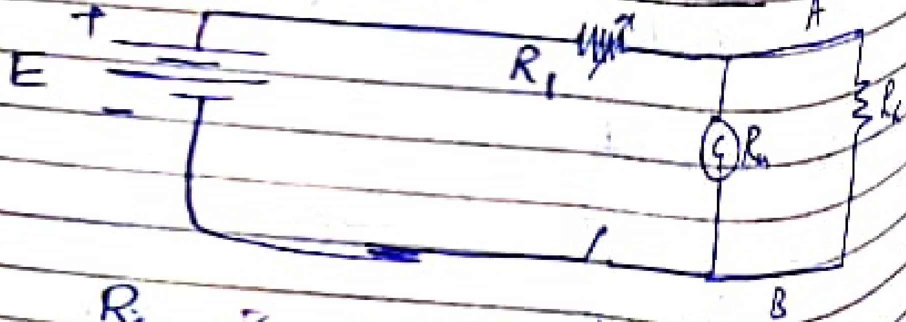
$R_s = (S \times V) - R_m$ given knowing of sensitivity, range and internal resistance

↓ Loading effect

Lecture 6

30/10/19

Shunt Type ohmmeter



R_1 is current limiting resistance.

Switch = open ⇒ so no current will flow from battery necessarily

→ Switch increases battery life.

Case 1 $R_x = 0$ ⇒ A and B = short
switch closed → no current through galvanometer
→ no deflection → that point marked as $0\,\Omega$.

Case 2 $R_x = \alpha$ ⇒ A and B = open ⇒ all current will flow through galvanometer and deflection may be full scale or s/w 0 and fsd
So varying R_1 ie decreasing R_1 so that galvanometer give full scale deflection. And then dont change R_1 and mark that point $\alpha\,\Omega$.

Case 3 R_X = some standard value
let $R_x = 50\,\Omega$
$R_x = 100\,\Omega$

When $R_x = \alpha$ → I_{fsd} was current through galvanometer in case 2.

so $I_{fsd} = \dfrac{E}{R_1 + R_m}$ → ①

we need value of R_1, for which galvanometer gives full scale deflection.

$$I_{fsd} R_1 + I_{fsd} R_m = E$$

$$R_1 = \frac{E}{I_{fsd}} - R_m \rightarrow ②$$

Now connecting some unknown resistance R_x so now I_m current will flow through galvanometer $I = \dfrac{E}{R_1 + R_m || R_x}$ → ③

Current divider rule $I_m = I\left(\frac{R_x}{R_m+R_x}\right)$

$$I_m = \frac{E}{R_1 + \frac{R_m R_x}{R_m+R_x}}\left(\frac{R_x}{R_m+R_x}\right)$$

$$\Rightarrow I_m = \frac{E R_x}{R_1R_m + R_1R_x + R_mR_x} \rightarrow ④$$

Divide ④ by ①

$$\frac{I_m}{I_{fsd}} = \frac{\cancel{E}\, R_x}{R_1R_m + R_1R_x + R_mR_x} \times \frac{R_1+R_m}{\cancel{E}}$$

$$\frac{I_m}{I_{fsd}} = \frac{R_x(R_1+R_m)}{R_x(R_1+R_m)+R_1R_m}$$

$$\frac{I_m}{I_{fsd}} = \frac{R_x}{R_x + \frac{R_1R_m}{R_1+R_m}}$$ → By ÷ing num and denom by R_1+R_m

$$\rightarrow \frac{I_m}{I_{fsd}} = \frac{R_x}{R_x+R_p} \rightarrow ⑤$$

Using this eq we can callibrate and as resistance is non linear so scale will be non uniform.

- How to differ by 2 shunt type ohmeters

→ using mid scale point.

A and B → $\boxed{R_x = R_h}$ so G will give half scale deflection ie if

$$I_m = \frac{I_{fsd}}{2}$$

using ⑤ we can find R_h

$$R_x = R_h$$

$$I_m = \frac{I_{fsd}}{2}$$

(5) ⇒ $\frac{0.5\, I_{fsd}}{I_{fsd}} = \frac{R_h}{R_h + R_p}$ ⇒ $0.5 = \frac{R_h}{R_h + R_p}$

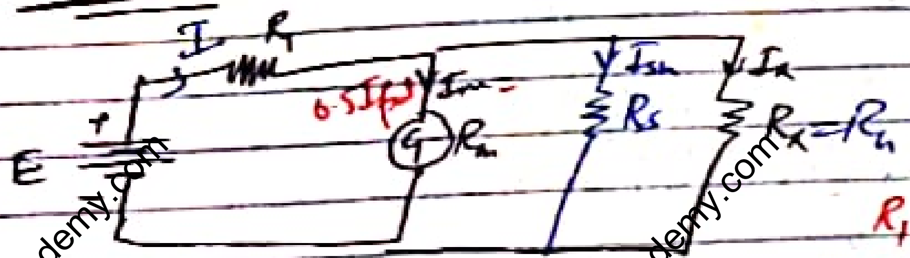
⇒ $R_h = R_p = \frac{R_m R_1}{R_m + R_1}$ = internal resistance.

Ex 4-8

$I_{fsd} = 10mA$

$R_m = 5\Omega$

$E = 3V$



In shunt type R_1 will always be relatively greater than R_m.

Modify the circuit by connecting an appropriate resistance or shunt resistance across galvanometer such that new R_{sh} should be such that at mid point of scale we have 0.5 Ω.
ie $R_h = 0.5\,\Omega$ after modification

(a) $R_{sh} = ?$ (b) $R_1 = ?$

(a) $R_x = R_h$ so meter will give ~~full~~ half scale deflection so $I_m = \frac{I_{fsd}}{2} = 5mA$

$E_m = I_m R_m = 5mA\,(5\Omega) = 25mV$

Find I_x ie $I_x = \frac{E_m}{R_h} = \frac{25mV}{0.5}$

⇒ $I_x = 50mA$

$I_{sh} = ?$ $R_{sh} = ?$

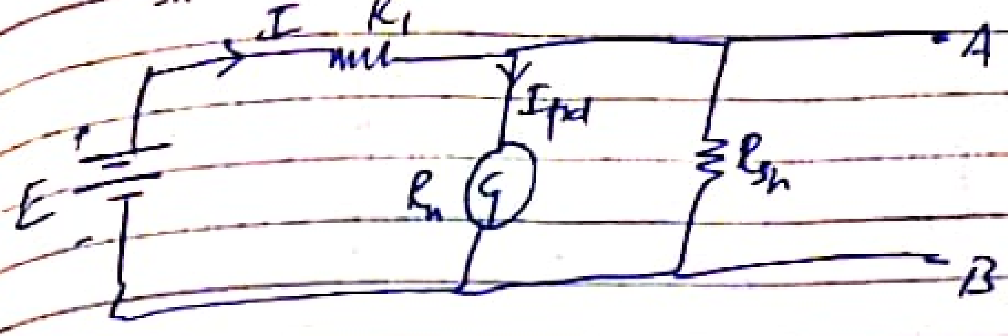


fig 2.

In both figures battery current is almost same because resistance b/w A and B is almost negligible.

In fig 2 current in meter become doubled as fig 1. also I_{sh} is doubled.

In fig 1 $I_{sh} = I_x - I_m$

$I_{sh} = 50 - 5 = 45$ mA

$R_{sh} = \frac{E_m}{I_{sh}} = \frac{25mV}{45mA} \Rightarrow R_{sh} = \frac{5}{9}\Omega$.

(b) $I = I_m + I_{sh} + I_x = 5 + 45 + 50$

$I = 100$ mA

KVL $E = E_1 + E_m$

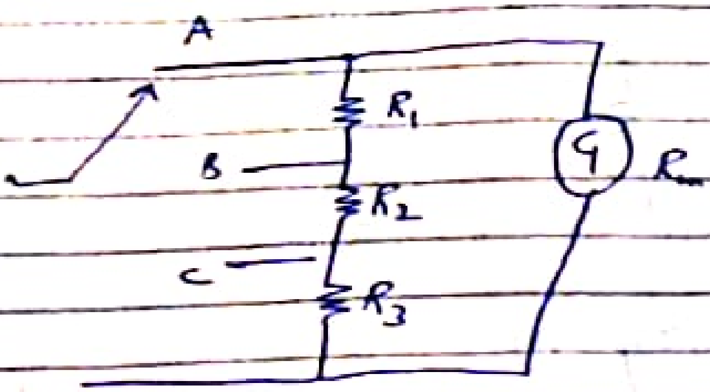
$E_1 = E - E_m = 3 - 25\times10^{-3}$

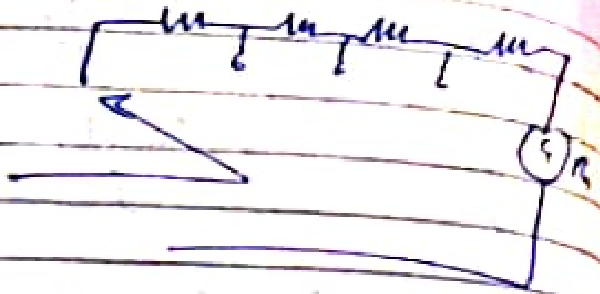
$E_1 = 2.975$ V.

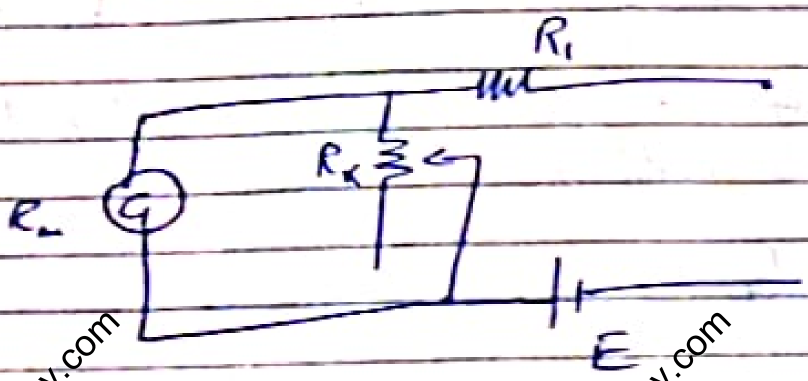
$E_1 = IR_1$

$\Rightarrow R_1 = \frac{E_1}{I} = \frac{2.975}{100\times10^{-3}} = 29.75\Omega$

Multimeter or VOM :



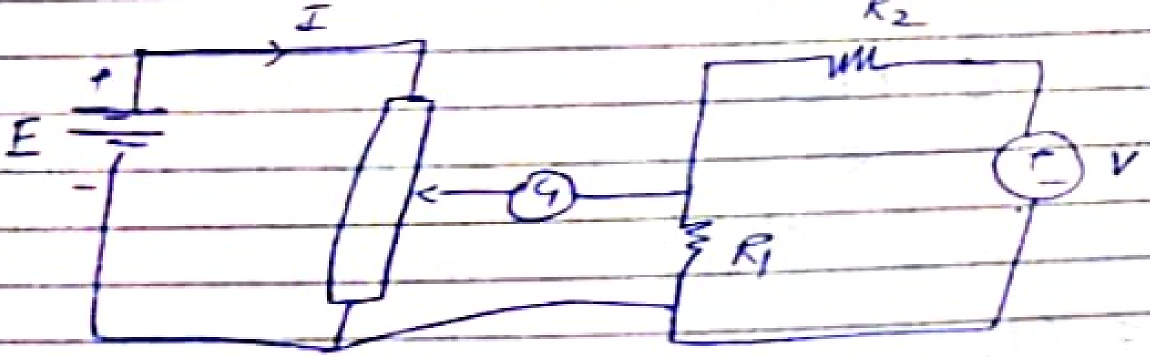




* Callibration of DC instrument

One instrument that can be used for callibration is potentiometer.

Potentiometer is a null type device



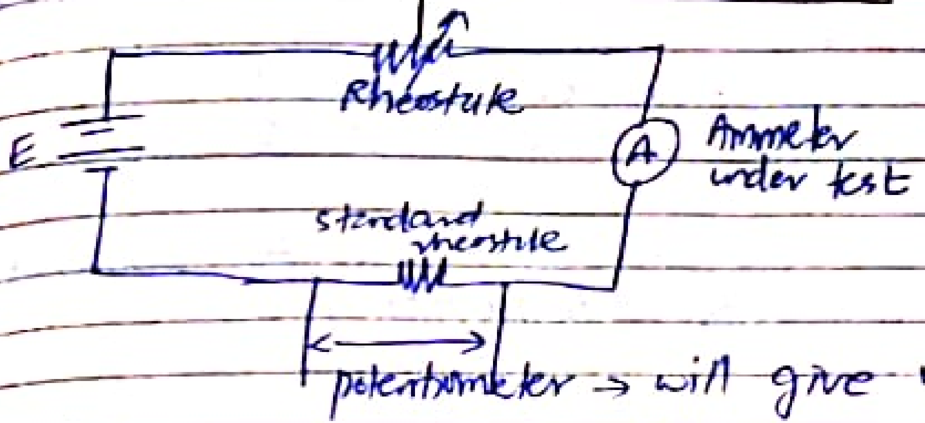
$R_{total} = \frac{\rho L}{A}$, $R_1 = \frac{\rho L_1}{A}$

$E = I R_{total}$, $E_1 = I R_1$

$E = I \frac{\rho L}{A}$, $E_1 = I \frac{\rho L_1}{A}$

$$\frac{E_1}{E} = \frac{l_1}{L} \rightarrow \boxed{E_1 = \frac{l_1}{L} E}$$

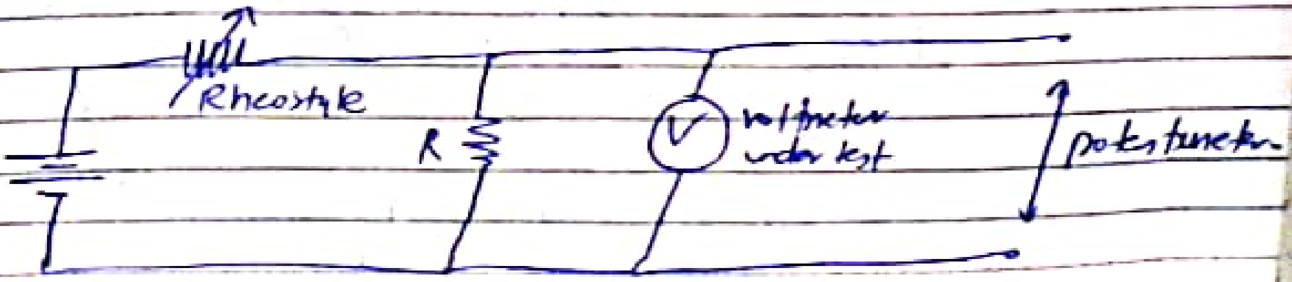
Callibration of DC Ammeter



potentiometer → will give voltage.

$I = V/R$ — Find different current values for different rheostate values and make the scale

Callibration of Voltmeter.



Alternating Current Indicating instrument

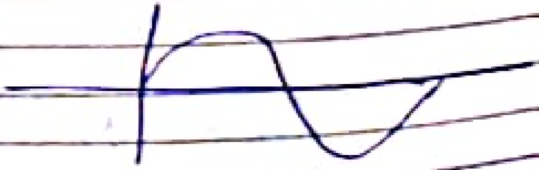
$T \propto I$

Pointers also have some inertia.

AC A

$f = 50$ Hz

$T = 20$ms



Electrodynamometer.

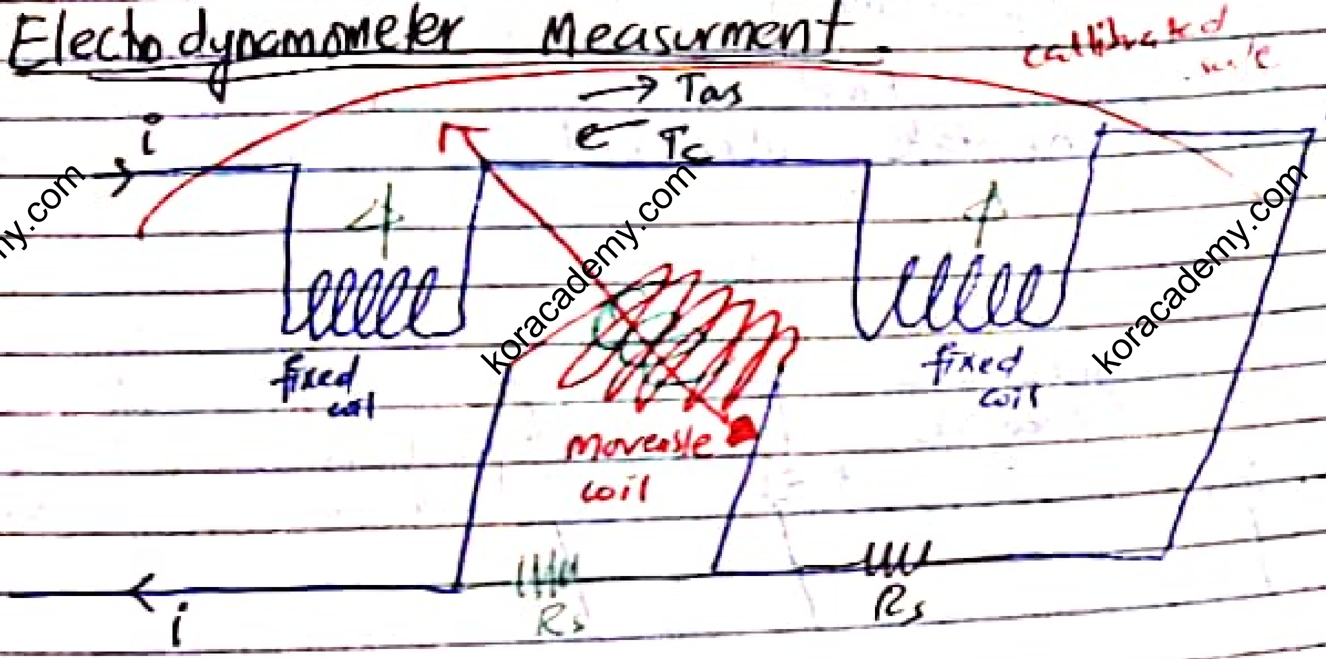
It can be used as AC voltmeter, AC ammeter.

We can also design power factor meter, power meter, frequency meter.

[It is also called transfer instrument
↳ callibration for DC is also valid for AC.

Disadvantage → can be used only for power line frequencies (low range frequencies).

Electrodynamometer Measurment



$i \to \phi$ $\qquad T \propto \phi i N$

$T \propto \phi i$ $\qquad$ N is constant

As $\phi \propto i \Rightarrow T \propto i^2$ $\qquad T_{inst} = K_1 i_{inst}^2$

instantanous → $T_{inst} \propto i_{inst}^2$ — ①

$T_{avg} = avg(T_{inst}) = avg(K_1 \, inst^2)$

$T_{avg} = \frac{1}{T} \int^{T} K_1 i_{inst}^2 \, dt$

$$T_{avg} = k_1 \frac{1}{T}\int_0^T i_{inst}^2\, dt$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i_{inst}^2\, dt}$$

$$\Rightarrow \boxed{T_{avg} = k_1 I_{rms}^2}$$

to make it stop → attach spring with pointer

$$\boxed{T_c = k_2 \theta}$$

Pointer will stop when $T_{avg} = T_c$

$$k_1 I_{rms}^2 = k_2 \theta \Rightarrow \theta = \frac{k_1}{k_2} I_{rms}^2$$

$$\rightarrow \theta = k I_{rms}^2 \Rightarrow \theta \propto I_{rms}^2$$

θ → deflection

Disadvantage?

- Scale is non uniform as equation is non linear
- very low sensitivity.
- high power consumption

to increase range → R_s → [illegible] must be coil

conductors of coil should be more thicker.

To convert to voltmeter add R_s as in series as shown

Lecture 7:

Form factor

Ex 4.9 , 4.10

Thermo instruments

Ex 1.1 117.02, 117.11, 117.09, 117.03.

$E_{avg} = \frac{117.02 + 117.11 + 117.09 + 117.03}{4} = 117.06$

Range = Max − Min   Range of error = Max − avg

117.11 − 117.06 = 0.05 V.

1.2 $R_1 = 18.7\ \Omega$ $R_2 = 3.624$ Ans

23.$\boxed{3}$24 ... < 50 = 23.$\boxed{3}$

1.3 $I = \boxed{3.18}$ A $R = 35.68\ \Omega$

$(3.18)(35.68) = 113.\boxed{4624}$.

3 4 ... < 50

Ans 113.

Ex 1.4

Add 826 ± $\boxed{5}$ + 628 ± $\boxed{3}$.

= 1454 → ± $\boxed{8}$ → worst case

How to calculate %age error?

