Communication systems

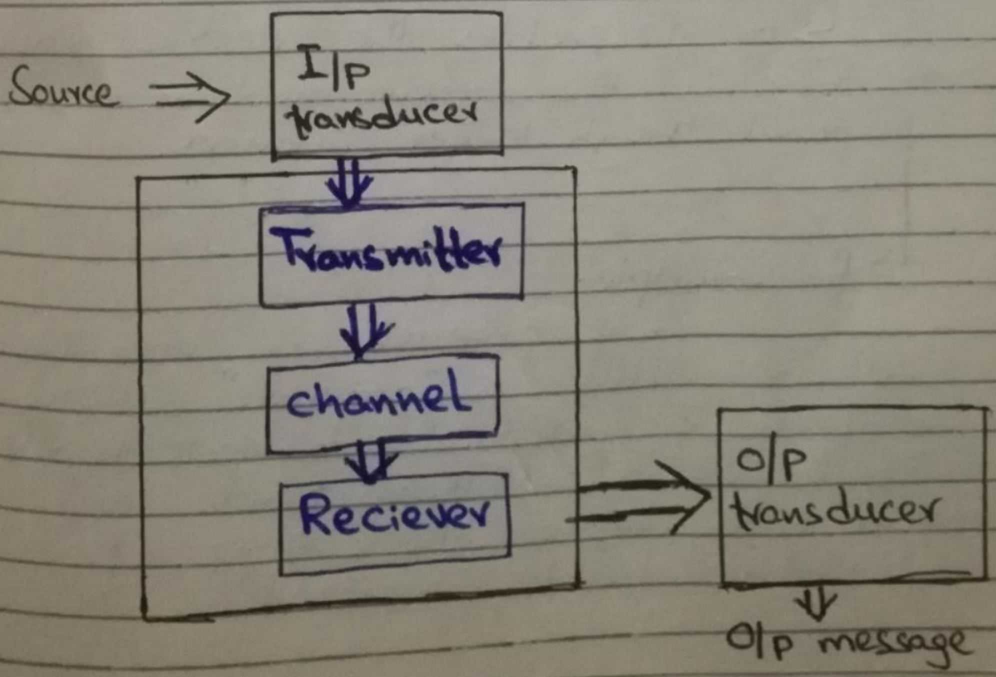
Lecture 1

What is Communication?

- Communication is the transfer of information from one point to another.

- In early days, Pigeons were used to transfer information. Then come era of wired N/W in which short distance communication was possible with similar data rates.

  ↳ Today with the development of electronic communication system, we can communicate at longer distance with high data rate and reliable connection



- Source: For example it can be an audio signal in the form of sound waves.

- I/P transducer: It convert the source to an electrical signal.

  → example is a microphone.

  source is a non-electrical signal.

Transmitter: It modifies the electrical signal in such a way that it can be passed through the channel.

→ or it enables the electrical signal to travel in the given medium and travel through longer distances

→ for example:

- if channel is air then the transmitter is "antenna" which ~~converts~~ transmit the electrical to electromagnetic signal which can then travel in air and hence the range of distance is also increased.

Can also cause distortion (changing shape of signal)

• Channel: It is the medium in which the signal travels

wired medium e.g optical fibre twisted pair, coaxial cable etc

wireless medium i.e free space

As signal move in this medium the signal loses energy which is called attenuation.

attenuation is the decrease in amplitude of a signal.

as amplitude of signal is related with power so it is the decrease in power of signal.

Note: In extreme cases the channel might act as a filter.

Noise is also added to the signal as it travels in the medium.

• Reciever: It undo nullify all the effects done by the transmitter and the medium (chanel)

it also undo the attenuation and the nullification of ~~noise~~ distortion

↳ the reciever is a complex circuit it consist of
- amplifier
- oscillator
- detector
- mixer

- O/P transducer: It converts the electrical signal to the desired signal which is required as an O/P message.
  ↳ for example load speaker

Modes of Communication:

1. Simplex:

- Communication is only in one direction
  ↳ for example: TV broadcast, Radio broadcast, Keyboard/monitor relation.
  ↳ In broadcast we have single sender and multiple recievers.
  ↳ ~~In multicast again we have single sender and multiple~~

↳ In multicast again we have one sender and multiple recievers but here information is send to a particular group of recievers.

↳ for enample teacher giving a lecture

2. Duplex :

• Sender and reciever both can send information to one another.