1.5 Subtract 628 ± 3 from 826 ± 5

1.6 ___

1.7 Sensitivity = 1000 Ω/V reads 100 V

on 150 scale

multiplier current 5 mA

$R_T = \frac{V_T}{I_T} = \frac{100}{5mA}$ = 20 kΩ

~~Lecture 6~~ : ~~30/10/19~~

~~Shunt type ohmmeter~~

Volt. resistance = sensitivity × full scale

$= 1000 \times 150 = 150\,k\Omega$.

parallel

$$\frac{R_T R_V}{R_T + R_V}$$

Voltmeter resistance = sensitivity × full scale deflection.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{20 \times 150}{170}$$

% error = $\frac{\text{actual} - \text{[illegible]}}{\text{actual}}$

$$R_T = \frac{40}{800\,mA} = 50\Omega.$$

$R_V = 150 \times 1000 = 150\,k\Omega$

$R_x = \frac{R_T R_V}{[illegible]}$

$\bar{x} = 12.0 + 12.2$ [illegible]

$12 + 1 - 12.05$

deviation = reading − average

[illegible] Reading? [illegible] $\bar{x} = \frac{\sum x}{n}$

error = σ
↳ std. dev

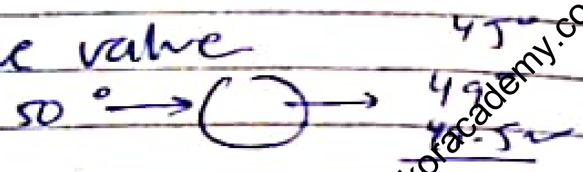
$$\sigma = \sqrt{\frac{\Sigma d^2}{n-1}}$$

0-150 voltmeter accuracy 1 percent full scale
V = 83 V limiting x gr error reading.

limiting error = 150 × 0.01 = 1.5 V

% error at FL = $\frac{1.5}{83}$ × 100 % p = 1.81 p

Three Things associated with a measuring device.
① Accuracy ② Precision ③ Sensitivity
↳ closeness with the value



②

let i/p is F = 10N

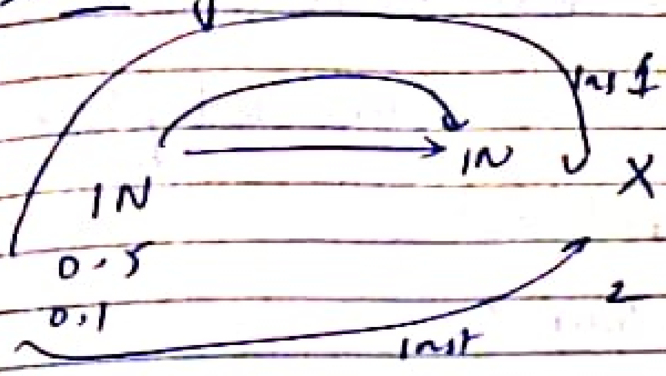
o/p	Instrument	Readings again and again				
	1	9	8	7	6	
	2	9	9	2	3	
	3	9	7	5	4	
	4	7	7	7	7	→ precise
	5	5	6	7	9	

Precise → o/p is reproduced
↳ almost similar o/ps

"Precision is not the guarantee of accuracy.

St Accuracy is the precision of precision.

⑤ Sensitivity.



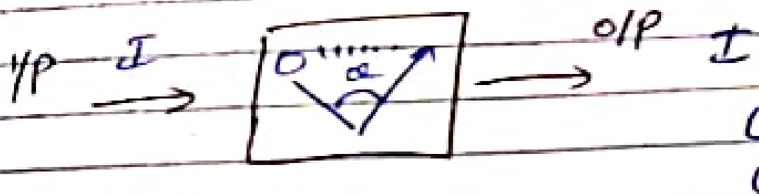
2 is more sensitive.

Responds to smaller change in i/p → more sensitive

Sensitive → to the variable for which it is designed

Resolution
Smallest change in i/p that a device can measure
eg ruler – 1mm.

DC Meter . → will give you average value



$\theta \propto I$

$\theta = KI$

↳ scale is linear

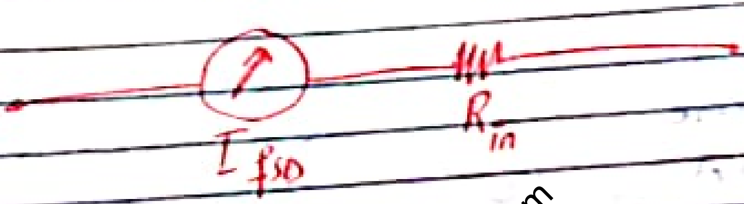
indicates on the scale

eg PMMC .
↳ measures DC not AC.

AC i/p → 0 o/p.

If $\theta \propto I^2$ → RMS measuring device.

→ An instrument is defined by the maximum current it can measure and its internal resistance.



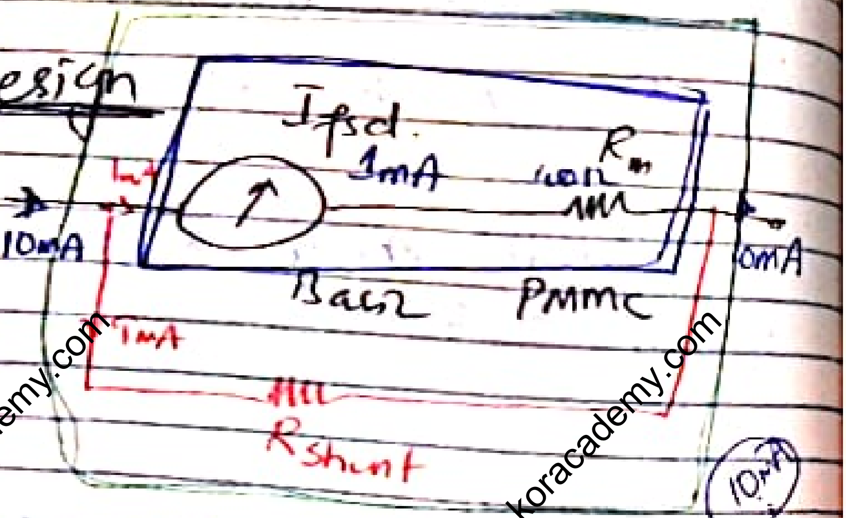
AC meter measures both DC and AC

$\theta \propto I^2 \rightarrow$ RMS indicating device.

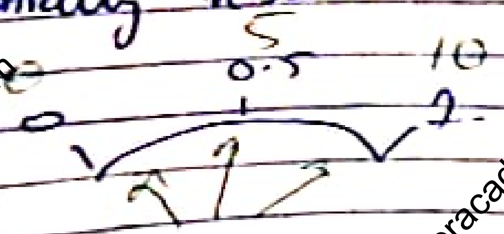
The scale is non linear.

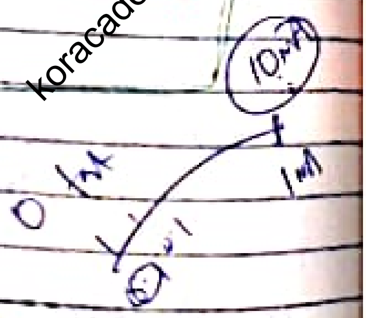
→ eg moving iron instrument.

DC Ammeter Design



Initially the scale





$R_s = ?$

$V_m = V_{sh}$

$I_m R_m = I_s R_s$

$\Rightarrow R_s = \frac{I_m R_m}{I_s}$

1mA × 100 = 9mA × R_s

$R_s = \frac{0.1}{9mA} = \frac{100}{9} = 11.11$

multiplying factor / scale calibration factor = $\frac{\text{Required value}}{\text{Actual value}}$

m

In this case

$\frac{10mA}{1mA} = 10$

As $I_m R_m = I_s R_s$

$$\frac{R_m}{R_s} = \frac{I_s}{I_m} = \frac{I - I_m}{I_m}$$

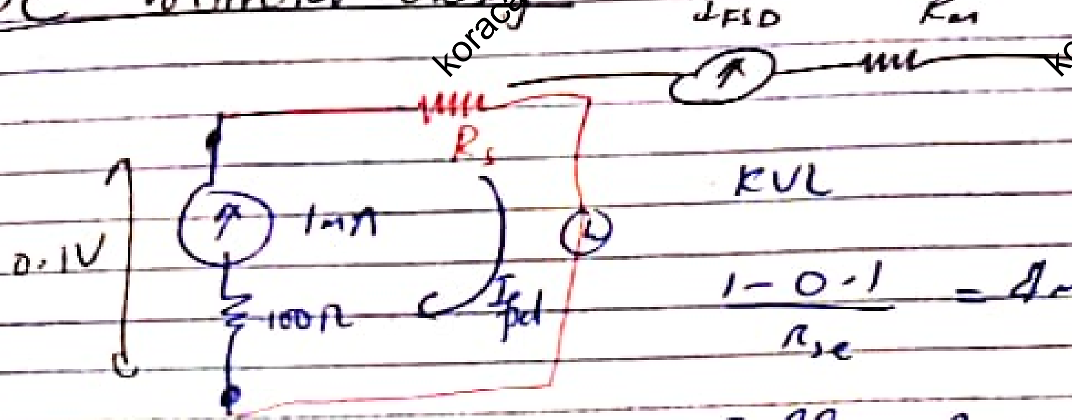
$$\frac{R_m}{R_s} = \frac{I}{I_m} - 1.$$

$$\frac{R_m}{R_s} + 1 = \frac{I}{I_m} = \text{multiplying factor}$$

$$M = \frac{R_m}{R_s} + 1$$

DC Voltmeter design.

Let



KUL

$$\frac{1 - 0.1}{R_{se}} = 1mA$$

$$\frac{0.99}{1m} = R_{se}$$

$\Rightarrow R_{se} = 0.99\ K\Omega$

generally

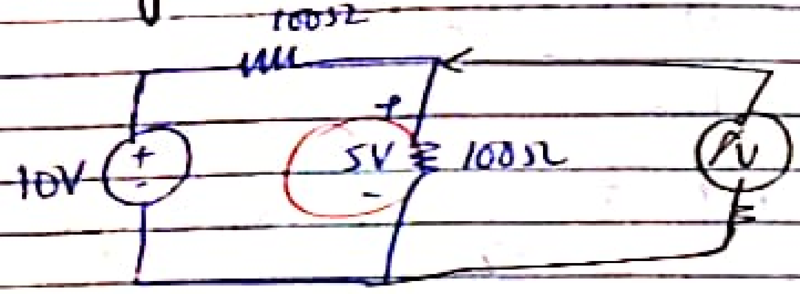
$$\frac{V_{FSD} - I_{FSD} R_m}{R_{se}} = I_{FSD}$$

$$V_{FSD} = R_{se} I_{FSD} + I_{FSD} R_m$$

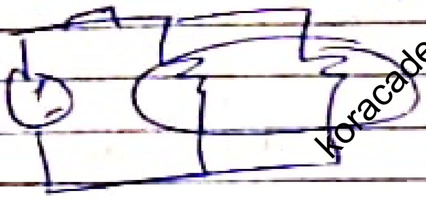
$$V_{FSD} = [R_{se} + R_m] I_{FSD}$$

$$\text{Sensitivity} = \frac{1}{I_{FSD}}$$

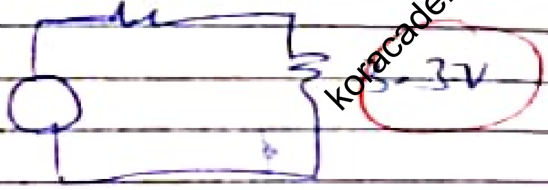
Loading Error:



resistance of voltmeter very high say 100 Ω



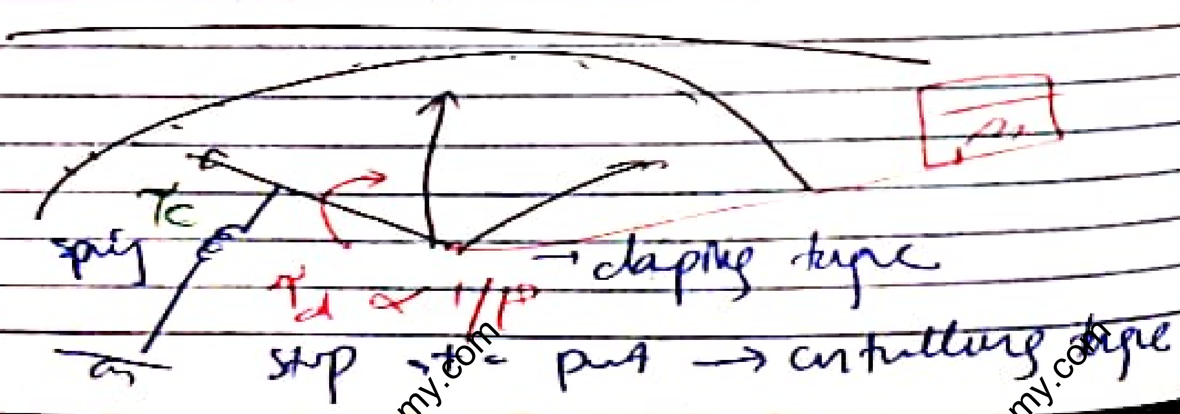
q



5 → 3.3
↳ loading error
↳ voltmeter resistance

$L.E \downarrow \rightarrow R_V > 10\, R_L$

$R_v \uparrow \rightarrow S \uparrow = \frac{\Omega}{V}$



$= \tau_c$

gravity control is also there
↳ vertical mounted instruments

damping torque?

vibration after $\tau_d = \tau_c$
↳ to stop
piston
→ air damping
↳ liquid damping
↳ eddy current damping → when there is high magnetic field

000, 000, 000, 000, 000

143, 010, 120, 009, 018 δ

σ

Addition 3 150.0 1
0.702 3

=

4.51 100 Tn error = $x_m - x_t$

3 1's % error = $\frac{x_m - x_t}{x_t} \times 100$

= 7.51

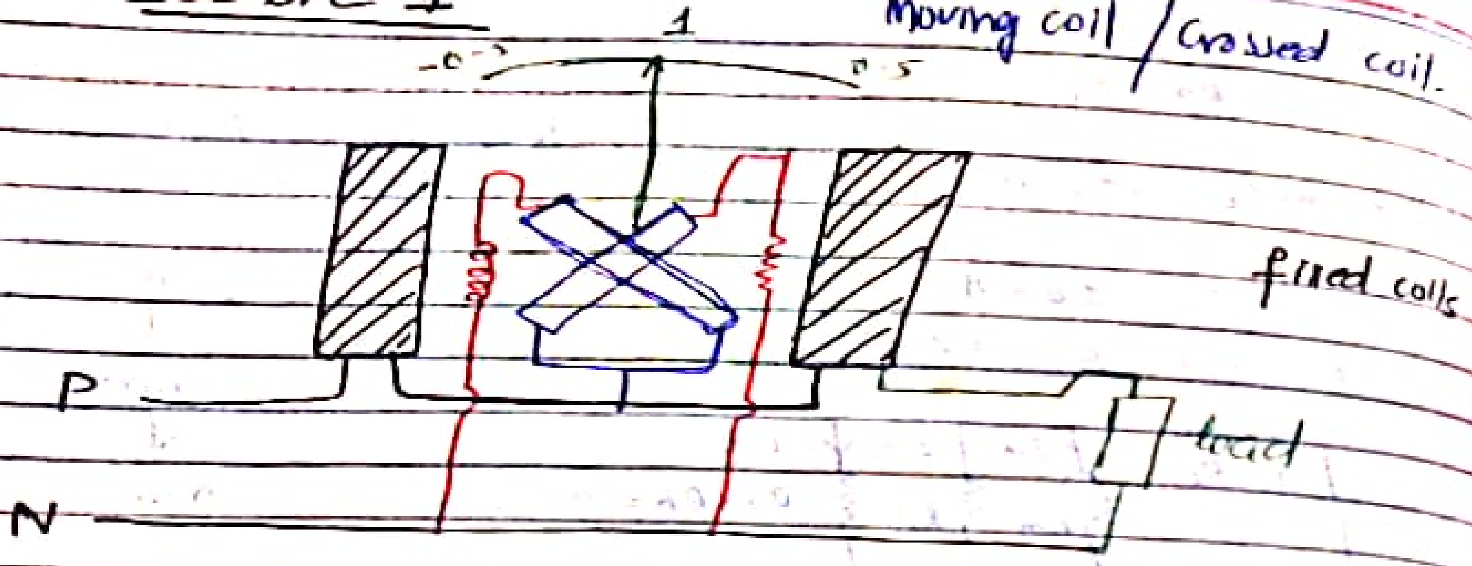
= 8

5, 3, 2 $\frac{5+3+2}{3}$

Probal error = 0.6745 σ

FINAL

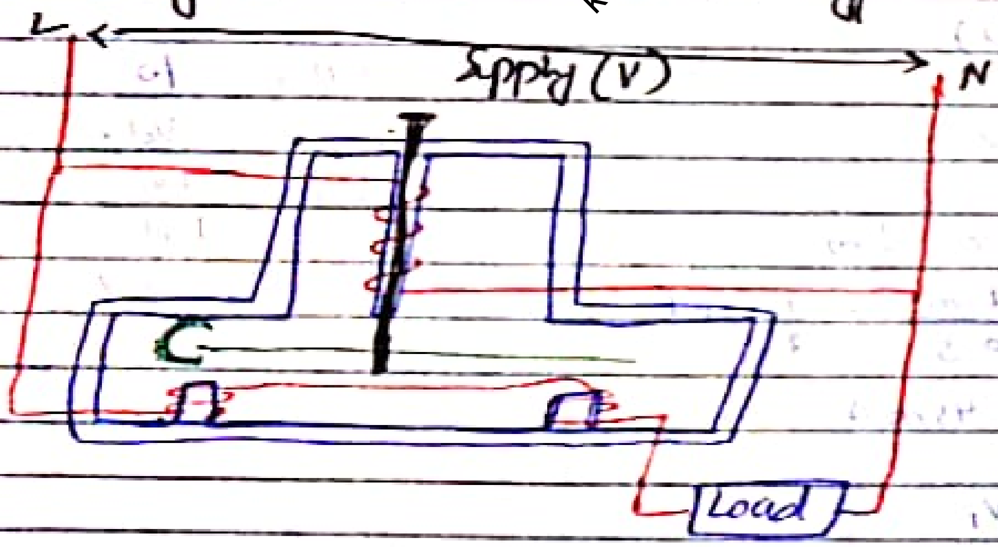
Lecture 1



Watt hour ~~readin~~ meter
↳ unit of energy

Kilowatt hour = Units.

Single Phase Induction Type Energy meter



↳ is an integrating type instrument
↳ (i) counting
(ii) clipping
(iii) cumulative addition

4 main parts
i) Driving system
↳ voltage coil, current coil and the core.
↳ has to produce driving torque.

Eddy

ii) Moving System
↳ Disk, shaft and bearings.

iii) Braking System
Basically U shaped permanent magnet.

iv) Registering / Counting system
made from gears

Working.

Voltage coil → highly inductive

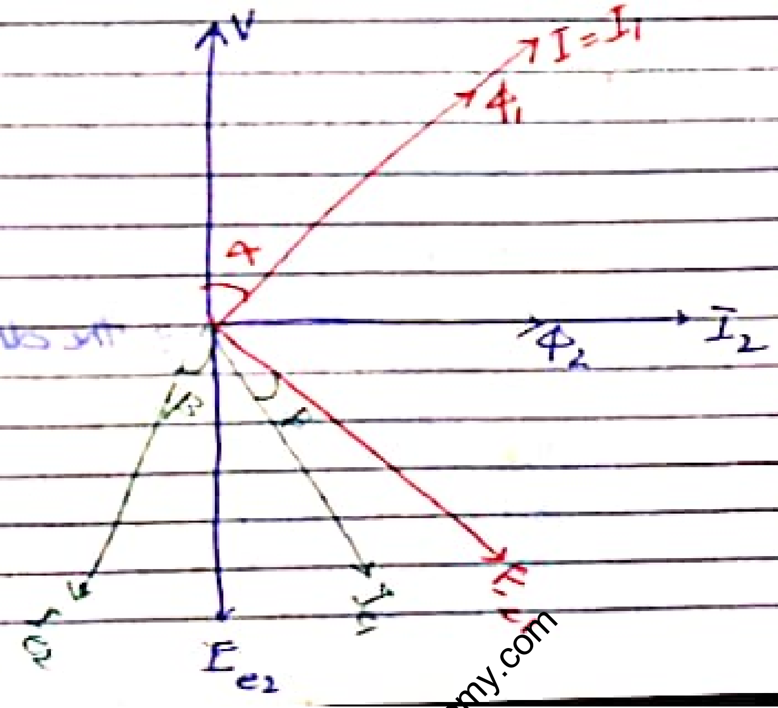
I_2 laggs V by 90°

Flux ϕ_2 produces. $\phi_2 \propto I_2$

Flux is always in phase with its current.

Induced emf E_{e2} laggs ϕ_2 by 90°

I_{e2} laggs E_{e2} → by angle ? → depends on perm fricts of the disc.



and T_2 are opposite in directions

$T_{inst} \propto IV$ $\qquad$ $T_{avg} \propto P$

viva

$T_{avg} \propto VI \cos \phi$

$T_{inst} \propto i_{e1} \phi_2$

$T_1 \propto I_{e1} \phi_2 \cos(\alpha + \beta)$

$T_2 \propto I_{e2} \phi_1 \cos(180° - \alpha + \beta)$

$$T_d = T_1 - T_2$$

$T_d \propto I_{e1} \phi_2 \cos(\alpha + \beta) - I_{e2} \phi_1 \cos(180 - \alpha + \beta)$

$T_d \propto IV \cos(\alpha + \beta) - VI \cos(180° - \alpha + \beta)$

$T_d = K_1 VI [\cos(\alpha + \beta) - \cos(180 - \alpha + \beta)]$

As $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$

$T_d = 2K_1 VI \cos\alpha . \cos\beta$

p.f of the [illegible]

$2K_1 \cos\beta = K_2$

$T_d = K_2 VI \cos\alpha$

$\rightarrow \boxed{T_d = K_2 P}$

Breaking torque.

$T_b \propto n$ speed of disc.

$$\boxed{T_b = K_3 n}$$

Point of action of these two torques are different.

$T_b = T_d$ → disc will rotate with uniform speed.

$$K_3 n t = K_2 P t$$

$$K_3 N = K_2 E$$

N → no. of revolutions
E → energy

$$N = \frac{K_2}{K_3} E \Rightarrow N = k E$$

$$N \propto E$$

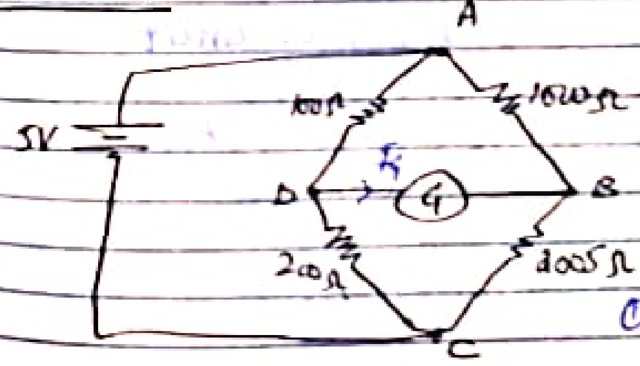
$K = \frac{N}{E} = \left(\frac{rev}{kW/Hour}\right)$
→ meter constant.

Lecture 2 04/12/19

Wheatstone Bridge

G has S = 10mm/μA

Ex 5.1



Internal resistor of G, $R_m = 100\Omega$

Deflection D = ?

Solving by thevenin eq circuit

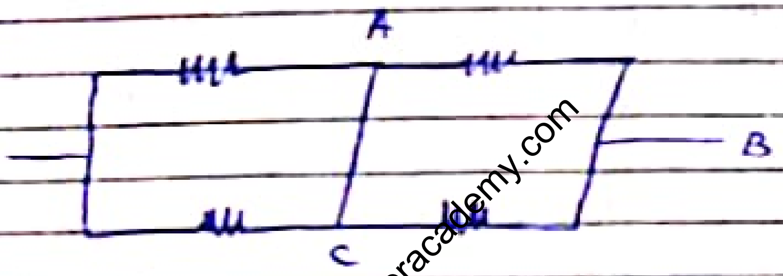
① Remove the element.

$$E_{Th} = E_B - E_D$$

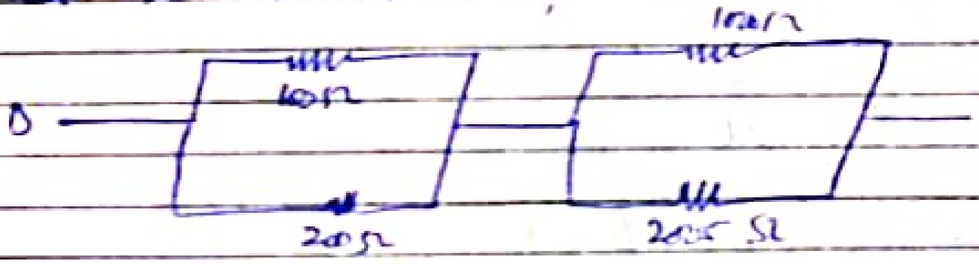
$$E_{Th} = 5 \times \frac{2005}{1000 + 2005} - 5 \times \frac{200}{100 + 200}$$

$$E_{Th} = 2.77 \text{mV}$$

① Find R_{Th}. open circuit resistence.
For This make all sarces zero first.



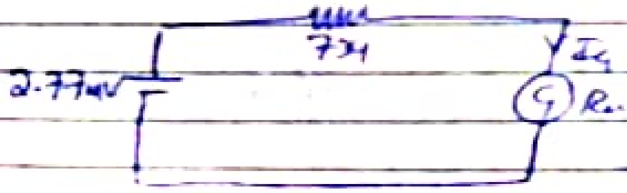
or can be redrawn as:



$$R_{Th} = 100 // 200 + 1000 // 2005$$

$$\Rightarrow R_{Th} = 734 \Omega$$

① Thevenin eq circuit



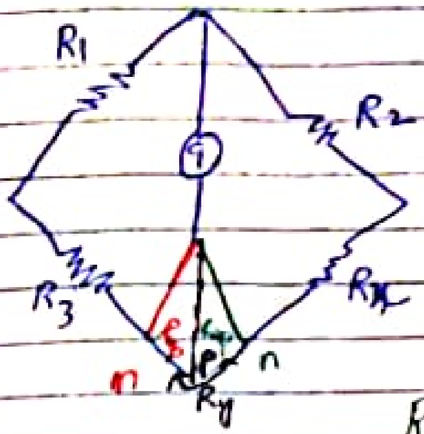
$$I_g = \frac{E_{Th}}{R_{Th} + R_m}$$

$$\boxed{I_g = 3.32 \mu A} \leftarrow = \frac{2.77 \times 10^{-3}}{734 + 100}$$

$$D = I_g \times S = 332 \times 10$$

$$\Rightarrow \boxed{D = 332 \text{ mm}}$$

Effects of connecting leads



$R_1(R_x+R_y) = R_2R_3$

$R_1R_x = R_2(R_3+R_y)$

If connected at point P.

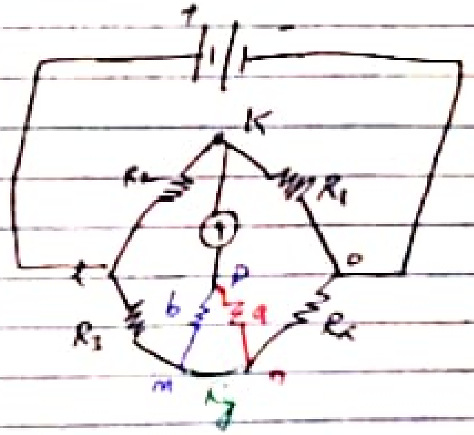
$R_1(R_x+R_{np}) = R_2(R_3+R_{mp})$

$R_1R_x + R_1R_{np} = R_2R_3 + R_2R_{mp}$

$R_1R_x = R_2R_3 + R_2R_{mp} - R_1R_{np} \to ①$

$\left.\begin{array}{l} \frac{R_{mp}}{R_{np}} = \frac{R_1}{R_2} \\ R_2R_{mp} = R_1R_{np} \end{array}\right\}$ $R_1R_x = R_2R_3$.

Kelvin Bridge



$\frac{a}{b} = \frac{R_1}{R_2} \to ①$

balanced

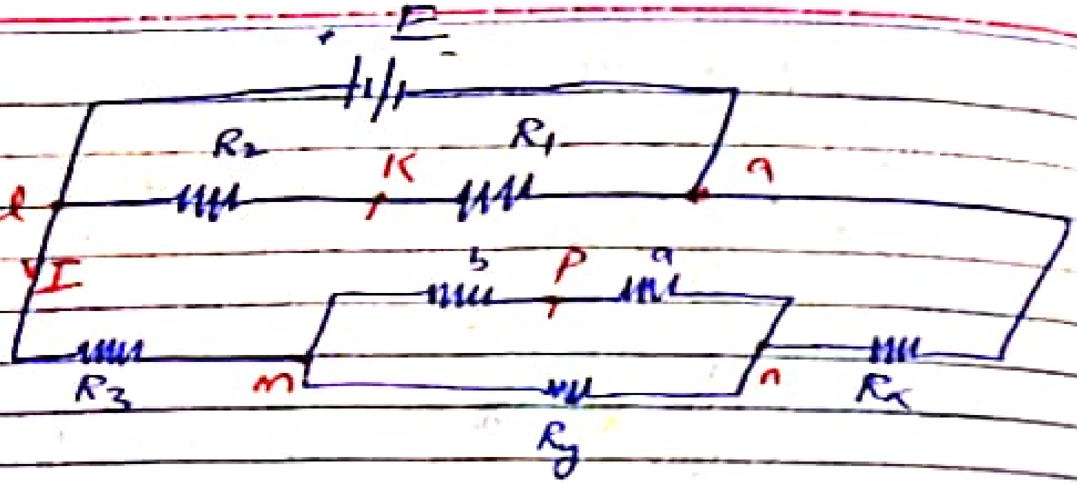
$E_k = E_p$

$E_t - E_k = E_t - E_p$

$E_{tk} = E_{tp} \to ②$

E_{tk}

$E_{tk} = E \times \frac{R_2}{R_1+R_2}$



$$E = I \times \left(R_3 + (a+b) \,||\, R_y + R_x \right) \to ④$$

$$E_{lk} = \frac{R_2}{R_1+R_2} \times I \times \left(R_3 + \frac{(a+b)R_y}{a+b+R_y} + R_x \right) \to ⑤$$

<u>E_{lp}</u>

$$E_{lp} = IR_3 + E_{mp} \to ⑥$$

$$E_{mp} = E_{mn} \times \frac{b}{a+b} \to ⑦$$

$$E_{mn} = I \times \left((a+b) \,||\, R_y \right) \to ⑧$$

$$E_{mp} = I \times \frac{(a+b)R_y}{a+b+R_y} \times \frac{b}{a+b}$$

$$E_{mp} = I \times \frac{bR_y}{a+b+R_y} \to ⑨$$

⑨ in ⑥ $$E_{lp} = I\left(R_3 + \frac{bR_y}{a+b+R_y} \right) \to ⑩$$

⑨ and ⑩ in ②

$$\Rightarrow \frac{R_2}{R_1+R_2} \times I \times \left(R_3 + \frac{(a+b)R_y}{a+b+R_y} + R_x\right) =$$

$$I \left(R_3 + \frac{bR_y}{a+b+R_y}\right)$$

$$\Rightarrow R_3 + \frac{aR_y}{a+b+R_y} + \frac{bR_y}{a+b+R_y} + R_x = \left(\frac{R_1+R_2}{R_2}\right)\left(R_3 + \frac{bR_y}{a+b+R_y}\right)$$

$$\Rightarrow \cancel{R_3} + \frac{aR_y}{a+b+R_y} + \cancel{\frac{bR_y}{a+b+R_y}} + R_x = \frac{R_1R_3}{R_x} + \frac{R_1}{R_2}\frac{bR_y}{a+b+R_y} + \cancel{R_3} + \cancel{\frac{bI}{a+b+R_y}}$$

$$\Rightarrow R_x = \frac{R_1R_3}{R_2} + \frac{R_1}{R_2} \cdot \frac{bR_y}{a+b+R_y} - \frac{abR_y}{b(a+b+R_y)}$$

$$R_x = \frac{R_1R_3}{R_2} + \cancelto{0}{\frac{bR_y}{a+b+R_y}\left(\frac{R_1}{R_2} - \frac{a}{b}\right)}$$

$$\Rightarrow \boxed{R_x = \frac{R_1R_3}{R_2}}$$

Lecture 3 11/12/19

AC Bridge applications

Source → AC
↳ Power line frequency 50 - 60 Hz
↳ RF = radio frequency.

3/

4/ V V V

1) Detector → Buzzer
→ Headphone
→ Amplifier
→ Electrogn

4) Balanced principle for balanced condition

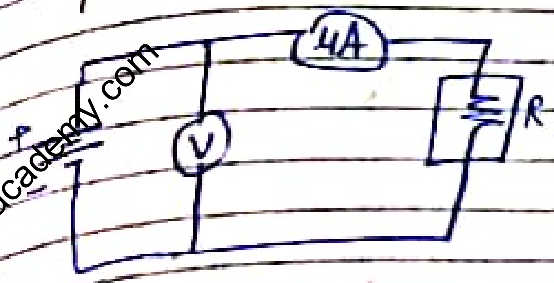
Impedences Z_1 Z_2
Z_3 Z_3

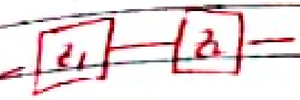
Ⓐ → I

Guarded Wheatstone Bridge

Ⓥ → V

$R = \frac{E}{I}$





$Z_1 + Z_2$

$\frac{Z_1 Z_2}{Z_1 + Z_2}$

$\frac{Y_1 Y_2}{Y_1 + Y_2}$

$Y_1 + Y_2$

$Z = R + jX$ R → resistance X → reactance

$X_L = j\omega L$ $X_C = \frac{-j}{\omega C}$

$Y = G + jB$

G = Conductance = 1/R

B = Susceptance

$= \frac{1}{}$

$\dashv\vdash \; B = +j\omega C$ $\quad$ $\text{—ᗰ—} \; B = -\frac{j}{\omega L}$

Lecture 4 $\quad$ 18/12/19

Schering Bridge

Wein Bridge

Wagner ground connection

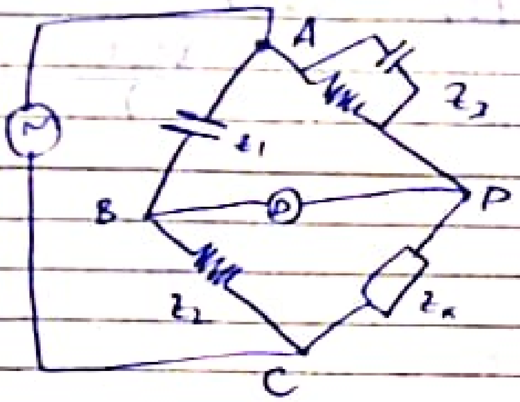
charge → tre coming
[illegible] [illegible]

Problem 5.5

$f = 1\ kHz$ $\quad$ AB: $C_1 = 0.2\mu F$ $\quad$ BC: $R_2 = 500\Omega$

CD: unknown $\quad$ DA: $R_3 = 300\Omega \parallel C_3 = 0.1\mu F$

CD: R and C or L connected in series. $\quad$ Balanced bridge.



Balanced so $Z_1 Z_x = Z_2 Z_3$

$$Z_x = \frac{Z_2 Z_3}{Z_1} = \frac{Z_2}{Z_1 Y_3}$$

$Z_1 = \frac{-j}{2\pi f C_1} = \frac{-j}{2\pi(1000)(0.2\times10^{-6})} = -j\,795.8\,\Omega$

$Z_2 = R_2 = 500\,\Omega$

$Y_3 = \frac{1}{R_3} + j2\pi f C_3 = \frac{1}{300} + j2\pi(1000)(0.1\times10^{-6}$

$Y_3 = (3.33\times10^{-3} + j\,628.32\times10^{-6})\,℧$

$\Rightarrow Z_x = \frac{500}{(-j795.8)(3.33\times10^{-3} + j628.32\times10^{-6})}$

$\Rightarrow Z_x = (34.4 + j\,182.2)\,\Omega$

~~Ans~~

+j → inductor
-j → capacitor

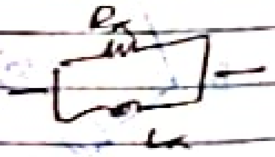
$Z_x = R_x + j\omega L_x$

$R_x = 34.4\,\Omega$

$2\pi f L_x = 182.2$

$L_x = \frac{182.2}{2\pi(1000)} = 29\,mH$

If in parallel,



$Y_x = \frac{1}{Z_x}$

$Y_x = G_x - jB_x$

$Y_x = \frac{1}{R_x} - \frac{j}{2\pi f L_x}$

Lecture 5 CT, PT

Lecture 6 24/12/19

Oscilloscope.

Lab12 To learn the construction and working of DSO.

Lecture 7 01/01/20

Transducer

Lecture 8 08/01/20

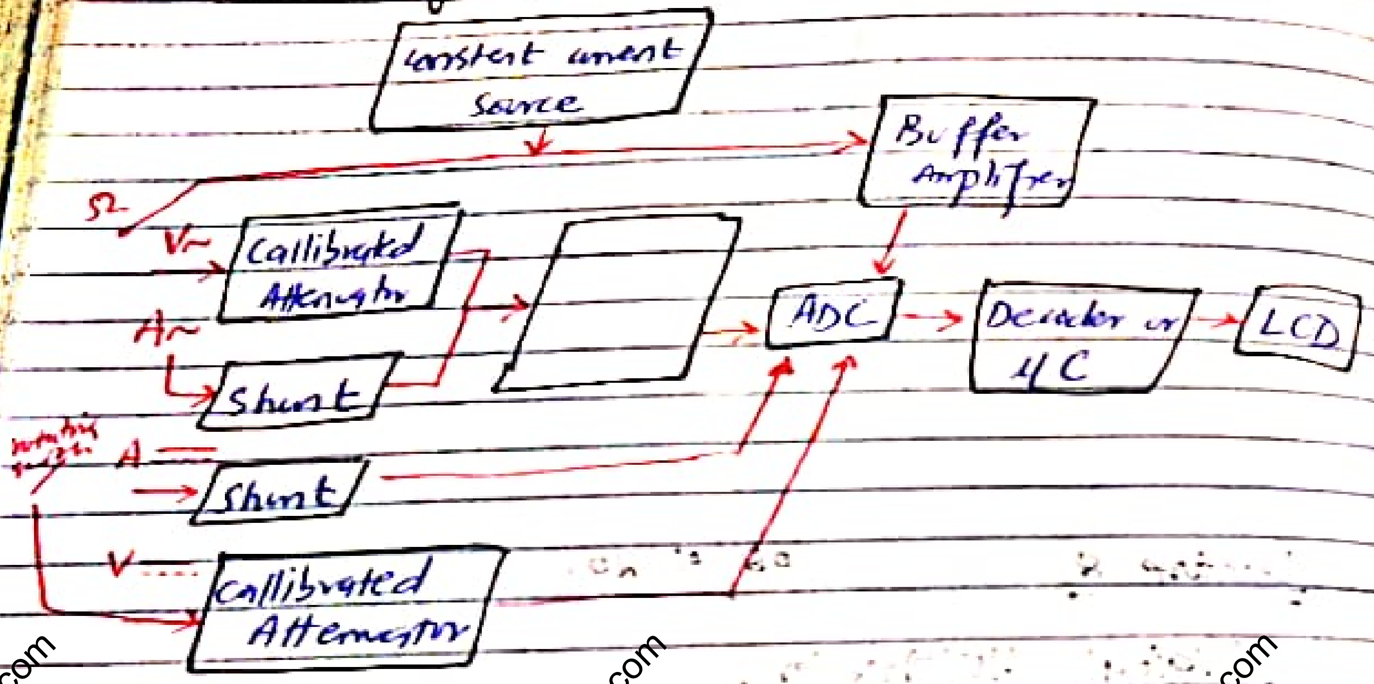
Digital Multimeter.

We will measure
(i) AC voltage (ii) DC voltage (iii) AC current
(iv) DC current (v) Resistance.

Principle? we will convert unknown quantity (except DC voltage) into an equivalent DC voltage → Then we will measure that DC voltage → And then we will callibrate to find the unknown quantities.

V~ AC voltage V ⎓ DC voltage
A~ AC current A ⎓ DC current
Ω Resistance.

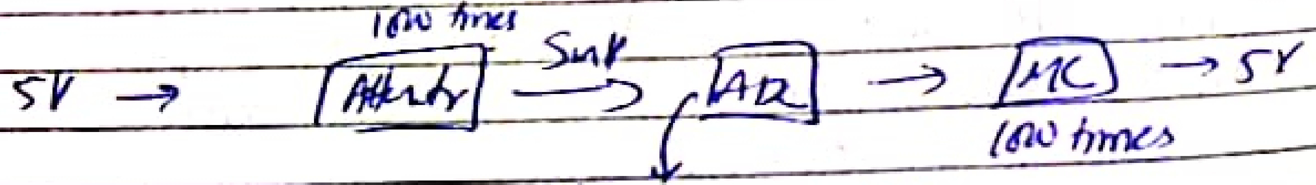
Block diagram.



DC voltage measurement.

i/p may be very high so we use callibrated attenuator (series resistance) so we connect very high resistance in series to galvanometer (kΩ, MΩ)

let i/p voltage is 5V.
→ attenuator 100 times = 5mV



let

1mV	101010…
2mV	01011…
⋮	
5mV	01010—

~~AC voltage~~ If in nano Volts then amplifier is used

AC voltage Measurement

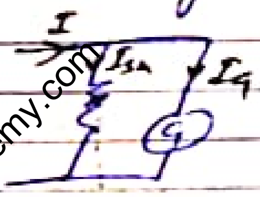
rotating switch $V\sim$

very high $\rightarrow \boxed{\text{Attenuator}} \rightarrow \boxed{\text{AC to DC}} \rightarrow \boxed{\text{ADC}} \rightarrow \boxed{\mu C}$

DC current

$A\sim \rightarrow \boxed{\text{Shunt}} \xrightarrow{\propto} \boxed{\text{ADC}} \rightarrow \boxed{\mu C} \rightarrow I$

we use R in parallel to galvanometer to convert to ammeter.



$I_{sh} \approx I$

$V_{sh} = I_{sh} R_{sh}$

$V_{sh} \propto I_{sh}$

R_{sh} is very low so V_{sh} so low so no need of attenuator. now $I = \frac{V_{sh}}{R_{sh}}$

AC current

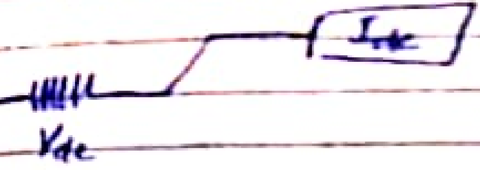
$A\sim \quad \boxed{\text{Shunt}} \xrightarrow{V\sim} \boxed{\text{AC to DC}} \rightarrow \boxed{\text{ADC}} \rightarrow \boxed{\mu C}$

↓

we may use current to voltage circuit.

Resistance measurement

→ Ω   Constant current source with provide DC cur

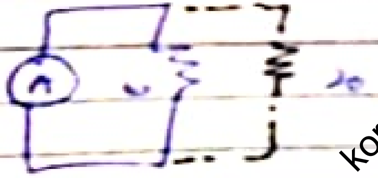


→ If voltage of the source is independent of the load then it is voltage source.

→ If the current applied by the source is not changing with change in load, then it is current source.

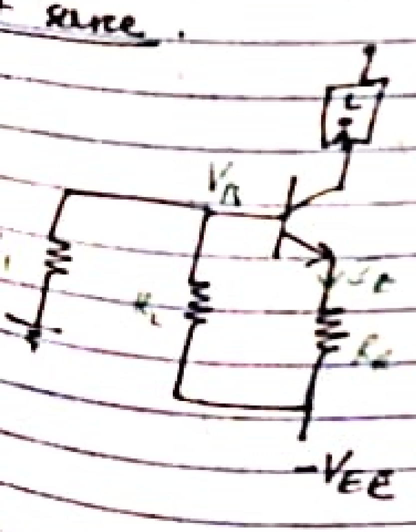
eg

V same



I same

Current source.