• Full Duplex:

• Sender and reciever can communicate at the same time i.e simultaneously

↳ For enample: Telephone system

• Half Duplex:

• The communication b/w sender and reciever occurs in both directions in half Duplex transmission, but only one at a time.

↳ The sender and reciever can both send and recieve the information, but only one is allowed to send at any given time.

↳ For enample: Walkie-talkie system

Frequency related questions that all wireless comm: engineers should know:

- What is the audible frequency range?

Ans: 200 Hz - 200 KHz

- What is frequency range of telephony (voice) signal?

Ans: 300 Hz - 3.4 KHz
↳ but in some 0 - 4.5 KHz
or 300 Hz - 4.5 KHz

- What is frequency range of video signal?

Ans: 0 - 4.5 MHz

- What is frequency range of amplitude modulation (AM)

Ans: 535 KHz - 1605 KHz
↳ in some books 550 KHz - 1720 KHz

• What is frequency range of frequency modulated (FM) signal?

Ans: 88MHz – 108MHz

• What is the frequency of Wifi signal if it is single band?

Ans: 2.4 GHz

↳ if dual band Wifi signal

Ans: 2.4 GHz & 5 GHz

↳ in some 2.4 GHz & 5.8 GH

• What is frequency of Bluetooth signal?

Ans: 2.45 GHz

• What is frequency of GSM?

Ans: GSM has two bands

900 MHz & 1800 MHz

↓ 2G

↓ 2.5G (H tech)

• What is micro-wave frequency range?

Ans: 300 MHz to 300 GHz

• What was frequency of 1G mobile communication:

Ans: 150 MHz

• 2G mobile comm: 900MHz
• 2.5G mobile comm: 1800 MHz
"H" tech

GSM base technology

• 3G mobile comm: 1.6GHz – 2GHz

• 4G mobile comm: 2GHz – 8GHz

• 5G mobile comm: 28GHz & above.

- In OP Amps dB and dBm are taken in terms of Voltage

• dB & dBm scale:

In context of power gain:

$$\boxed{P_{(dB)} = 10 \log_{10}\left(\frac{P_{out}}{P_{in}}\right)} \quad \text{—— (A)}$$

• if ratio is

$P_{out} = 2 \times P_{in}$

from (A) $\Rightarrow P_{(dB)} = 3dB$

and so on

ratio $\frac{P_{out}}{P_{in}}$	$P_{(dB)}$
2	3 dB
4	6 dB
8	9 dB
10	10 dB
100	20 dB
1000	30 dB
10^4	40 dB
10^5	50 dB

here gain is = +tive

b/c $P_{out} > P_{in}$

• if $P_{out} < P_{in}$ then ratio:

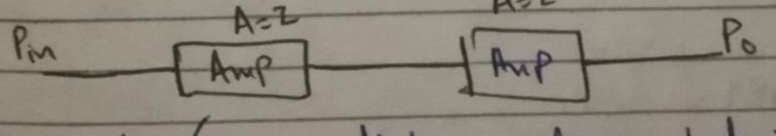
$P_{out} = \frac{1}{2} \times P_{in}$

from (A) $\Rightarrow P_{dB} = -3dB$

and so on

ratio	$P_{(dB)}$
1/2	−3 dB
1/4	−6 dB
1/8	−9 dB
1/10	−10 dB
1/100	−20 dB
1/1000	−30 dB

• Suppose Amplifiers:



• In cascade we multiply values but in dBs.

3dB + 3dB = 6dB

b/c $\log a \times b = \log a + \log b$

Conversion:

let gain of amplifier in dB is 13dB i.e

$P(dB) = 13dB$ convert to linear scale.

Sol:

(i) we can use equation (A)

(ii) OR as

$$P(dB) = 13dB$$
$$= 10dB + 3dB$$
$$= 10 \times 2$$
$$P = 20$$

so in linear scale $P = 20$ and it is amplifying it 20 times.

dB in context of voltage ratio/gain:

$$\boxed{A_v(dB) = 20 \log_{10}\left(\frac{V_{out}}{V_{in}}\right)} \longrightarrow (B)$$