Suppose base input impedance r_i of BJT is much larger than R_1 and R_2.

choose R_1, R_2 that base current is greater.

$$V_B = \frac{R_1}{R_1 + R_2}(-V_{EE})$$

$$V_{BE} = V_B - V_E = 0.7$$

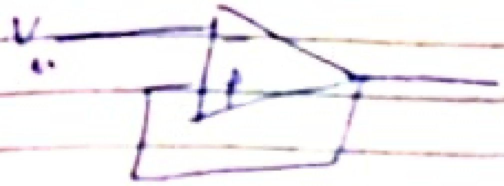
$$V_E = V_B - 0.7$$

$$I_E = \frac{V_E + V_{EE}}{R_E} \approx I \quad (\text{constant})$$

as the quantities are constant

Buffer Amplifier.

op amp is a three port device
(i) i/p (ii) o/p (iii) Power



In -ve feedback
$V^+ = V^-$

If we connect V_{in} to ground
then $V_{out} = 0$, It is virtual ground.

$V^+ = V_{in}$ $V^- = V_{out}$ $V_{out} = V_{in}$

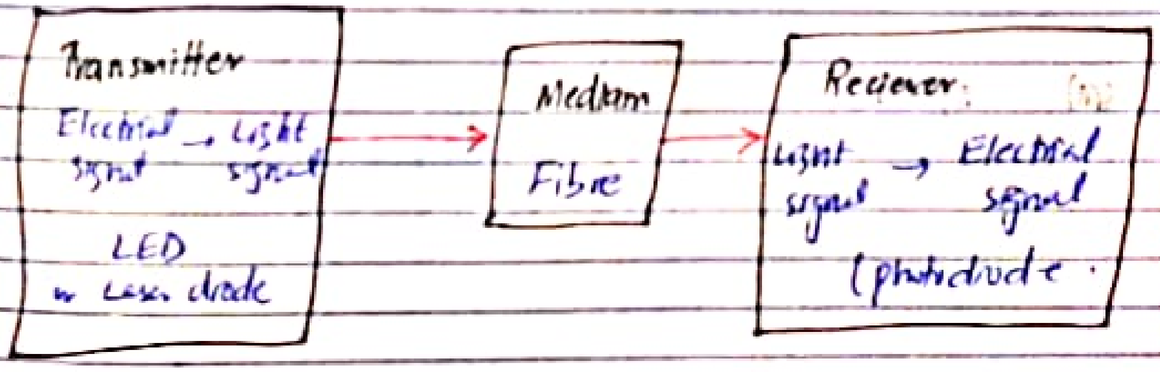
$A_f = 1$

- Impedance matching
  i/p impedance almost ∞. o/p almost 0

② Isolation.
Isolate one part of circuit from another part.

$V_0 = I_{out} R$.

Fibre optic Measurement

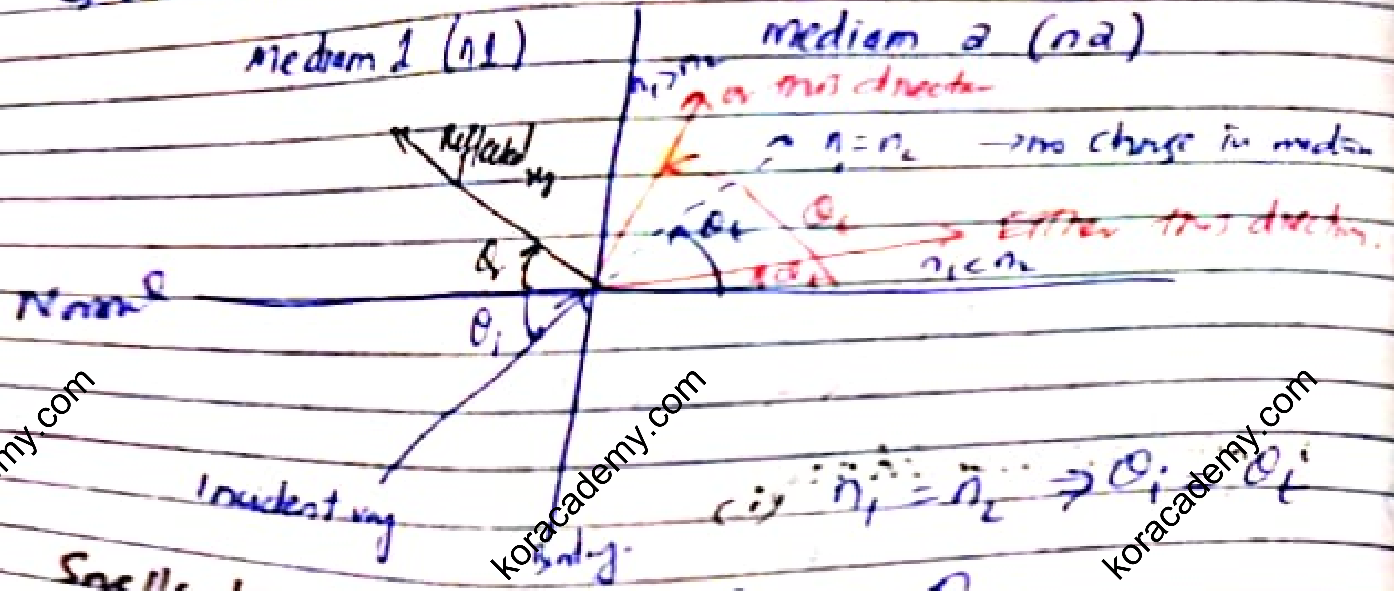


Fibre optics work on the principle of total internal reflection.

$n \rightarrow$ refractive index.

$n = 1$ for air

↳ For all other mediums is greater than 1.



(i) $n_1 = n_2 \Rightarrow \theta_i = \theta_t$

Snells law of reflection: $\theta_i = \theta_r$

Snells law of refraction: $n_1 \sin\theta_i = n_2 \sin\theta_2$

(ii) $n_1 > n_2$ ⟶ $\theta_i \uparrow \rightarrow \theta_t \uparrow$

when $\theta_t = 90°$

where $\theta_i = \theta_c$

↓

if $\theta_i > \theta_c$

↓

all the ray is reflected

↓

total internal reflection

$\sin\theta_i < \sin\theta_t$

$\theta_i < \theta_t$

Bend away from the normal

(iii) $n_1 < n_2$

$\sin\theta_1 > \sin\theta_t$

$\theta_1 > \theta_t$

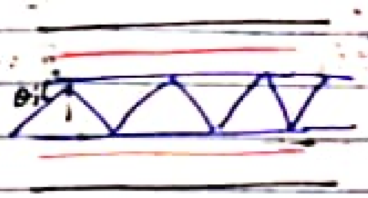
Bend towards the normal

Fibre can are made of glass or plastic (polymer).

For long range communication → glass
short range → plastic
medium range → both

Inner cylinder core → n_1
outer cylinder cladding → n_2

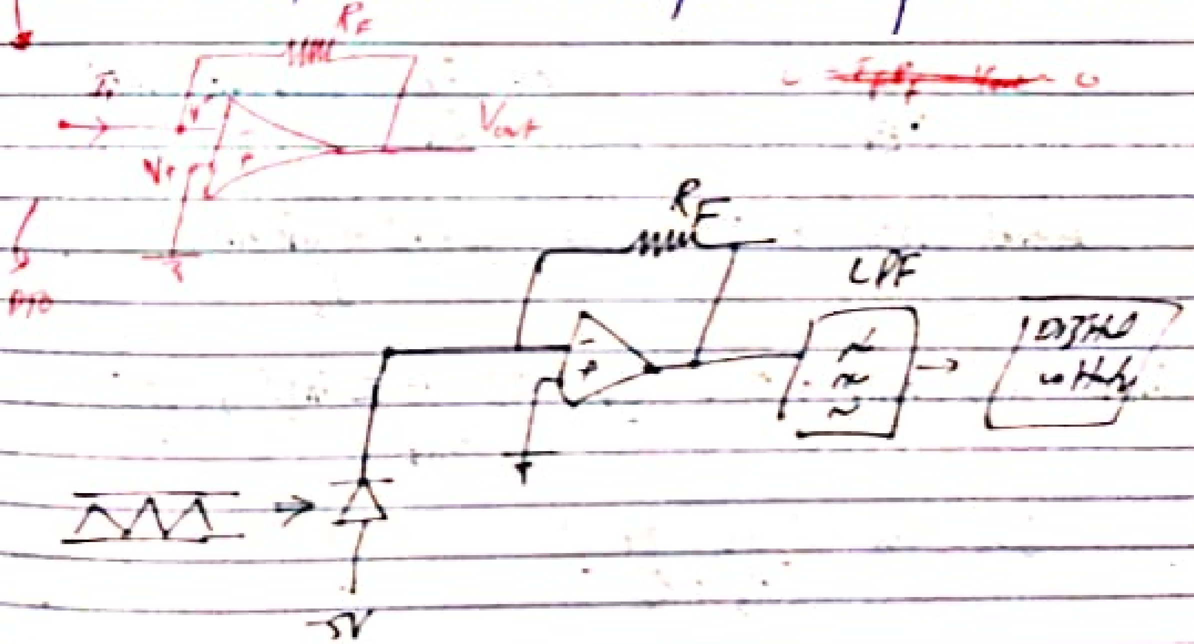
For protection → an outer layer of coating → Jacket



Current to voltage conv should have the following two properties.

(i) $V_{out} = K I_i$

(ii) V_{out} should be independent of the load.

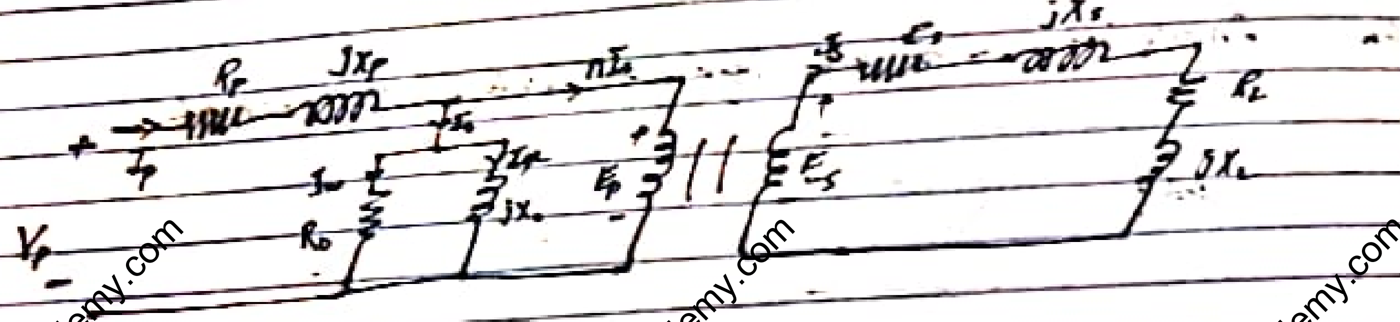


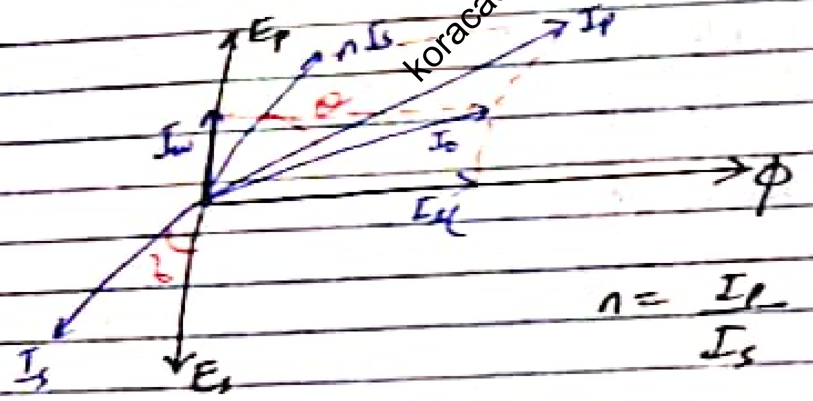
Trans impedance amplifier

-ve feedback $V^- = V^+ \Rightarrow I_f = I_i$

$0 - V_{out} = I_f R_f$ $\quad V_{out} = -I_f R_f$

if $K = -R_f$ $\Rightarrow V_{out} = K I_i$





$n = \frac{I_p}{I_s}$ = nominal ratio

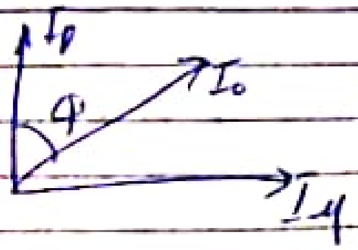
Actual ratio → $R = n + \frac{I_w \cos\delta + I_x \sin\delta}{I_s}$

Phase angle error → $\theta = \frac{I_x \cos\delta - I \sin\delta}{n I_s} \times \frac{180^\circ}{\pi}$

$\delta = \tan^{-1}\left(\frac{X_s + X_L}{R_s + R_L}\right)$

Example: A ring core type CT has a ratio of 200/10 n = 200, when operating at a rated primary current with a secondary burden of non inductive resistive value of 2Ω ... $f = 0°$ $R_c = 2\Omega$ $I_0 = 2A$. $Pf = 0.3$

$I_s = 10A$ Ratio error = ?



$$I_w = I_0 \cos\varphi.$$
$$= 2 \times 0.3.$$
$$I_w = 0.6\ A.$$

$$R = n + \frac{I_w}{I_s}$$
$$= 200 + \frac{0.6}{10}$$
$$\Rightarrow R = 200.06.$$

$$\% \text{ R.E} = \left| \frac{n - R}{R} \times 100 \right| = 0.03\%$$

Phase angle

$$\theta = \frac{I_\mu}{n I_s} \times \frac{180°}{\pi}$$

$$\varphi = \cos^{-1} 0.3 = \_\_\_$$
$$I_0 \sin\varphi = I_\mu$$

$$\theta = 0.055966°$$

Actual phase angle

$$180° - \theta.$$

Ex 6: A bar type C.T. ($n_p = n_1 = 1$) has $n_s = 400$. Impedance of secondary circuit: $2 + j1.5 = Z_s$ with 4 A flowing in the secondary. Magnetizing MMF is 80 A-turns and the iron loss is 1 Watt. Determine the ratio error and phase angle error.

$N_1 = 1 \rightarrow$ bar magnet $\quad N_2 = 400$

$Z_s = 2 + j1.5\ \Omega \quad I_s = 4A \quad MMF = 80\ A\,T$

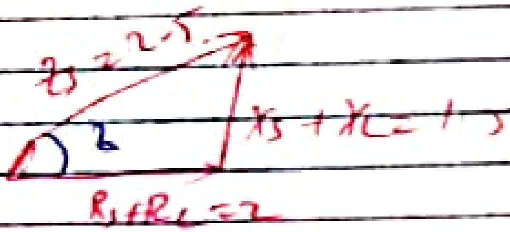
Iron loss = 1 W

Ratio error = ? $\quad \theta = ?$

$$n = \frac{N_2}{N_1} = \frac{400}{1} = 400$$

$$|Z_s| = \sqrt{(1.5)^2 + (2)^2} = 2.5$$

$$\cos\delta = \frac{2}{2.5} = 0.8$$



$$\sin\delta = \frac{1.5}{2.5} = 0.6$$

$$MMF = I_\mu N_1 \Rightarrow I_\mu = \frac{MMF}{N_1} = 80A$$

$I_w = ?$

$$\text{Iron loss} = I_w^2 R_o = I_w I_w R_o = I_w E_p$$

$$1 = I_w E_p \Rightarrow \boxed{I_w = \frac{1}{E_p}}$$

$E_p = ?$ First E_s. $\quad E_s = I_s Z_s$

$$= (4)(2.5) = 10V$$

Now $E_p = \frac{E_s}{n} = \frac{10}{400} = \frac{1}{40}$ V

Now $I_w = \frac{1}{E_p} \Rightarrow I_w = 40A$

$$R = n + \frac{I_w \cos\delta + I_\mu \sin\delta}{I_s}$$

$$= 400 + \frac{(40 \times 0.8) + (80 \times 0.6)}{4}$$

$$\Rightarrow R = 420$$

% Ratio error $\left| \frac{n - R}{R} \times 100 \right| = 4.76\%$

$$\theta = \frac{I_\mu \cos\delta - I_w \sin\delta}{n I_s} \times \frac{180^\circ}{\pi}$$

$$\Rightarrow \theta = 1.43^\circ$$

EMI GATE ACADEMY

Wheatstone Bridge

Bridge network

DC bridge	AC bridge
→ DC supply ($\omega = 0$)	→ AC supply ($\omega \neq 0$)
→ Only resistive components	→ R, L, C
→ Phasors are not involved as V and I are in phase	→ Phasors are involved.

Why bridge is used?

For measurement of R, L, C, power factor, quality factor, etc.

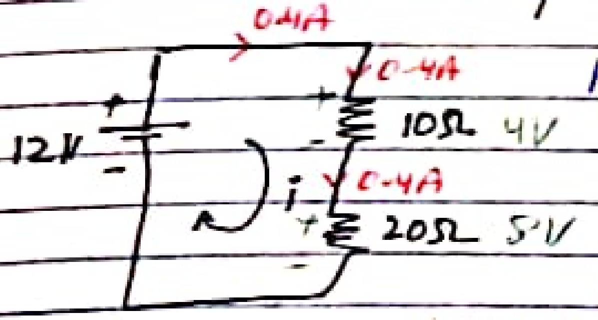
Major application of bridge is the callibration of ammeter or voltmeter.

Wheatstone bridge is the basic DC bridge.

Invented by charles wheatstone in 1833.

It is a series parallel combination of 4 resistors which gives zero difference voltage at balanced condition.

Assume a simple network;



KVL $-12 + i10 + i20 = 0$

$30i = 12$

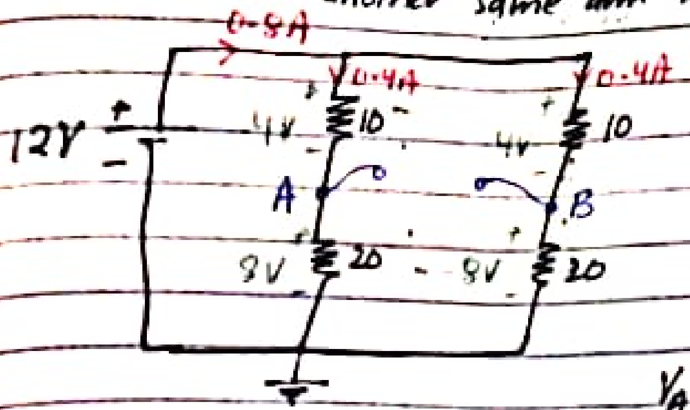
$\Rightarrow i = \frac{12}{30} = 0.4\ A$

Voltage drop;

$10 \times 0.4 = 4V$

$20 \times 0.4 = 8V$

Connect another same arm in parallel.



$10+20 \parallel 10+20$

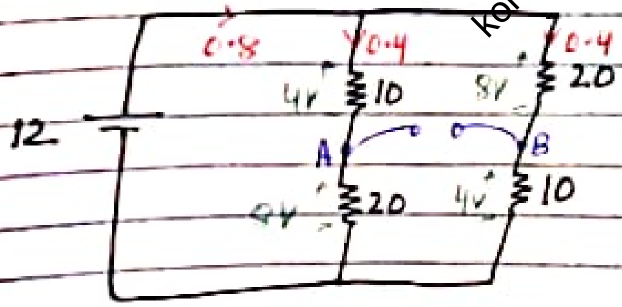
Bk equivalent resistance is half ie 30/2.
current drawn is double ie 0.8

$V_A = 8V \quad V_B = 8V$

$V_{AB} = V_A - V_B = 8 - 8 = 0V$
↓
Balanced condition.

Bridge?
4 arms and 4 nodes.

Source and detector are always connected on two opposite nodes in a bridge network.



$V_A = 8V$
$V_B = 4V$

$V_{AB} = V_A - V_B = 4V$
Bridge unbalanced

Balancing is judged by voltages.

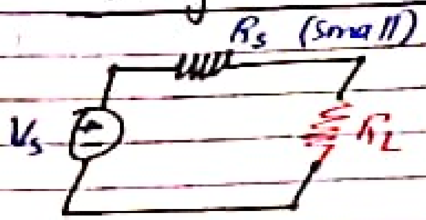
How is voltage modified in terms of current?
Difference voltage is the open circuit voltage of terminals A and B.
↓
thevenin voltage

Install an instrument to detect balanced condition.

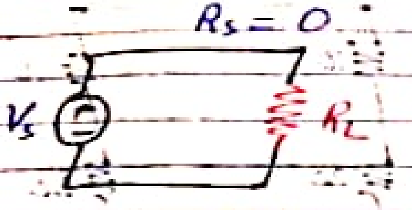
Basic electrical quantities: Voltage, current

Basic instruments: Voltmeter, Ammeter, galvanometer.
↓ voltage measurement
↓ current measurement
↓ current detection

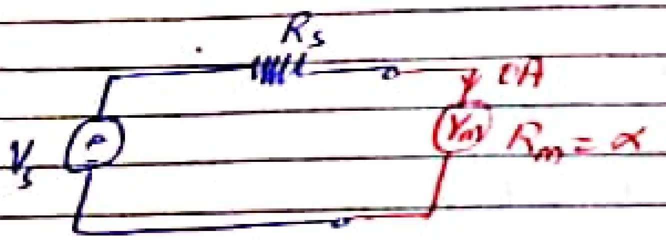
Voltage Source and Voltmeter

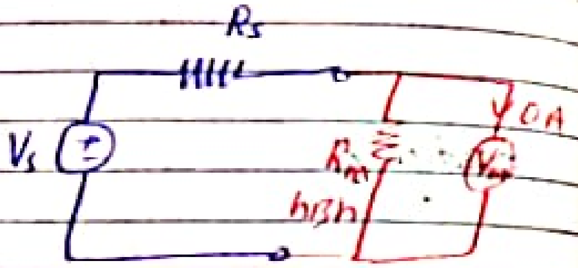


Practical voltage source



Ideal





Ideal voltmeter.

(ii) Practical voltmeter.