- if ratio is:

if $V_{out} = 4\, V_{in}$

$A_v(dB) = 20 \log_{10} 4$

$A_v(dB) = 12dB$

Also

$V_{out} = 2\, V_{in}$

$A_v(dB) = 6dB$

Note: dB(V) will be double of the dB(P) scale

ratio	dB(P)	dB(V)
2	3dB	6dB
4	6dB	12dB
8	9dB	18dB
10	10dB	20dB
100	20dB	40dB
1000	30dB	60dB
10000	40dB	80dB
10^5	50dB	100dB

- Suppose we hape OP Amp having $A_v = 10^5$ then $A_v(dB) = ?$

$$A_v(dB) = 20 \log_{10}(10^5)$$
$$A_v(dB) = 100\,dB$$

- **dBm:**
  - It is used mostly for power
  - dB is a ratio but dBm is w.r.t to some refrence power i.e 1mW.

- $$\boxed{P(dBm) = 10 \log_{10}\left(\frac{Power}{1mW}\right)} \quad \text{—— (c)}$$

- If we want to convert $1W \rightarrow dBm$

$$P(dBm) = 10 \log_{10}\left(\frac{1W}{1mW}\right)$$
$$P(dBm) = 30$$

- So power in dBm is

$1W \longrightarrow 30\,dBm$

- So if power is given then

$$\boxed{P(dBm) = P(dB) + 30} \longrightarrow (D)$$

Eg

$$P(dB) = P(dBm) - 30$$

- **What is signal?**

- Signal is something that carries information, but this definition is not enough in communication and electronics.

↳ Signal should be detected measured, must contain information e.g traffic lights, temperature etc

- Signal must vary w.r.t some independent variable e.g space or time.

- Audio, speech, video, stock market values, image etc are all signals.

• What is ~~sys~~ system?

• System is an entity that transforms or processes a set of signals (inputs) and yeilds another set of signals (O/Ps)

↳ It may consist of physical/hardware components such as electrical and mechanical systems, or it may be an algorithm that ~~takes~~ some inputs, does processing and yeilds outputs (software part)

↳ For example Amplifier is a system, as it takes input, amplifies it and takes it out as an output.

Size of signal?

• The size of signal indicates the largeness or strength of the signal.

↳ This strength is measured in terms of energy or power.

• Energy signal:
It has finite energy. Condition for finite energy is that signal amplitude → 0 as $|t| \to \infty$.

↳ When energy is finite, power is zero
i.e $P = E/t = \frac{\text{finite value}}{\infty} = 0$

∴ So, energy signal has zero power. $P = 0$

- So we have

$$E_g = \int_{-\infty}^{\infty} g^2(t)\, dt \quad \text{—— (i)}$$

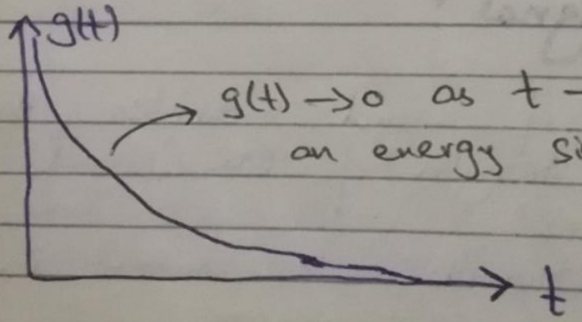
energy of signal

- Now if we have an energy signal then eq (i) will be some finite value

↳ i.e $0 < E_g < \infty$

or

$$0 < \int_{-\infty}^{\infty} g^2(t)\, dt < \infty$$



- If we have a complex signal:

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2\, dt \quad \text{—— (ii)}$$

Note: Power signal = $E = \infty$
energy signal : $P = 0$

Power signal: It is a signal having finite power.

↳ When power is finite, the energy is infinite

i.e $P = \frac{E}{t}$

$E =$ finite value $\times \infty$

$E = \infty$

∴ So energy of a power signal is ∞

- Power is given by

$$\underbrace{P_g}_{\text{power of a signal}} = \lim \frac{1}{T} \int_{-T/2}^{T/2} g^2(t)\,dt \quad \text{—— (iii)}$$

Note: Most of the periodic signals are Power signals

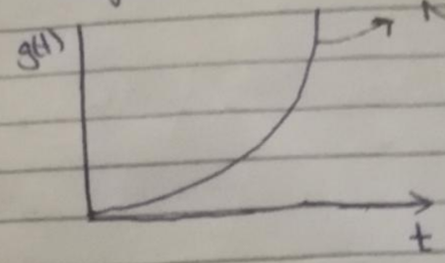
- There are also niether energy nor power signals (NENP).

↳ e.g $g(t) = \tan t \longrightarrow$ NENP

it is periodic but

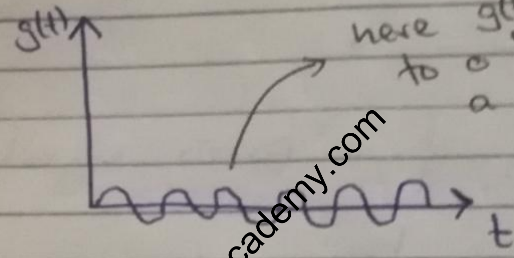
b/c its amplitude $\rightarrow \infty$ as the time $\rightarrow \infty$

- NENP signal:



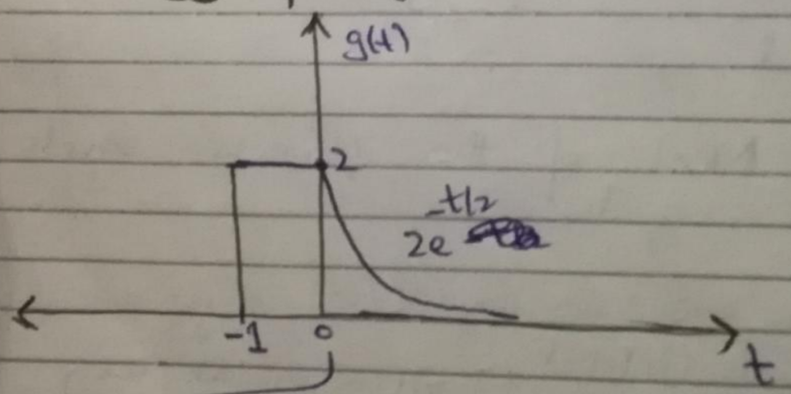
NENP b/c $g(t) \to \infty$ as $t \to \infty$

- Power signal:



here g(t) doesn't approach to 0 or ∞ so it is a power signal

- Find energy of signal?



It is an energy signal b/c when t approch to ∞ g(t) approach to zero

we know

$$E_g = \int_{-\infty}^{\infty} g^2(t)\,dt$$

$$E_g = \int_{-1}^{0}(2)^2\,dt + \int_{0}^{\infty}\left(2e^{-t/2}\right)^2 dt$$

$$E_g = 4\int_{-1}^{0} dt + 4\int_{0}^{\infty} e^{-t}\,dt$$

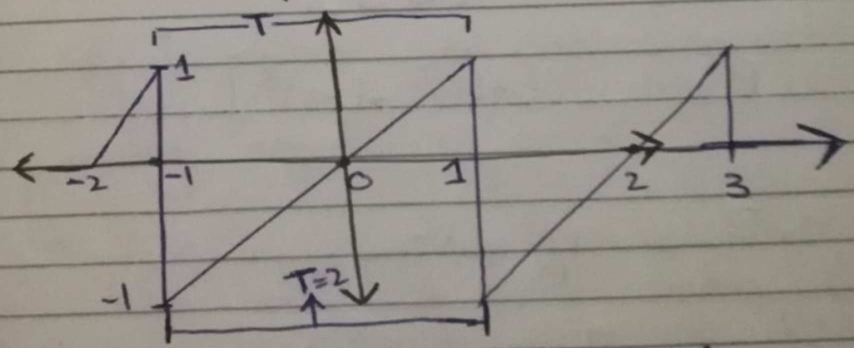
$$E_g = 4[t]_{-1}^{0} + 4\left[\frac{e^{-t}}{-1}\right]_{0}^{\infty}$$

$$E_g = 4[0-(-1)] + \frac{4}{-1}\left[e^{-\infty} - e^{-0}\right]$$

$$E_g = 4 - 4[0-1]$$

$E_g = 4 + 4 = 8\,J$ which is a finite value so it is an energy signal

• Find Power of the signal?