↳ (i) $-V_s + 0 \times R_s + V_m = 0$

$\Rightarrow \boxed{V_m = V_s}$

↓ Thevenin / open circuit voltage as $I = 0$

(ii) Voltage divider rule

$$V_m = V_s \times \frac{R_m}{R_m + R_s} \doteq V_s \times \frac{High}{High + low}$$

$$\Rightarrow V_m \approx V_s \times \frac{high}{high} \Rightarrow \boxed{V_m \approx V_s}$$

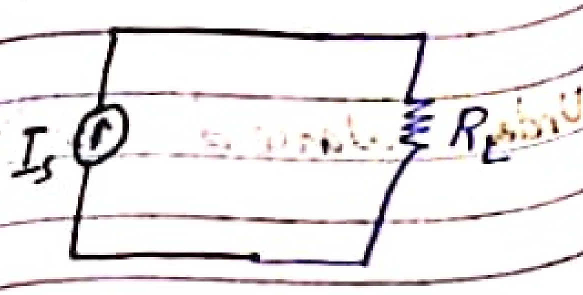
↓ Approximately Thevenin voltage.

Voltage source ⇒ internal impedance in series ⇒ very small
ideally $R_s = 0$

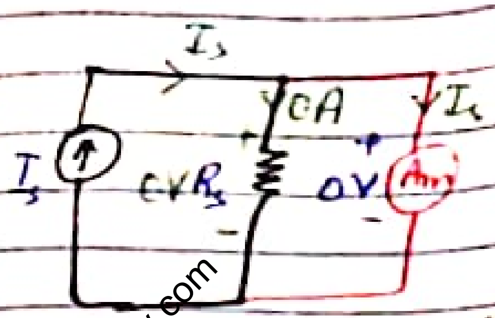
voltmeter ⇒ internal impedance in parallel ⇒ very high.
ideally infinite ⇒ current $= 0$
↳ so open circuit voltage
$R_m = \infty$

Current Source and Ammeter

Practical current source



Ideal



(ii) Ideal ameter

(i) Practical Ammeter

(i) current divider

$A_m = I_s \times \frac{R_s}{R_s + R_m} = I_s \times \frac{high}{low + high}$

$A_m = I_s \cdot \frac{high}{high}$ ⇒ $\boxed{A_m \simeq I_s}$

(ii) $\boxed{A_m = I_s}$ Ideal ammeter always measures the short circuit current ie Norton current.

current source ~~Ammeter~~ → internal impedance in parallel → very high
ideally infinite.

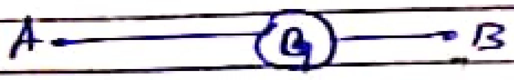
~~Current~~ Ammeter → internal impedance in series → very low
Ideally $R_i = 0$ → voltage = 0 → so short circuit current.

So we can connect either voltmeter, ammeter or galvanometer b/w points A and B.

Under balanced condition;

V = 0 Volt.

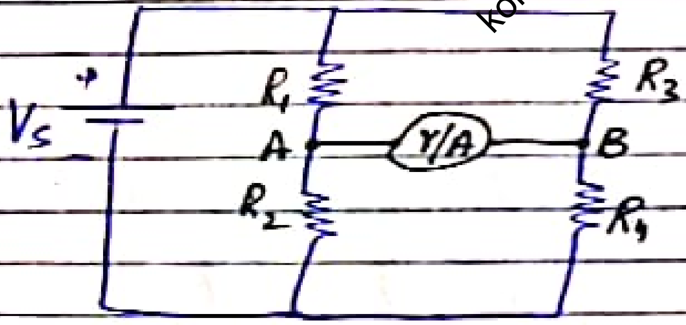
I = 0 Amp → measured



I = 0 Amp → detected.

We can calculate the exact balanced point with the help of thevenin equivalent circuit.

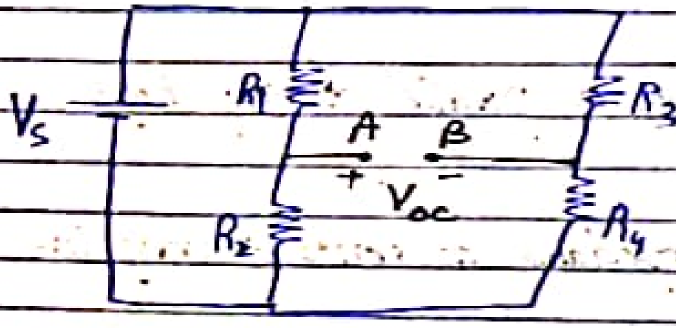
Thevenin Equivalent of Bridge Network



We calculate the thevenin equivalent circuit across the terminals AB

Calculation of Thevenin voltage.

Open circuit A and B.



$$V_{oc} = V_A - V_B = V_{Th}$$

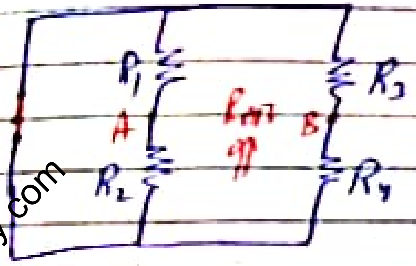
$$V_{Th} = \frac{V_s \times R_2}{R_1 + R_2} - \frac{V_s \times R_4}{R_3 + R_4}$$

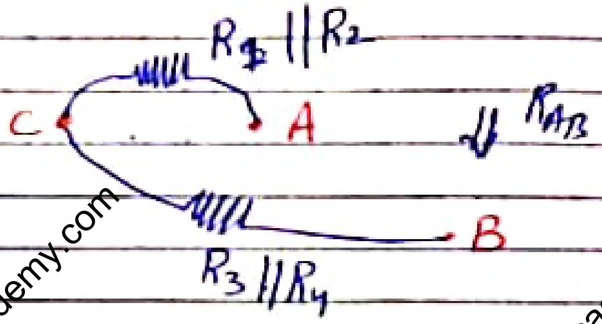
$$V_{Th} = V_s \left[\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right]$$

$$V_{Th} = V_s\left[\frac{R_2R_3 + \cancel{R_2R_4} - R_4R_1 - \cancel{R_2R_4}}{(R_1+R_2)\,(R_3+R_4)}\right]$$

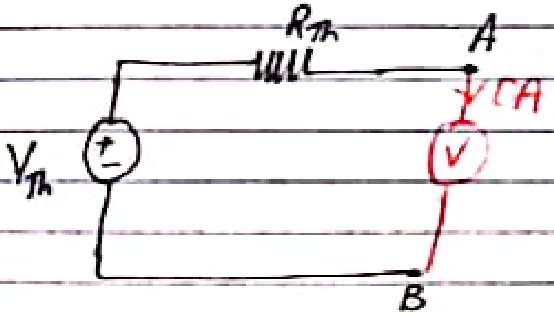
$$\boxed{V_{Th} = V_s\left[\frac{R_2R_3 - R_4R_1}{(R_1+R_2)(R_3+R_4)}\right]}$$

Calculation of Thevenin resistance

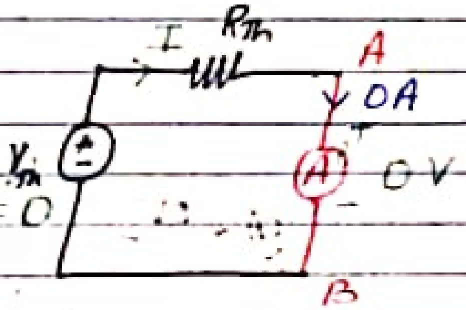




$$\boxed{R_{AB} = R_{Th} = (R_1 || R_2) + (R_3 || R_4)}$$



(or)



$-V_{Th} + 0 \times R_{Th} + V = 0$

$-0 + I R_{Th} + 0 = 0$

$\boxed{V = V_{Th} = 0\,V}$

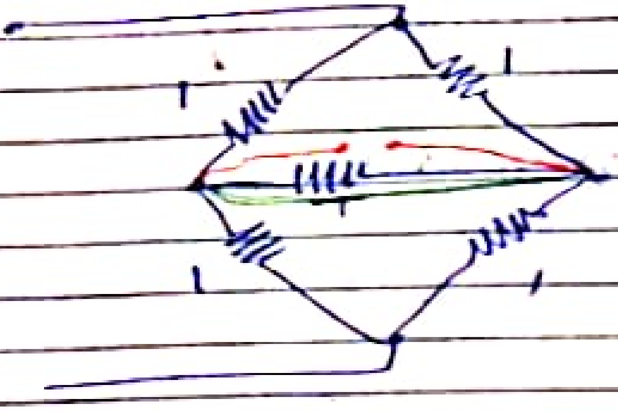
If balanced

$\Rightarrow \boxed{I = 0\,A}$

Under balanced condition, the detector of bridge network is both open circuit and short circuit at the same time. (In case of resistive load)

$V_{th} = 0 \Rightarrow R_2 R_3 - R_4 R_1 = 0$

$\Rightarrow \boxed{R_4 R_1 = R_2 R_3}$ → condition for balancing



If open circuited

$R_{th} =$

$=$

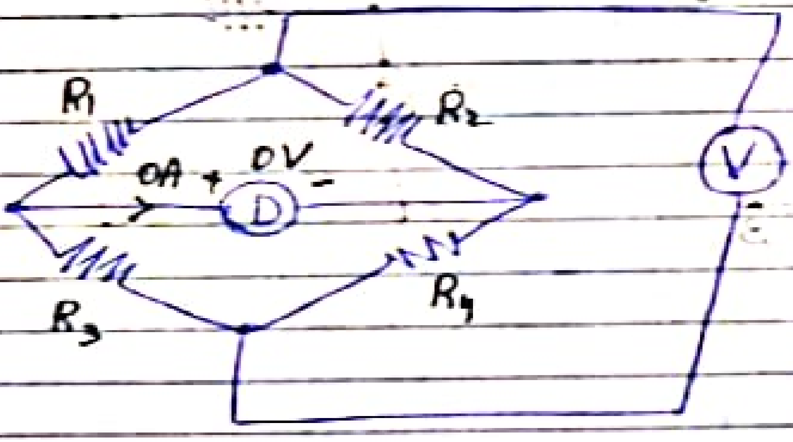
$\Rightarrow R_{th} = 1\Omega$

If short circuited

$R_{th} = (1 \| 1) + (1 \| 1)$

$R_{th} = 0.5 + 0.5 = 1$

The wheatstone bridge is as;



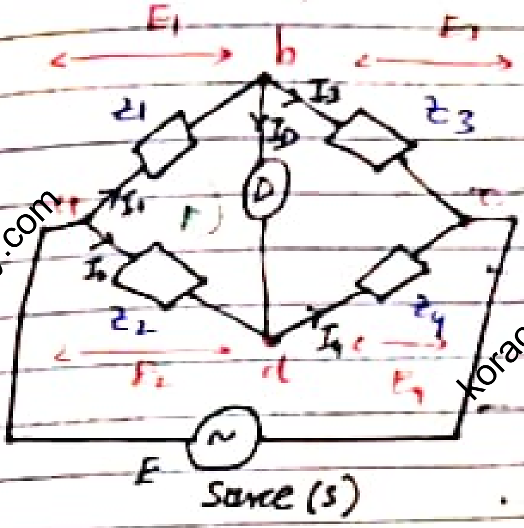
$\boxed{R_2 R_3 = R_4 R_1}$

Why are AC bridges used?

i. For calculating R, L, C, power factor (p.f), dissipation factor (df) and loss tangent (LT) without using voltmeter and ammeter.

ii. It is also used for providing phase shifting and providing feedback path to oscillators.

Basic Bridge network



4 Nodes (a, b, c, d)
4 Arms (ab, ad, bc, cd)
D: detector
S: source

Source and Detector are connected in opposite nodes.

If source and detector are connected in the same node, the bridge will never come to balanced condition.

General criteria for bridge balance;
(i) Detector current should be zero. ie $I_D = 0$
(ii) Potential difference b/w points b and d should be zero. ie $V_{bd} = 0$

- Mathematical meaning of conditions;

$$I_1 = I_3 \, , \quad I_2 = I_4$$

$$E_1 = E_2 \quad \Rightarrow \quad I_1 Z_1 = I_2 Z_2 \rightarrow (A)$$

$$I_1 = I_3 = \frac{E}{Z_1 + Z_3}$$

$$\boxed{I_2 = I_4 = \frac{E}{Z_2 + Z_4}}$$

$$① \Rightarrow \frac{E}{Z_1+Z_3}(Z_1) = \frac{E}{Z_2+Z_4}(Z_2)$$

$$\Rightarrow \frac{Z_1}{Z_1+Z_3} = \frac{Z_2}{Z_2+Z_4}$$

$$\Rightarrow Z_1(Z_2+Z_4) = Z_2(Z_1+Z_3)$$

$$\Rightarrow \cancel{Z_1 Z_2} + Z_1 Z_4 - \cancel{Z_1 Z_2} - Z_2 Z_3 = 0$$

$$\Rightarrow \boxed{Z_1 Z_4 = Z_2 Z_3}$$

Similarly also for admittances;

$$\boxed{Y_1 Y_4 = Y_2 Y_3}$$

If impedances are given in complex form;

$Z_1 = |Z_1| \angle \theta_1$ $\quad Z_2 = |Z_2| \angle \theta_2$ $\quad Z_3 = |Z_3| \angle \theta_3$

$Z_4 = |Z_4| \angle \theta_4$

$$\Rightarrow |Z_1| \angle \theta_1 \times |Z_4| \angle \theta_4 = |Z_2| \angle \theta_2 \times |Z_3| \angle \theta_3$$

$$\Rightarrow \boxed{|Z_1||Z_4| \angle \theta_1 + \theta_4 = |Z_2||Z_3| \angle \theta_2 + \theta_3}$$

$$|Z_1||Z_4| = |Z_2||Z_3|$$

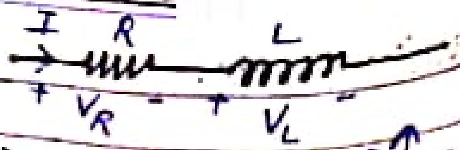
↳ magnitude criteria

$$\angle \theta_1 + \angle \theta_4 = \angle \theta_2 + \angle \theta_3$$

↳ phase criteria

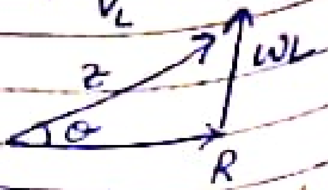
ARM COMBINATIONS

(1) Series R and L



(i) impedance angle (θ)

$Tan\,\theta = \frac{\omega L}{R}$



$$\rightarrow \boxed{\theta = Tan^{-1}\left(\frac{\omega L}{R}\right)}$$

(ii) Power Factor is cos φ

$$\boxed{Cos\,\phi = \frac{R}{Z}}$$

→ this formula is valid only for series combination

(iii) Quality Factor (Q)

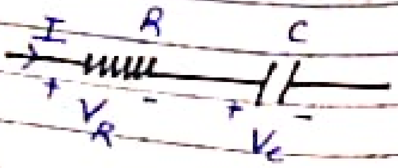
$$Q = \frac{V_L}{V_R} = \frac{storing}{dissipating} = \frac{IX_L}{IR} = \frac{X_L}{R}$$

$$\boxed{Q = \frac{\omega L}{R}}$$

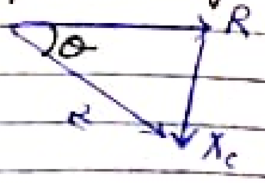
(iv) Dissipation Factor is inverse of quality factor

$$\rightarrow \boxed{D = \frac{R}{\omega L}}$$

(2) Series R and C



(i) impedance angle



$Tan\,\theta = \frac{1/\omega C}{R}$

$$\boxed{\theta = Tan^{-1}\left(\frac{1}{\omega CR}\right)}$$

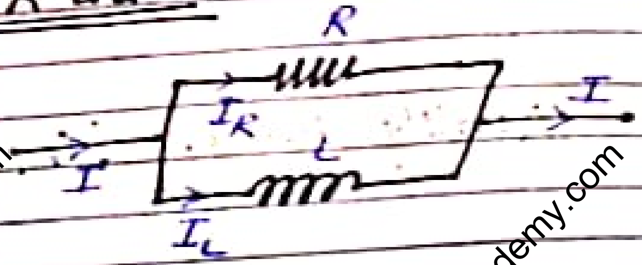
(ii) Power factor $\boxed{Cos\,\phi = \frac{R}{Z}}$

iii. Quality Factor. $Q = \frac{V_C}{V_R} = \frac{I X_C}{I R} = \frac{X_C}{R}$

$\rightarrow \boxed{Q = \frac{1}{\omega C R}}$

iv. Dissipation factor. $D = \frac{1}{Q} \rightarrow \boxed{D = \omega C R}$

3 Parallel Combination of R and L



(i) Impedance angle

As $Z = R \| j\omega L$

$Z = \frac{(R)(j\omega L)}{R + j\omega L}$

$\boxed{\theta = 90^\circ - \tan^{-1}\left(\frac{\omega L}{R}\right)}$

ii. Power Factor

As $I = I_R + I_L = \frac{V}{R} + \frac{V}{j\omega L} = V\left[\frac{1}{R} + \frac{1}{j\omega L}\right]$

$\frac{V}{I} = \frac{1}{(1/R + 1/j\omega L)} \xrightarrow{\text{convert}} Z\angle\phi$

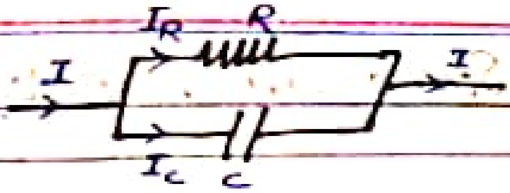
↳ $\cos\phi$

(iii) Quality Factor $Q = \frac{I_L}{I_R} = \frac{V/X_L}{V/R} = \frac{1/X_L}{1/R}$

$\boxed{Q = \frac{R}{X_L}} = \frac{R}{\omega L}$

(iv) Dissipation Factor, $\boxed{D = \frac{1}{Q} = \frac{\omega L}{R}}$

4 Parallel R and C



(i) Impedance Angle

$$Z = R \parallel \frac{1}{j\omega C} = \frac{(R)(1/j\omega C)}{R + 1/j\omega C}$$

$$\boxed{\angle Z = -90^\circ - \tan^{-1}\left(\frac{1}{\omega CR}\right)}$$

(ii) Power Factor

$$I = I_R + I_C = \frac{V}{R} + \frac{V}{1/j\omega C} = V\left[\frac{1}{R} + j\omega C\right]$$

$$\frac{V}{I} = \left(\frac{1}{1/R + j\omega C}\right) \xrightarrow{\text{convert}} Z\angle\phi \to \cos\phi$$

(iii) Quality Factor

$$Q = \frac{I_C}{I_R} = \frac{V/(1/j\omega C)}{V/R} = \boxed{\omega CR}$$

(iv) Dissipation Factor $\boxed{D = \frac{1}{Q} = \frac{1}{\omega CR}}$

Arm combination		Q Factor
Series RL	→	$Q = \omega L/R$
Parallel RL	→	$Q = R/\omega L$
Series RC	→	$Q = 1/\omega CR$
Parallel RC	→	$Q = \omega CR$

$$Q_{series} = \frac{1}{Q_{parallel}}$$

Source And Detector

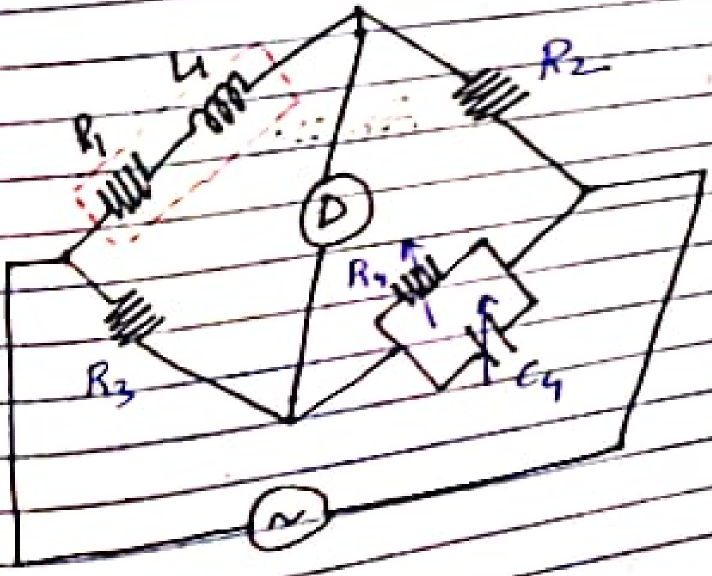
→ For measurment at low frequency, power line may act as source of supply to bridge network.

→ For measurment at high frequency, electronic oscillator may act as source of supply to bridge.

→ Source and detector are always placed in opposite nodes.

	Detector	Frequency
i	DC Galvanometer	0Hz (DC)
ii	Vibration galvanometer	2-100 Hz
iii	Telephone and headphone galvanometer	100 - ~~200~~ 2KHz
iv.	Tunable Amplifier Galvanometer	$f > 2kHz$
v.	CRO	In MHz.

Maxwell's Bridge



$R_1, L_1 \rightarrow$ unknown.

Rest all parameters are known.

$Z_1 = R_1 + j\omega L_1$

$Z_2 = R_2$

$Z_3 = R_3$

$Z_4 = R_4 \,||\, C_4$

$$= \frac{R_4 (1/j\omega C_4)}{R_4 + \frac{1}{j\omega C_4}}$$

$$Z_4 = \frac{R_4}{\cancel{j\omega C_4}} \div \frac{1 + j\omega R_4 C_4}{\cancel{j\omega C_4}}$$

$$\boxed{Z_4 = \frac{R_4}{1 + j\omega R_4 C_4}}$$

By opposite impedances; $Z_1 Z_4 = Z_2 Z_3$

$$\Rightarrow (R_1 + j\omega L_1)\left(\frac{R_4}{1 + j\omega R_4 C_4}\right) = R_2 R_3$$

$$\Rightarrow (R_1 + j\omega L_1) R_4 = R_2 R_3 (1 + j\omega R_4 C_4)$$

$$\Rightarrow \boxed{R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega R_2 R_3 R_4 C_4}$$

L.H.S = R.H.S $\Rightarrow$ Real = Real
Imaginary = Imaginary

Real

$$R_1 R_4 = R_2 R_3 \Rightarrow \boxed{R_1 = \frac{R_2 R_3}{R_4}}$$

Imaginary

$$\cancel{j\omega} L_1 \cancel{R_4} = \cancel{j\omega} R_2 R_3 \cancel{R_4} C_4$$

$$\boxed{L_1 = R_2 R_3 C_4}$$

$L_1 = f(C_4)$
$R_1 = f(R_4)$

Q factor of unknown arm is $Q = \frac{\omega L_1}{R_1}$ → put L_1 and R_1

$$Q = \frac{\omega \cancel{R_2 R_3} C_4 R_4}{\cancel{R_2 R_3}} \Rightarrow \boxed{Q = \omega C_4 R_4}$$

$Q \propto C_4$ → disadvantage

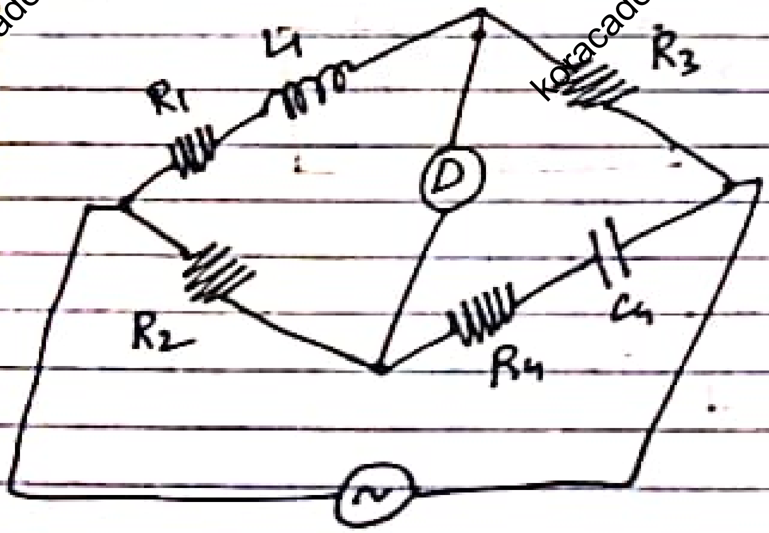
medium range of Q factor ↵

$Q < 1$ → Anderson Bridge
$1 < Q < 10$ → Maxwell's bridge
$Q > 10$ → Hay's bridge

Key Points in Maxwell's Bridge

→ Used for determination of medium range of Q factor.
→ At balanced condition R_1, L_1 are independent of supply frequency.
→ To achieve faster response, R_4 and C_4 should be variable.
→ Maxwell's bridge should be used for measurement of R, L, Q factor and dissipation factor.

HAY's BRIDGE



$$Z_1 = R_1 + j\omega L_1$$

$$Z_2 = R_2$$

$$Z_3 = R_3$$

$$Z_4 = R_4 + \frac{1}{j\omega C_4}$$

At balanced condition,

$$Z_1 Z_4 = Z_2 Z_3$$

$$(R_1 + j\omega L_1)\left(R_4 + \frac{1}{j\omega C_4}\right) = R_2 R_3$$

$$(R_1 + j\omega L_1)\left(\frac{j\omega C_4 R_4 + 1}{j\omega C_4}\right) = R_2 R_3$$

$\Rightarrow (R_1 + j\omega L_1)(j\omega C_4 R_4 + 1) = R_2 R_3 j\omega C_4$

$\Rightarrow j\omega C_4 R_4 R_1 + R_1 + j\omega L_1 - \omega^2 L_1 C_4 R_4 = j\omega R_2 R_3 C_4$

$\Rightarrow \boxed{j\omega C_4 R_4 R_1 + R_1 + j\omega L_1 = \omega^2 L_1 C_4 R_4 + j\omega C_4 R_2 R_3}$

Real = Real Imaginay = imajiny

$R_1 + j\omega(L_1 + C_4 R_1 R_4) = \omega^2 L_1 C_4 R_4 + j\omega C_4 R_2 R_3$

Real $\boxed{R_1 = \omega^2 L_1 C_4 R_4}$

Im

$j\cancel{\omega}(L_1 + C_4 R_1 R_4) = j\cancel{\omega} C_4 R_2 R_3$

$L_1 = C_4 R_2 R_3 - C_4 R_1 R_4$

$L_1 = C_4(R_2 R_3 - R_1 R_4)$

Problem: one unknown is a function of another
ie R_1 of L_1

Putting values $L_1 + C_4 R_4(\omega^2 L_1 C_4 R_4) = C_4 R_2 R_3$

$L_1 + \omega^2 L_1 C_4^2 R_4^2 = C_4 R_2 R_3$

$L_1(1 + \omega^2 \cancel{L_1} C_4^2 R_4^2) = C_4 R_2 R_3$

$\boxed{L_1 = \frac{C_4 R_2 R_3}{1+\omega^2 C_4^2 R_4^2}}$ put this in R_1

$$R_1 = w^2 C_4 R_4 \times \frac{C_4 R_2 R_3}{1+w^2 C_4^2 R_4^2}$$

$$\boxed{R_1 = \frac{w^2 R_2 R_3 R_4 C_4^2}{1+w^2 C_4^2 R_4^2}}$$

Now $Q = \frac{1}{w^2 C_4^2 R_4^2}$

If Q is high

$$1+w^2 C_4^2 R_4^2 = 1 + \left(\frac{1}{Q}\right)^2 \simeq 1$$

$$\Rightarrow \boxed{L_1 \simeq C_4 R_2 R_3}$$

$$\boxed{R_1 \simeq w^2 R_2 R_3 R_4 C_4^2}$$

For unknown arm, $Q = \frac{wL_1}{R_1}$ → put exact values

$$Q = \cancel{w} \frac{\cancel{C_4 R_2 R_3}}{\cancel{1+w^2 C_4^2 R_4^2}} \times \frac{\cancel{1+w^2 C_4^2 R_4^2}}{w^{\cancel{2}} \cancel{R_2} \cancel{R_3} R_4 C_4^{\cancel{2}}}$$

$$\boxed{Q = \frac{1}{w R_4 C_4}} \Rightarrow Q \propto \frac{1}{C}$$

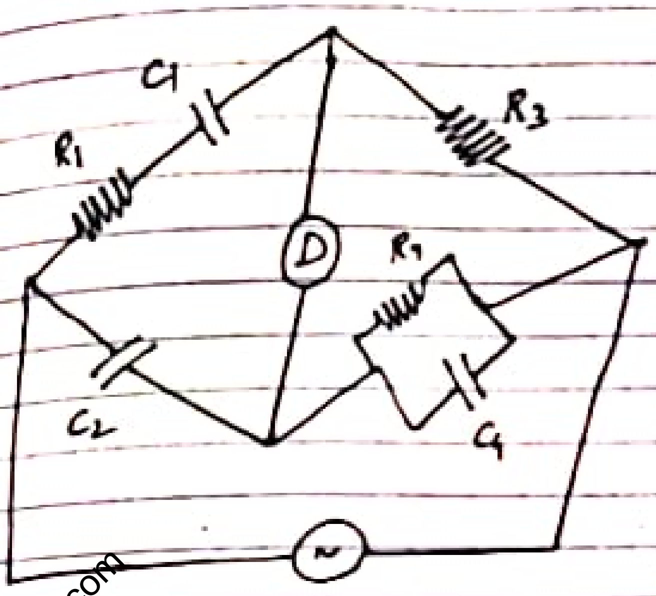
disadvantage? R_1 is a function of frequency

From equations of R_1 and L_1, R_4 and C_4 are variable

Boundness? Vary R_4 first and then C_4.

↳ so slow response.

SCHERING BRIDGE



$$z_1 = R_1 + \frac{1}{j\omega C_1}$$

$$z_2 = \frac{1}{j\omega C_2}$$

$$z_3 = R_3$$

$$z_4 = R_4 \,//\, {}^{1}/_{j\omega C_4}$$

$$\Rightarrow z_4 = \frac{R_4}{1 + j\omega R_4 C_4}$$

Balanced condition;

$$z_1 z_4 = z_2 z_3$$

$$\Rightarrow \left(R_1 + \frac{1}{j\omega C_1}\right)\left(\frac{R_4}{1 + j\omega R_4 C_4}\right) = R_3\left(\frac{1}{j\omega C_2}\right)$$

$$\Rightarrow \left(R_1 + \frac{1}{j\omega C_1}\right) R_4 = \frac{R_3}{j\omega C_2}\left(1 + j\omega R_4 C_4\right)$$

$$\Rightarrow \boxed{R_1 R_4 + \frac{R_4}{j\omega C_1} = \frac{R_3}{j\omega C_2} + \frac{\cancel{j\omega} R_4 R_3 C_4}{\cancel{j\omega} C_2}}$$

Comparing real $\cancel{R_1} R_4 = \frac{\cancel{R_4} R_3 C_4}{C_2}$

$$\boxed{R_1 = \frac{R_3 C_4}{C_2}}$$

Imaginary $\frac{R_4}{j\omega C_1} = \frac{R_3}{j\omega C_2} \rightarrow \frac{R_4}{C_1} = \frac{R_3}{C_2}$

$$\boxed{C_1 = \frac{R_4 C_2}{R_3}}$$

Quality Factor $Q_1 = \frac{1}{\omega C_1 R_1}$

Dissipation factor, $D_1 = \omega C_1 R_1$

$$D_1 = \omega \left(\frac{R_4 \cancel{C_2}}{\cancel{R_3}}\right)\left(\frac{\cancel{R_3} C_4}{\cancel{C_2}}\right)$$

$$\Rightarrow \boxed{D_1 = \omega R_4 C_4}$$

$$\omega C_1 R_1 = \omega R_4 C_4$$

DESAUTY's BRIDGE

Desauty's bridge

↓ Unmodified

→ Used for perfect capacitor.

↓

$C \neq 0, R = \alpha$

↓ Modified

Imperfect C.

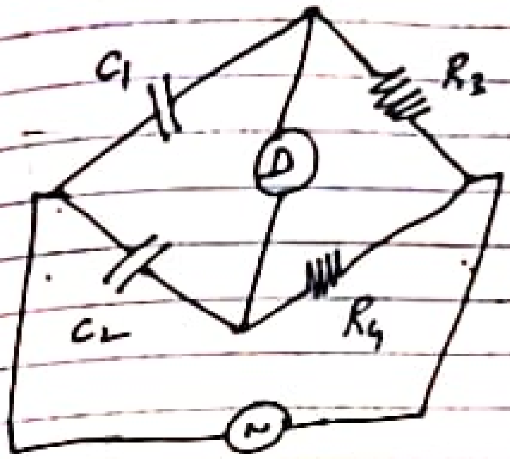
↓

$R \neq 0, C \neq 0$

Unmodified DeSauty's Bridge

lossless C.

C_1 is unknown.



$Z_1 = 1/j\omega C_1$

$Z_2 = 1/j\omega C_2$

$Z_3 = R_3$

$Z_4 = R_4$

Opposite impedances $Z_1 Z_4 = Z_2 Z_3$

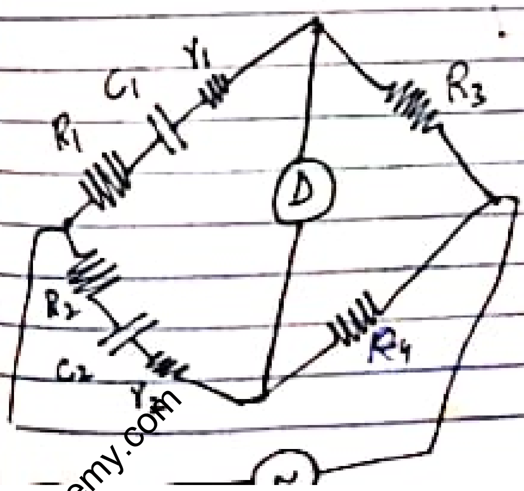
$$\left(\frac{1}{j\omega C_1}\right) R_4 = \left(\frac{1}{j\omega C_2}\right) R_3$$

$$\Rightarrow \frac{R_4}{j\omega C_1} = \frac{R_3}{j\omega C_2} \Rightarrow \boxed{C_1 = \frac{R_4 C_2}{R_3}}$$

only real or only imaginary term?
B/c only one variable here ie C_1.

Modified DeSauty's Bridge

$r_1, r_2,$ losses.



$$Z_1 = R_1 + r_1 + \frac{1}{j\omega C_1}$$

$$Z_2 = R_2 + r_2 + \frac{1}{j\omega C_2}$$

$Z_3 = R_3$, $Z_4 = R_4$

Opposite impedances : $Z_1 Z_4 = Z_2 Z_3$

$$\Rightarrow \left(R_1 + r_1 + \frac{1}{j\omega C_1}\right) R_4 = \left(R_2 + r_2 + \frac{1}{j\omega C_2}\right) R_3$$

$$\Rightarrow \boxed{R_1 R_4 + r_1 R_4 + \frac{R_4}{j\omega C_1} = R_2 R_3 + r_2 R_3 + \frac{R_3}{j\omega C_2}}$$

Comparing real

$$(R_1 + r_1) R_4 = (R_2 + r_2) R_3$$

$$\Rightarrow R_1 + r_1 = \frac{(R_2 + r_2) R_3}{R_4}$$

$$\Rightarrow \boxed{R_1 = \frac{(R_2 + r_2) R_3}{R_4} - r_1}$$

Comparing imaginary,

$$\frac{R_4}{j\omega C_1} = \frac{R_3}{j\omega C_2} \Rightarrow \frac{R_4}{C_1} = \frac{R_3}{C_2}$$

$$\Rightarrow \boxed{C_1 = \frac{R_4 C_2}{R_3}}$$

Quality Factor, $Q_1 = \frac{1}{\omega C_1 R_1}$

Dissipation factor, $D_1 = \omega C_1 R_1$

↓

put values of C_1 and R_1

$$D_1 = \frac{\omega R_4 C_2}{R_3'}\left[\frac{(R_2+r_2)R_3' - r_1 R_4}{R_4}\right]$$

$$D_1 = \omega C_2 (R_2 + r_2) - \frac{\omega R_4 C_2 r_1}{R_3}$$

$$D_1 = \frac{\omega C_2}{R_3'}\left[(R_2 + r_2)R_3' - r_1 R_4\right]$$

$$D_1 = \underset{\downarrow}{\omega C_2 R_2} + \omega C_2 r_2 - \omega C_2 r_1 R_4$$

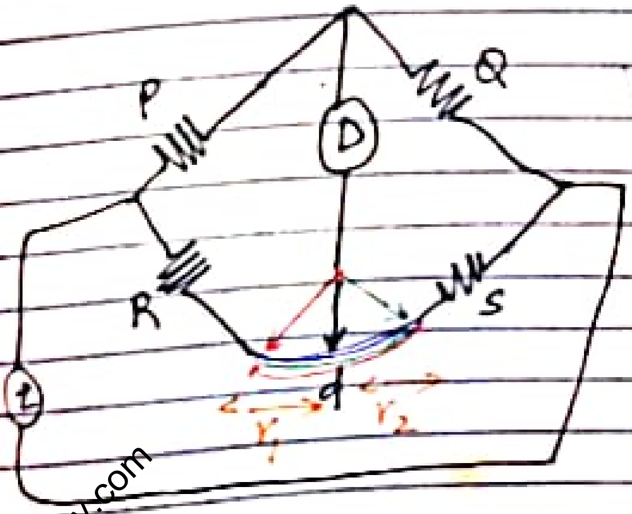
$$D_1 = D_2 + \omega C_2 r_2 - \omega C_2 r_1 R_4$$

Dissipation factor of one capacitor depends on D of another, so we cannot measure D_1 and hence cannot measure Q_1.
So this bridge is only used for calibration of imperfect capacitance.

Kelvin's Bridge

for unknown resistance.

Very low resistances cannot be measured by wheatstone.
lead resistance!
S → resistance
R → resistance



Connect at d to nullify lead resistances.

$$\frac{r_1}{r_2} = \frac{P}{Q}$$

opposite impedances; $P(S+r_2) = Q(R+r_1)$

$$\Rightarrow R+r_1 = \frac{P}{Q}(S+r_2) \rightarrow ①$$

As $\frac{r_1}{r_2} = \frac{P}{Q}$

$$\Rightarrow \frac{r_1}{r_1+r_2} = \frac{P}{P+Q}$$

and $\frac{r_2}{r_1+r_2} = \frac{Q}{P+Q}$

let $r_1+r_2 = r$

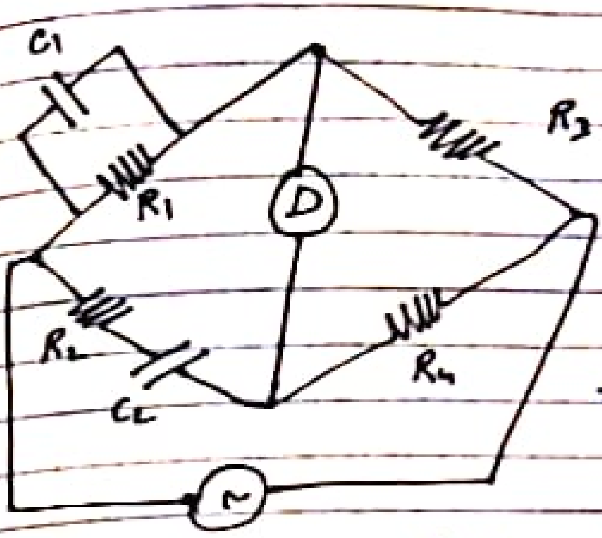
$$① \Rightarrow R + \left(\frac{P}{P+q}(r)\right) = \frac{P}{Q}\left(S + \frac{Q}{P+q}\cdot r_1\right)$$

$$\Rightarrow R + \frac{Pr}{P+q} = \frac{PS}{Q} + \frac{P\cancel{Q}r}{(P+q)\cancel{Q}}$$

$$\Rightarrow R + \frac{\cancel{Pr}}{P+Q} = \frac{PS}{Q} + \frac{\cancel{Pr}}{P+q}$$

$$\boxed{R = \frac{PS}{Q}}$$

WEIN BRIDGE



$$Z_1 Z_4 = Z_2 Z_3$$

$$\rightarrow (R_1 \parallel C_1) R_4 = R_3 \left(R_2 + \frac{1}{j\omega C_2}\right)$$

$$\Rightarrow \left(\frac{R_1}{1+j\omega C_1 R_1}\right) R_4 = R_3 R_2 + \frac{R_3}{j\omega C_2}$$

$$\Rightarrow \frac{R_1 R_4}{1+j\omega C_1 R_1} = \frac{R_3 R_2\, j\omega C_2 + R_3}{j\omega C_2}$$

$$\Rightarrow R_1 R_4 (j\omega C_2) = (R_3 R_2\, j\omega C_2 + R_3)(1+j\omega C_1 R_1$$

$$\Rightarrow R_1 R_1 (j\omega C_2) = R_3 + j\omega C_1 R_3^2 + R_3 R_2\, j\omega C_2 + R_3$$
$$(-1)\omega^2 C_1 C_2 R_3 R_2 R_1$$

Equate real part to zero.

$$R_3 + (-1)\omega^2 . R_2 R_3 C_2 . R_1 C_1 = 0$$

$$\cancel{R_3} = \omega^2 R_2 \cancel{R_3} C_2 R_1 C_1$$

$$\Rightarrow \boxed{\omega = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}}$$

CRO

Basic voltmeter
↳ electronic voltmeter
↳ responsible quantity is voltage.

CRO is a voltage sensitive element.
↳ we have a display
↳ Electronic image plotter

Test circuit checks the unknown signal.

Probe and cable working as transmitting media
↳ will take signal into vertical amplifier.
its o/p now goes in two ~~parts~~ ways ↲
↳ horizontal plate.

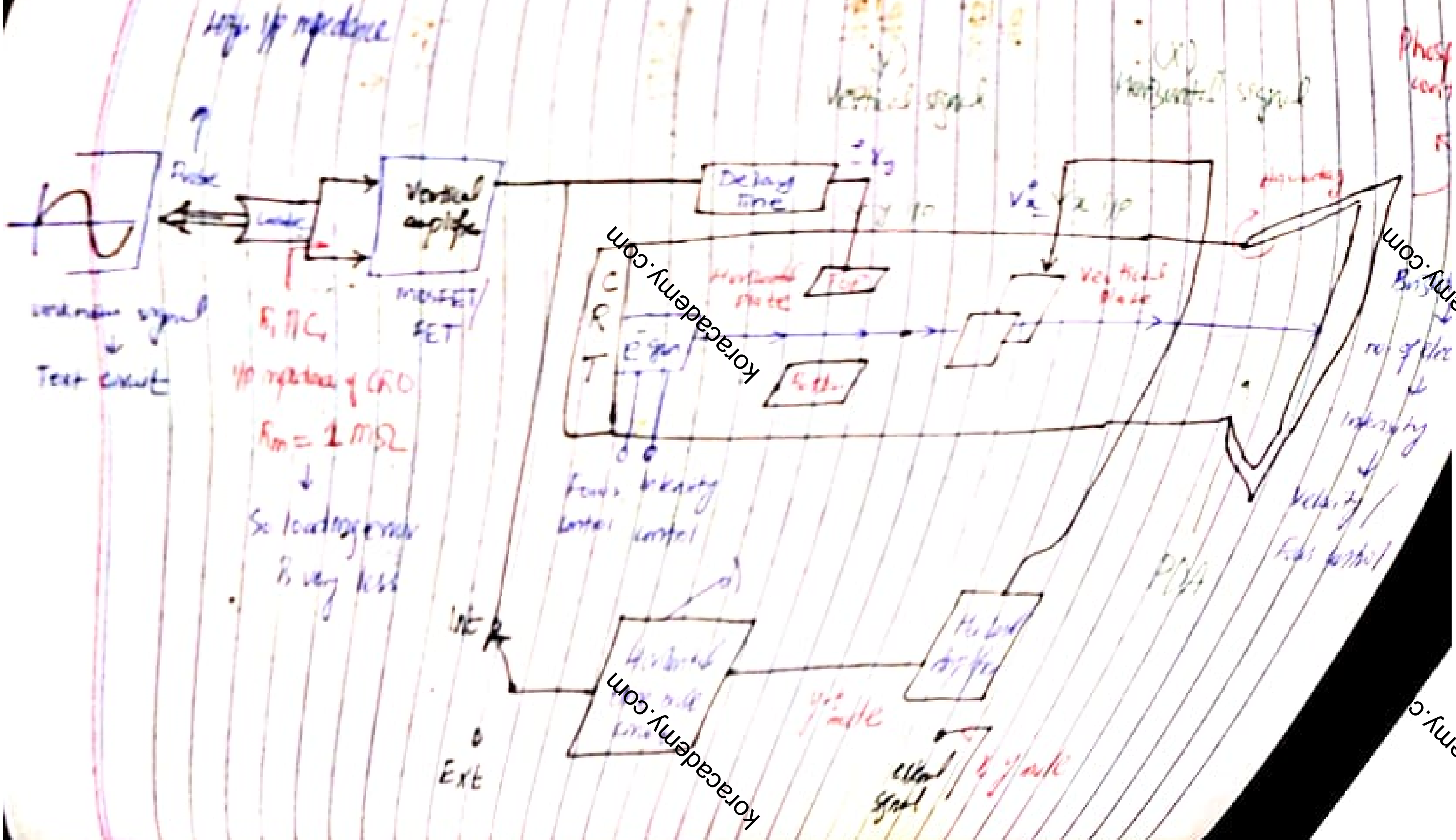
CRT → electron gun → we can control its intensity and focus.