• We know that for periodic signal:

$$P = \lim \frac{1}{T}\int_{-T/2}^{T/2} g^2(t)\,dt$$

from graph $T=2$

• Now for straight line

$$y = mx + c$$

here

$$g(t) = mt + 0 \Rightarrow g(t) = mt$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow m = \frac{1-(-1)}{1-(-1}$$

$$m = \frac{2}{2} \Rightarrow m = 1$$

$$g(t) = t$$

So

$$P = dt \frac{1}{2}\int_{-1}^{1} t^2 \, dt$$

$$P = \frac{1}{2} \left[\frac{t^3}{3}\right]_{-1}^{1}$$

$$P = \frac{1}{6} \left[(1)^3 - (-1)^3\right]$$

$$P = \frac{1}{3} \text{ watt}$$

Classification of signals:

- Energy and power signals
- Continous and discrete signals → Time axis
- Analog and digital signals → Amplitude axis
- Periodic and non-periodic signal
- Deterministic and random signals.

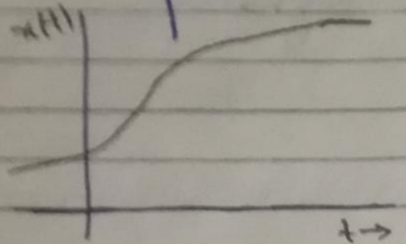
Continous time signals are defined for continous values of time.

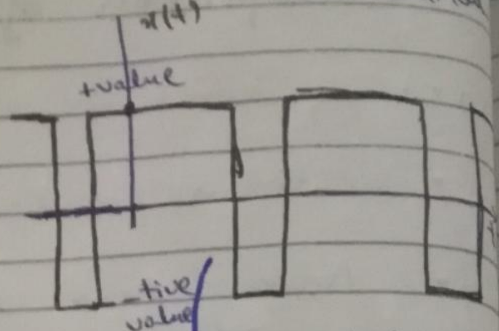
↳ While **discrete time signals** are defined for discrete values of time.

A signal whose amplitude can take any value in a continous range is an **analog signal.**

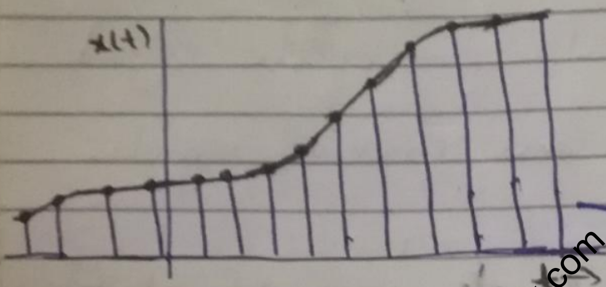
↳ while a signal whose amplitude can take only a finite number of values or quantized values is called **digital signal.**

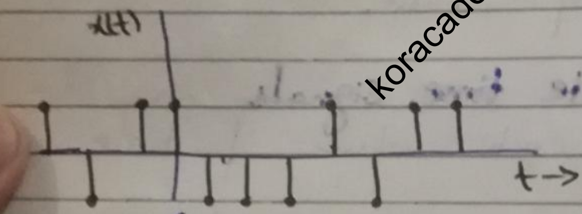
• continous signal
analog signal { b/c any value can be taken on amplitude axis i.e amplitude axis is continous





b/c t axis is continous
• continous signal
• digital signal
b/c amplitude axis only has 2 values i.e finite values





b/c time is defined for discrete values
• discrete signal
• analog signal
b/c the amplitude axis (x(t)) can take any value.

• Discrete signal
• digital signal b/c amplitude axis has only two values

for discrete:
$g[n] = g[t+n]$, $n = 1, 2, 3 \ldots$

- A **periodic signal** is one which repeats after certain intervals of time. i.e $g(t) = g(t+T)$ T = time period

- **Deterministic signals** can be expressed in mathematical or graphic form. for example sine wave.

  → **Nondeterministic signal**: (random signal or ~~prop~~probabilistic signal).

  → it can't be expressed in mathematical or graphical form but it can be described in probability terms like mean, variance, standard deviation and distributions. For example noise signal can be given by gaussian distribution.

Lecture #2

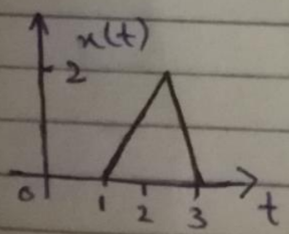
Operation performed on signal: or time transformation of signals:

- **Time shift:** It is shifting of signal w.r.t time.

Time shift → time delay, time advance.