We generally measure different parameters in y-t mode.

We generate a signal in horizontal time based circuit (whose time period is in our control) and take it into vertical plates through horizontal amplifier.

In x-y mode we provide external signal and it is used when we have to measure phase difference b/w two signals & frequency.
y-t mode is Amplitude Time mode.
x-y mode has x and y two different signals.

The basic principle of CRO is thermionic emission.

High i/p impedance
Probe
Vertical amplifier
MOSFET/FET
unknown signal
↓
Test circuit
R, || C,
i/p impedance of CRO
Rin = 1 MΩ
↓
So loading error is very less
Delay line
Vertical signal (Y)
Horizontal signal (X)
Vy i/p
Vx X i/p
C
R
T
Horizontal plate
Vertical plate
e⁻ gun
Focus control
Intensity control
Int
Ext
Horizontal amplifier
Y-t mode
X-Y mode
External signal
PDA
Phosphor coating
Bias
ray of elec
↓
intensity
↓
velocity /
focus control

PDA → Post deflection accelerator.
If electron speed is low, it speedens it to get a sharp image on the screen.

Modes of CRO

Y-t mode

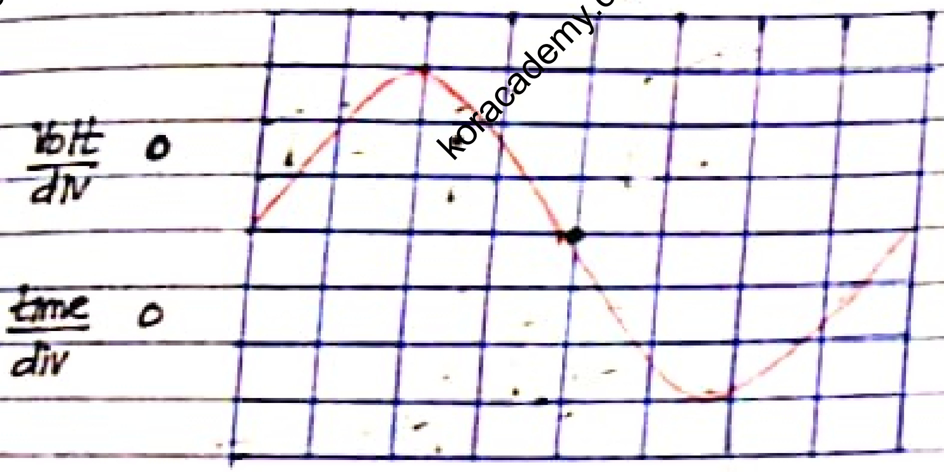
→ RMS
→ Peak
→ Average value

x-y mode

→ Phase
→ Frequency

SCREEN

y setting = volt/div → voltage
x setting = time/div → time



- origin this is an 8x8 | screen.
  ↳ total rows x columns.

Two knobs, volt/div and time/div.

eg if volt/div is 2V, it means that each division is of 2V.

Consider the curve.

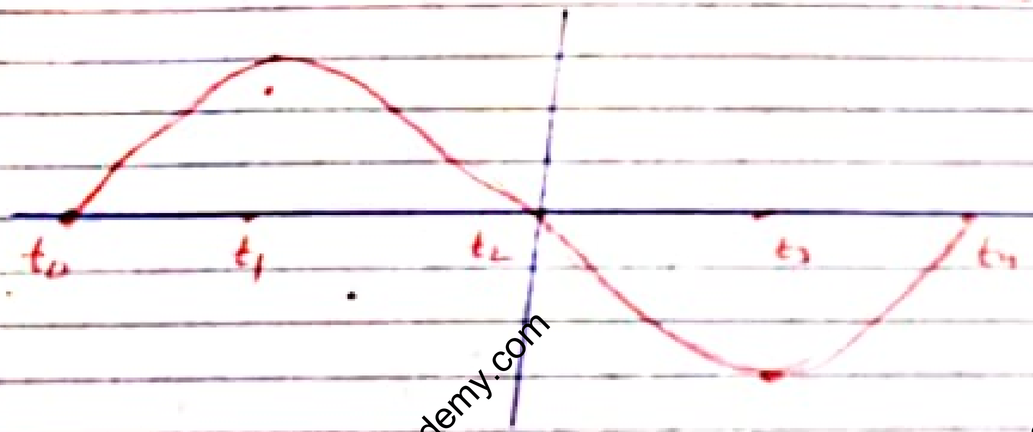
$$\text{peak value} = 3 \text{ div} \times 2 \frac{\text{volt}}{\text{div}} = 6 \text{ volts}$$

$$\boxed{y\text{-}t \text{ plot} = S \times V_y}$$

t	V_x	V_y
t_0	-2	0
t_1	-1	+2
t_2	0	0
t_3	-1	-2
t_4	+2	0

→ 0 × 2 = 0
→ 2 × 2 = 4 div
→ 0 × 2 = 0
→ -2 × 2 = -4 div
→ 0 × 2 = 0

→ we can also find the exact values as sensitivity is known



Conclusions from y-t mode

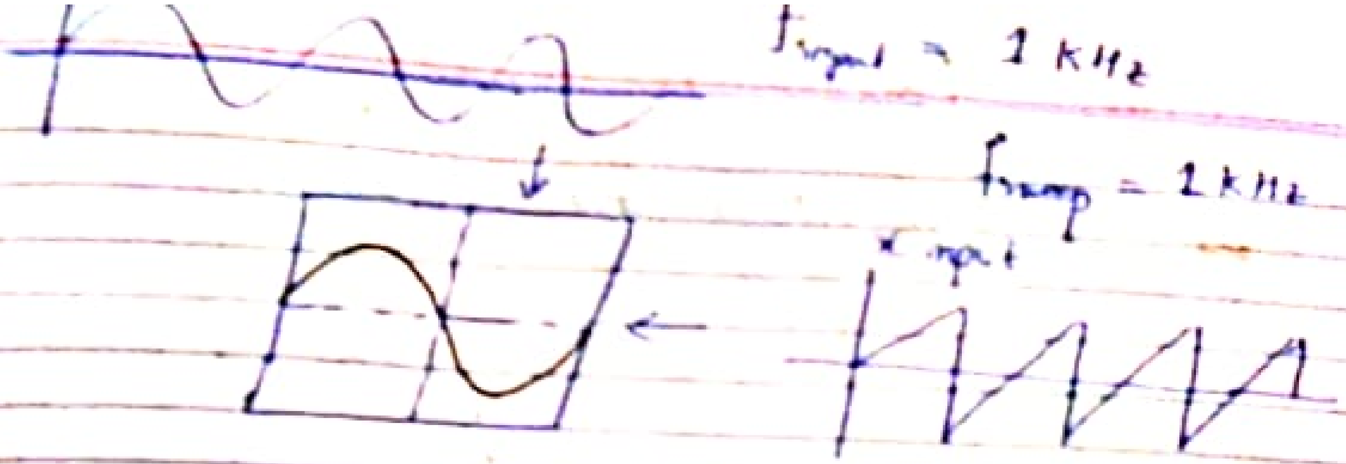
y input → unknown signal
x input → sweep signal / unknown signal

↓ yt mode ↓ x-y mode

↓ Rms, avrge, peak, time period ↓ Phase, frequency

CRO will display the signal definitely but what portion will it display is important.

↳ At least one complete cycle shall be displayed.



If n is the no. of cycles to be displayed;

$$\boxed{n = \frac{f_{signal}}{f_{sweep}}}$$

eg. Here $n = 1\text{ kHz} / 1\text{ kHz} = 1$

Similarly we can also calculate using time period;

$$n = \frac{1}{T_{sig}} \div \frac{1}{T_{sweep}} = \frac{1}{T_{sig}} \times T_{sweep}$$

$$\Rightarrow \boxed{n = \frac{T_{sweep}}{T_{signal}}}$$

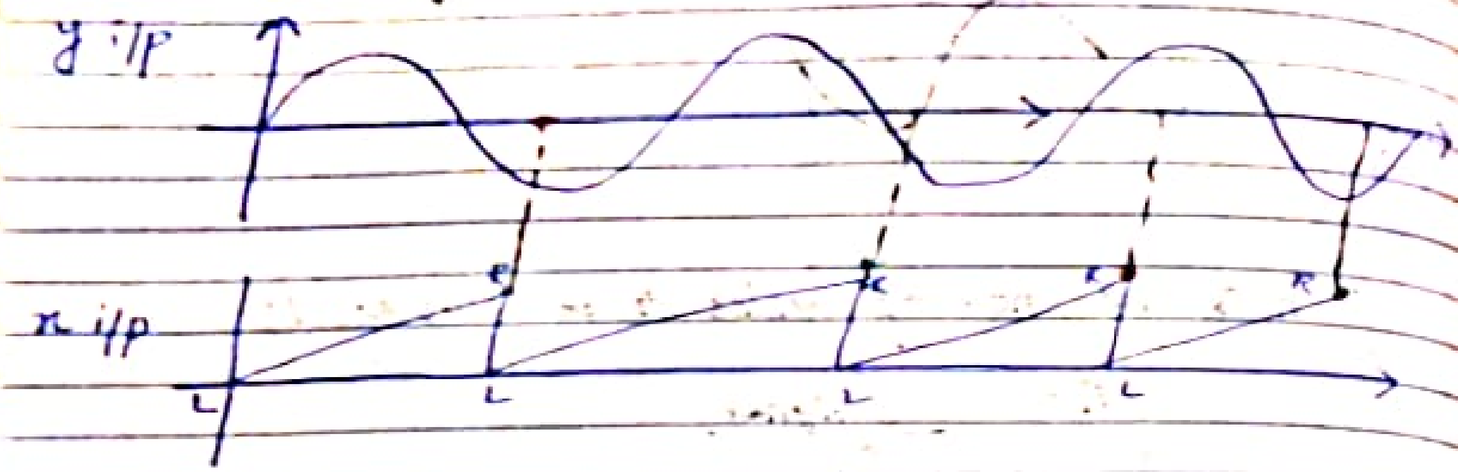
To display more cycles than 1,

$n > 1$ $\quad \frac{f_{signal}}{f_{sweep}} > 1 \Rightarrow f_{signal} > f_{sweep}$

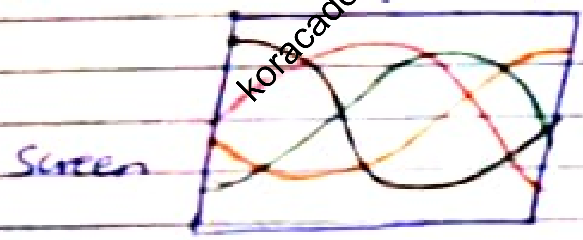
or similarly $\quad \frac{T_{sweep}}{T_{signal}} > 1 \Rightarrow T_{sweep} > T_{signal}$

C F_{signal} Vs F_{sweep}

1 Unsynchronized and untriggered



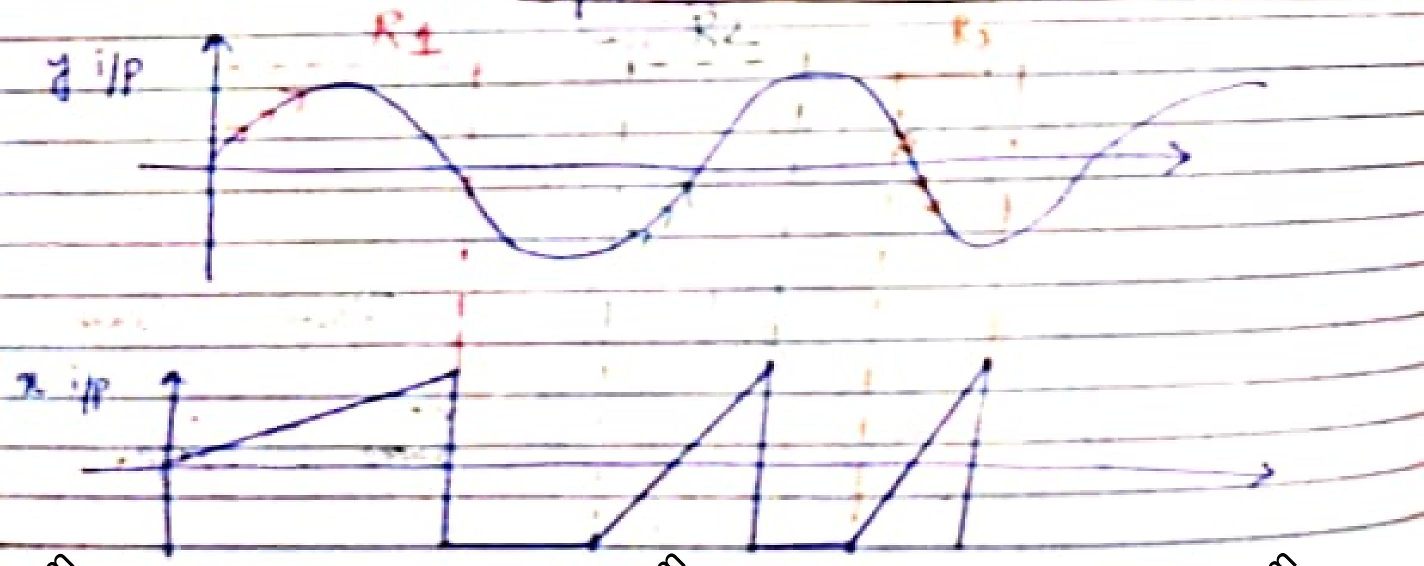
Oscilloscope displays the portion of y i/p for which the x i/p is increasing from left to right.



So we have a jumpled o/p.

2. Trigger Setting Are made

Slope = +ve Voltage level = 0 volts

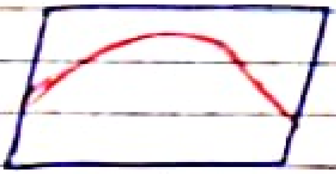


R1 satisfies conditions i.e slope is +ve & init voltage = 0

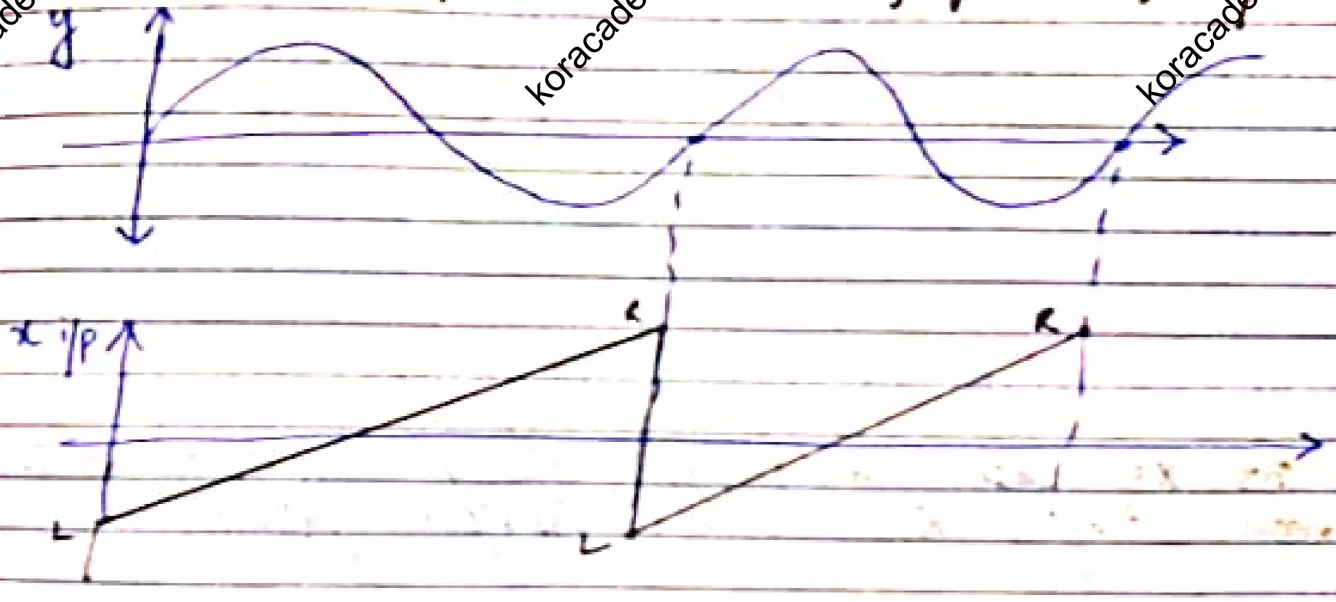
R2 has slope +ve but not voltage 0 so this will not be on the display.

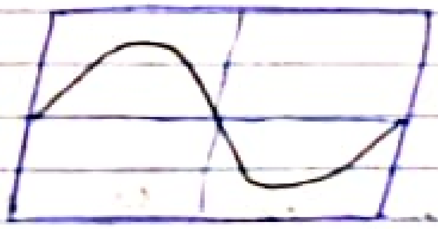
R3 does not satisfy any condition

So R_1 satisfies both conditions and hence it is steady state o/p to be displayed on the screen.



3. Trigger settings are made and with this we adjust time/div to make $f_{signal} = f_{sweep}$.



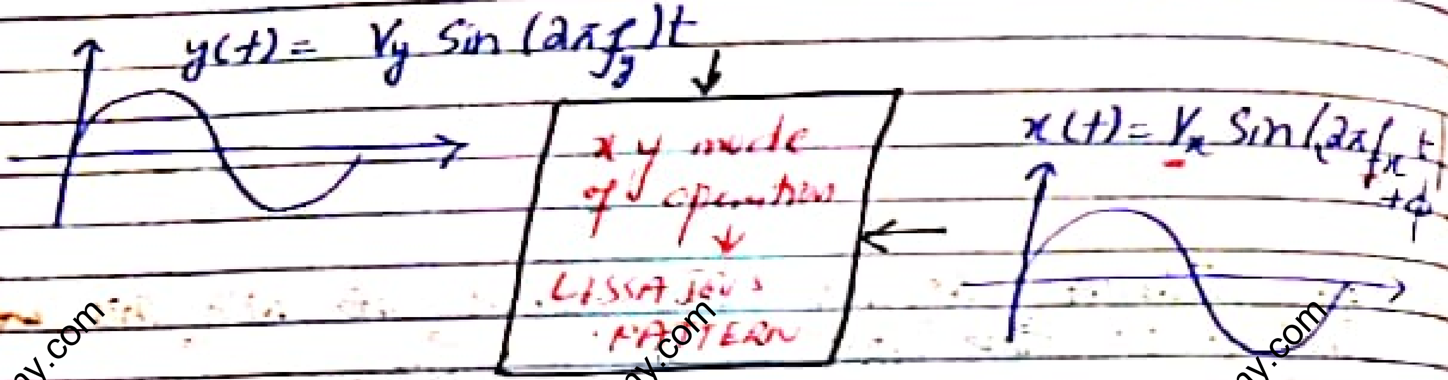


steady state o/p.

slope = +ve
initial voltage level = 0
$f_{sweep} = f_{signal}$
↳ synchronization

XY Mode of operation

Y input : Unknown signal
X input : externally applied known signal
↓
we know its phase and frequency etc.



In xy mode we are not using CRO as image plotter.
↳ No ramp on x (sweep signal) → no linear rise → no y signal plot on screen against rise

V_x and f_x are known.
V_y and f_y are unknown.

In this xy case, a pattern will be generated on the screen of CRO called Lissajous pattern.

With Lissajous pattern, we will carry out;
(i) Phase measurement.
(ii) Frequency measurement.

$V_y > V_x$ slope > 45°
$V_y = V_x$ slope = 45°
$V_y < V_x$ slope < 45°

Phase Measurment Using Lissajous Pattern

Y i/p → $V_y(t) = V_y \sin(2\pi f_y t + \phi)$ → unknown

x i/p → $V_x(t) = V_x \sin(2\pi f_x t)$ → known

Case 1 $\phi = 0°$

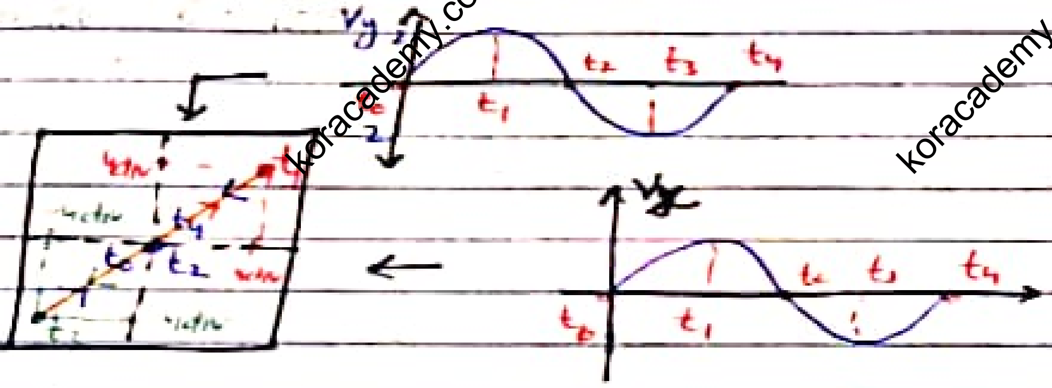
Assume $V_y = V_x = 2V$

$f_y = f_x = 50\ Hz$

$\omega_y = \omega_x = 2\pi \times 50 = 314$ rad/s

→ $V_y(t) = 2 \sin 314t$

$V_x(t) = 2 \sin 314t$

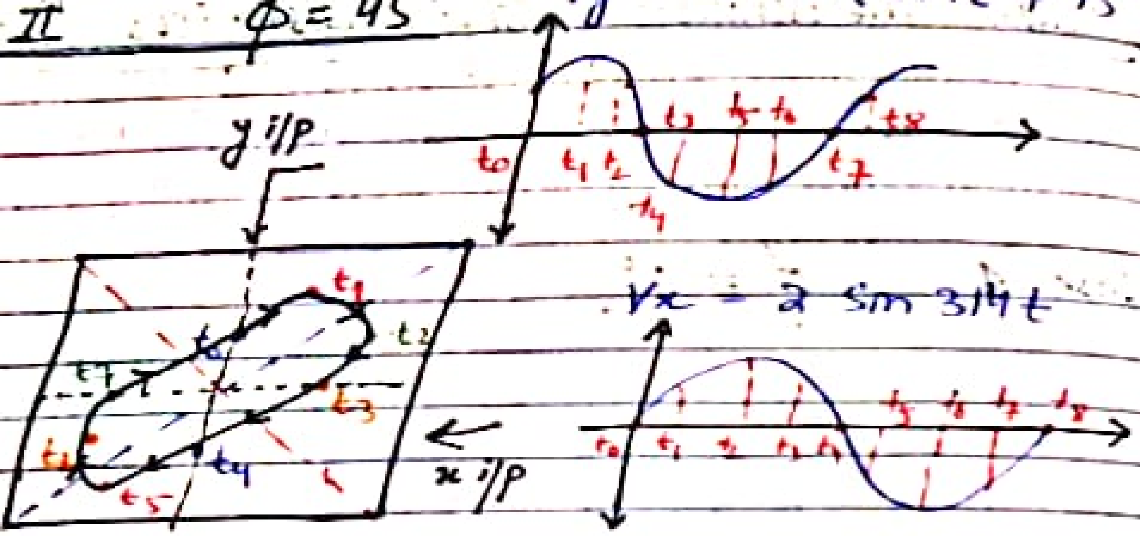


Say sensitivity $s = 2\ \frac{div}{V}$

t	x i/p	y i/p	
t_0	0	0	
t_1	2	2	→ 2 div/volt × 2 volt = 4 div
t_2	0	0	
t_3	−2	−2	→ 2 div/volt × −2 volt = −4 div.
t_4	0	0	

- Lissajous pattern for $\phi = 0°$
  ↳ is a straight diagonal line which is in 1st and 3rd quadrant. with slope 45°

Case II $\phi = 45°$



t	V_x	V_y
t_0	0	$2/\sqrt{2}$
t_1	$2\sqrt{2}$	2
t_2	2	$\sqrt{2}$
t_3	$\sqrt{2}$	0
t_4	0	$-\sqrt{2}$
t_5	$-\sqrt{2}$	-2
t_6	-2	$-\sqrt{2}$
t_7	$-\sqrt{2}$	0
t_8	0	$\sqrt{2}$

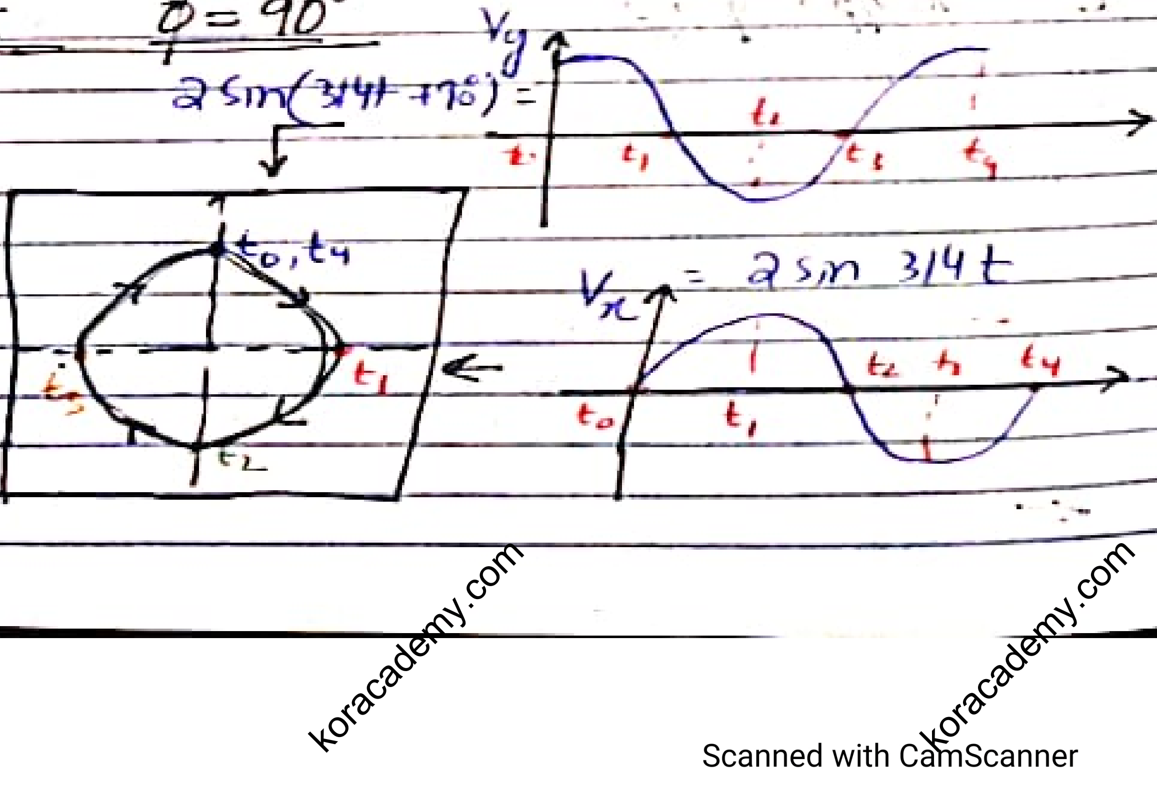
An ellipse with clockwise rotation is produced.

Major axis → I and III quadrant
Minor axis → II and IV quadrant

→ From this we conclude that phase shift is positive.

$0 < \phi < 90°$

Case III $\phi = 90°$

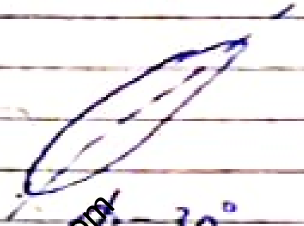


t	V_x	V_y
t_0	0	2
t_1	2	0
t_2	0	-2
t_3	-2	0
t_4	0	2

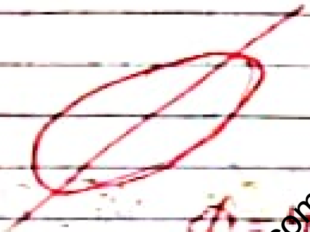
If phase difference is 90°, the Lissajous pattern is a perfect circle with clockwise rotation.

Conclusions From Lissajous pattern

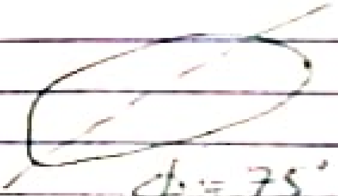
$\phi = 0°$
1st and 3rd quadrant
$V_x = V_y$ slope = 45°
$V_y > V_x$ slope > 45°
$V_y < V_x$ slope < 45°



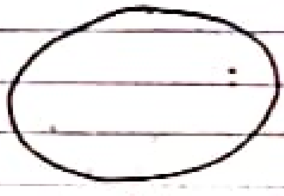
$\phi = 30°$
major axis in 1st and 3rd quadrant
clockwise rotation.



$\phi = 45°$
→ ″
→ ″
→ ″



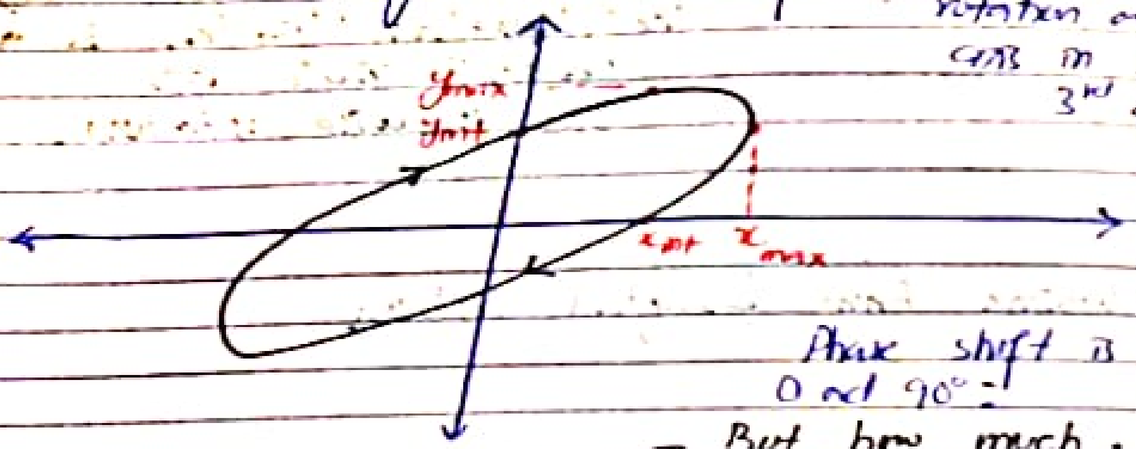
$\phi = 75°$
″
″
similar



$\phi = 90°$
clockwise rotation
Circle → special case of ellipse.

For straight line $y_{mt} = 0$
For ellipses $0 < y_{mt} < y_{max}$
For circle $y_{mt} = y_{max}$

Consider a generalized ellipse ; with clock wise rotation and major axis in 1st and 3rd quadrant



Phase shift is b/w 0 and 90° .

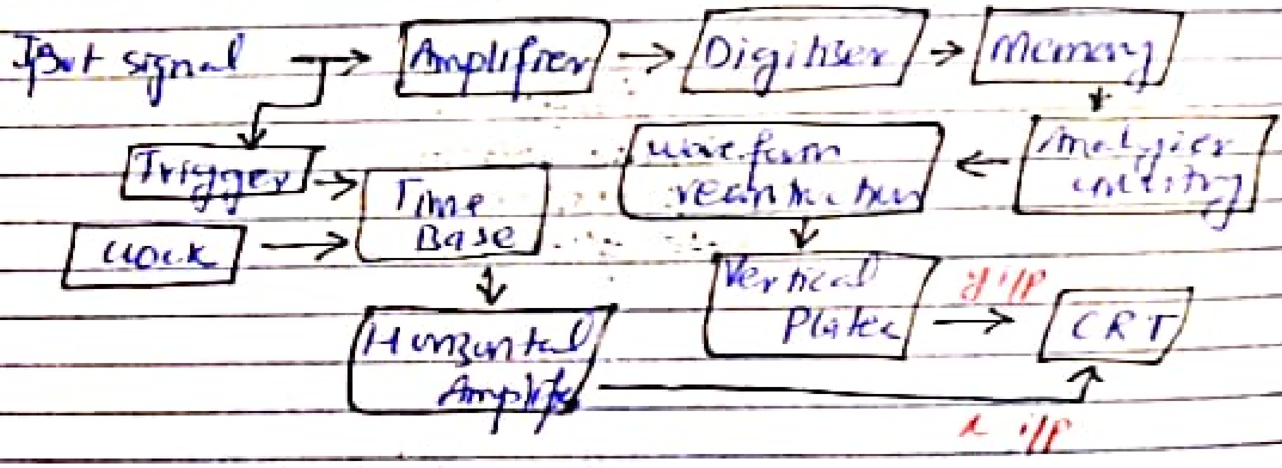
But how much ,

$$\boxed{\phi = \sin^{-1}\frac{x_{int}}{x_{max}}}$$

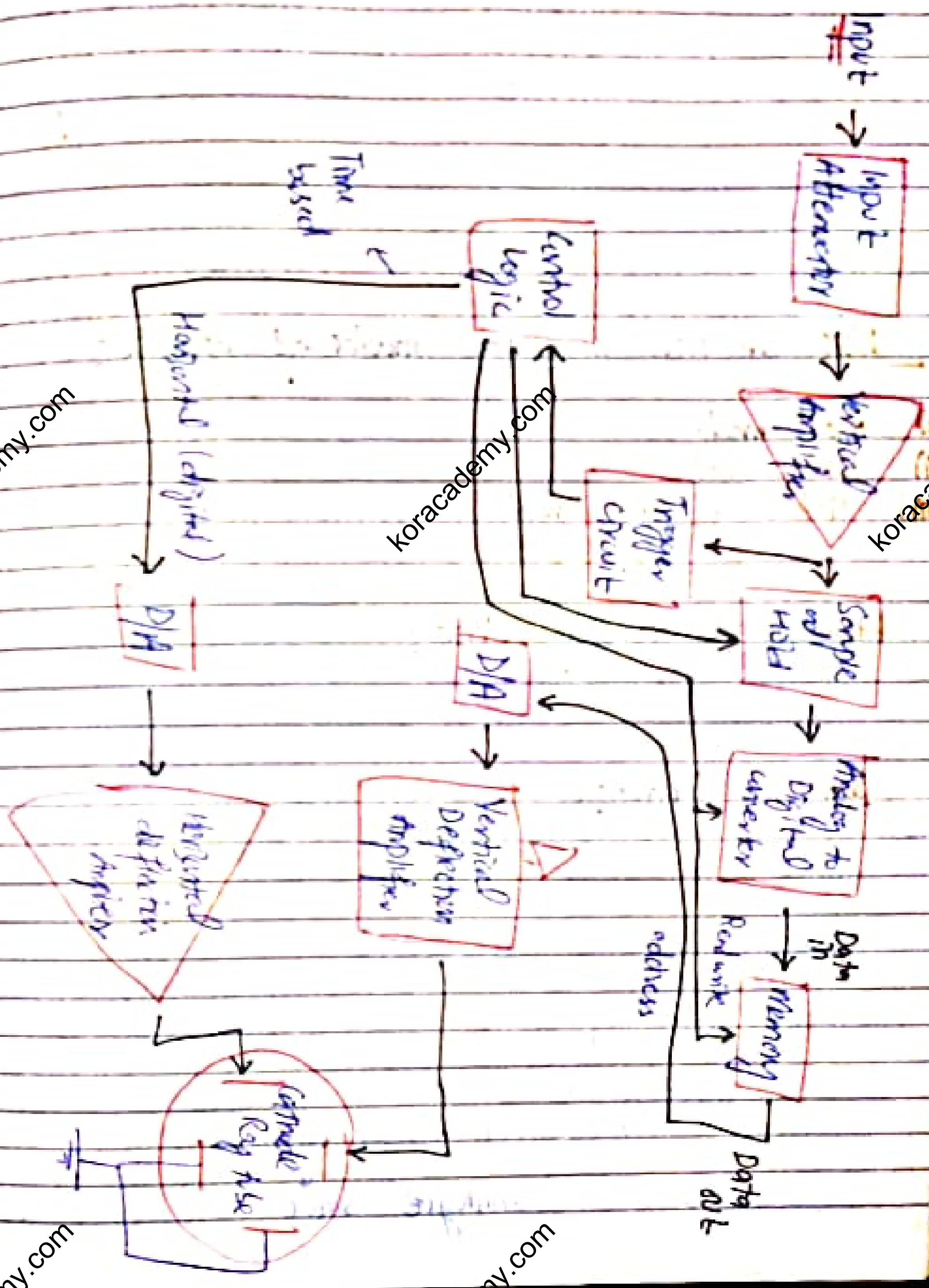
$$\boxed{\phi = \sin^{-1}\frac{y_{int}}{y_{max}}}$$

Digital Storage Oscilloscope (DSO)

Stores a signal in digital format.



A more detailed block diagram is as;



no distortion only amplification

→ The i/p signal is applied to the amplifier and attenuator section.

→ The attenuated signal is then applied to vertical amplifier.

→ Next it is given to Analog to digital converter.
↓
Create a data set that is stored in the memory.

→ Data set is processed by a microprocessor (control logic) and then sent to the display.

DSO can work in three modes of operation

(i) Roll mode → very fast varying signals are displayed.
(ii) Store mode → waveforms are stored in memory.
(iii) Hold or save mode → some part of waveform is held and then stored in the memory.

How is digitising done?

Digitising occurs by taking a sample of the i/p waveform at periodic intervals.

Sampling theorem. Sampling rate must be at least twice as fast as highest frequency in the i/p signal.

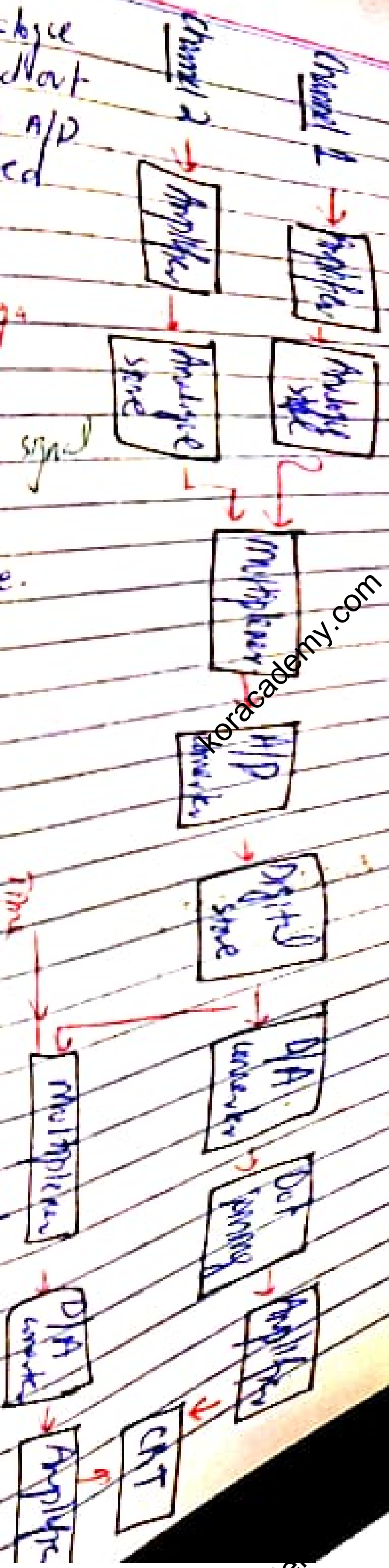
If sampling theorem is not satisfied, an effect called "ALiasing" occurs → analog signal is not properly converted to digital.

If the condition is satisfied → Resolution of A/D converter is decreased. → to eliminate
We introduce an analogue store.

→ When i/p signals are stored in analogue store register, They can be readout at a much slower rate to the A/D converter and the results are stored in digital store

→ Allows operation at upto 100 mega samples per second.

→ The oscilloscope will not read a signal once it is doing digitising.
↳ disadvantage of analogue store.

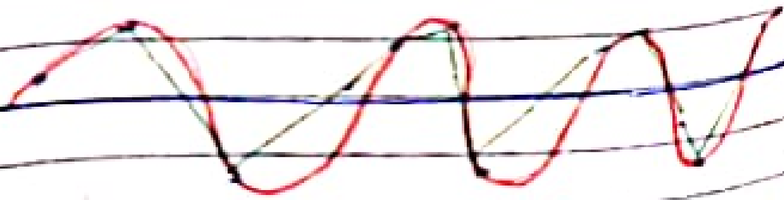


Waveform Deconstruction

(i) Linear interpolation –
(ii) Sinusoidal interpolation.

↳ dots are joined by a straight line

↳ dots are joined by a sine wave.



→ A DSO is an instrument which stores a digital copy of the waveform in the digital memory which it analyses further using digital signal processing techniques.

→ It captures non repetitive signals and displays it on the screen until the device gets reset.

→ In DSO, the signals are received, stored and displayed on the screen.

• The maximum frequency measured on the DSO depends on two things

(i) Sampling rate of the scope

(ii) Nature of the converter

• The traces in DSO are bright, highly defined, and displayed within seconds.

Advantage : Display visual as well as numerical values by analysing the stored traces.

The display traces can be magnified and also we can change the brightness of the traces and minute detailing can be done.

→ On the screen the i/p voltage is displayed on the perpendicular axis versus time.

→ DSO can display 3D figure or multiple waveforms for comparison purpose.

→ DSO can capture and store the electronic events.

→ DSO is widely used b/c of its advanced features like storage, display, fast trace rates and remarkable bandwidth.

DSO	Digital voltmeter
→ Shows the graphical representation of the signals for visual diagnosis and it helps to find out the unexpected voltage sources.	Only records the voltage fluctuations which further require diagnosis.