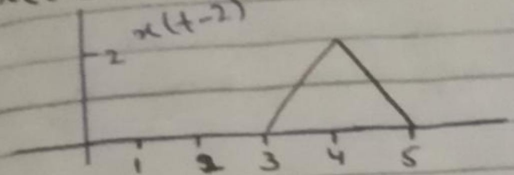
- let '$x(t)$' be the orignal signal and let 't_0' be some time greater then zero:
  - $x(t-t_0)$ is the time delayed signal
    - shift to right
  - $x(t+t_0)$ is the time advance signal
    - shift to left

for example:



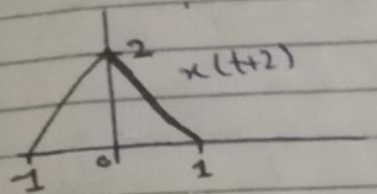
- for $x(t-2)$

- $x(t-2)$



$\therefore t-2 = 1$
$t = 3$
$\therefore t-2 = 3$
$t = 5$
$\therefore t-2 = 2$
$t = 4$

- $x(t+2)$



$\therefore t+2 = 1$
$t = 1-2 = -1$
$\therefore t+2 = 3$
$t = 1$
$\therefore t+2 = 2$
$t = 0$

Note: Time shifting is used in many signal processing applications.

↳ for example a time delayed version of the signal is used when performing auto-correlation: A statistical concept which measures a degree of similarity b/w a given signal and a lagged version of itself.

$\Longrightarrow$

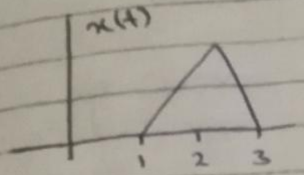
Time scaling: The compression or expansion of a signal in time is known as time scaling.

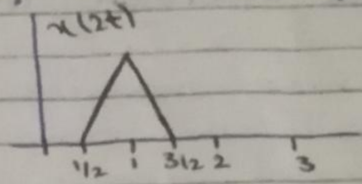
- If $x(t)$ is the orignal signal then for $a > 1$:
  - $\rightarrow$ $x(t/a)$ is enplanded / slowed down signal
  - $\rightarrow$ $x(at)$ is compressed / hurried forward signal

OR for $x(at)$
- $\rightarrow$ for $0 < a < 1 \longrightarrow$ enpand
- $\rightarrow$ for $a > 1 \longrightarrow$ compress

for example:

- if $x(t)$ represents an audio tape recording then $x(2t)$ represents same audio tape being compressed in time so it plays as twice the speed. i.e 2x speed
  - $\rightarrow$ Eg if $x(t/2)$ represents the same tape being enpanded in time, so it is slowed down and plays half the speed.





$\therefore 2t = 1$

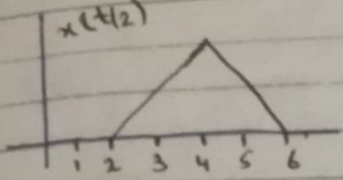
$t = 1/2$

$\therefore 2t = 2$

$t = 1$

$\therefore 2t = 3$

$t = 3/2$



$\therefore t/2 = 1$

$t = 2$

$\therefore t/2 = 2$

$t = 4$

$\therefore t/2 = 3$

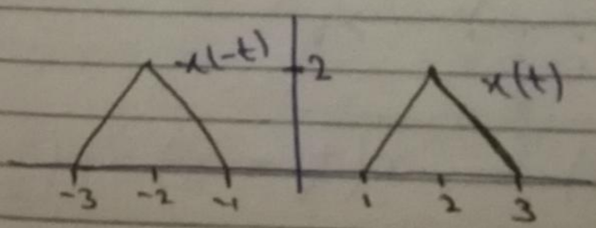
$t = 6$

Time inversion: It is also called time reversal or time folding or flipping signal.

→ It is special case of time scaling with $a = -1$

→ if $x(t)$ is original signal, then $x(-t)$ is the inverted signal.

→ so in this case we get mirror image of about the vertical axis.



Time inversion in convolution:

- Time reversal is an important step when computing the convolution of signals.

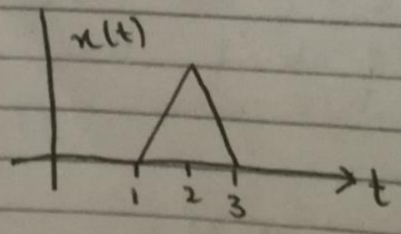
Convolution:

In convolution one signal is kept in its orignal state/position is fixed while the other is mirror imaged and slided along the former signal to obtain the result.

↳ convolution is performed to find the response of a system and is used in many image processing and signal processing procedures.

- Order of operation:
  - first shift
  - then scale
  - finally invert

Exp: $x(t)$ is given find $x\left(\frac{t+3}{2}\right)$

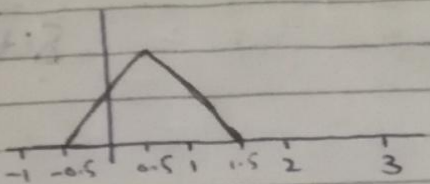


Sol: $x\left(\frac{t+3}{2}\right) = x\left(\frac{t}{2} + 3/2\right) = x\left(\frac{t}{2} + 1.5\right)$

+tive so advancing shift left

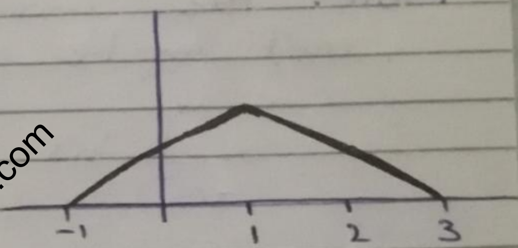
- graph $x\left(\frac{t}{2}+1.5\right)$

⇒ **Shifting:**



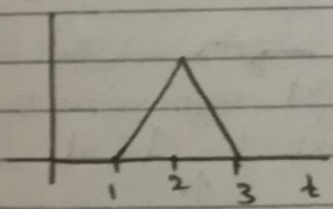
$\therefore t+1.5=1$
$t=-0.5$
$\therefore t+1.5=3$
$t=1.5$

⇒ **Scaling:**

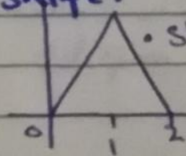


$\therefore t/2=-0.5$ $\quad\therefore t/2=0.5$
$t=-1$ $\quad t=1$
$\therefore t/2=1.5$
$t=3$

Exp: $x(t)$ is given
find $x\left(\frac{(-2t+3)}{3}\right)$
↳ $x\left(-\frac{2}{3}t+1\right)$

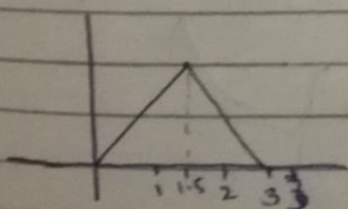


⇒ **shift:**



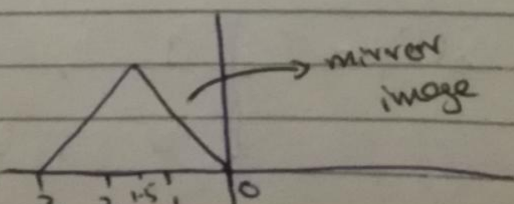
• shifting $x(t+1)$

⇒ **scaling** $x\left(\frac{2}{3}t\right)$



$\therefore \frac{2t}{3}=1 \Rightarrow t=3/2$
$\therefore 2t/3=2 \Rightarrow t=3$

⇒ **inversion** $x(-t)$



1) Unit Impulse signal:

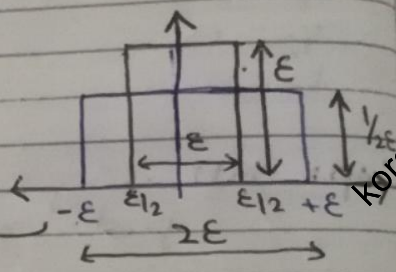
↳ Also known as Dirac Delta and is represented by $\delta(t)$.

• Definition of unit impulse signal:

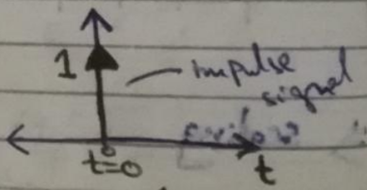
$$x(t) = \begin{cases} \infty, & t=0 \\ 0, & \text{else} \end{cases}$$

Note: Area under the curve of unit impulse signal is one/unity

$$\int_{-\infty}^{\infty} \delta(t) = 1$$



So ~~[illegible]~~ a signal whose ~~[illegible]~~ width approaches to zero and hight approaches to infinity while area is constant = 1 it is an impulse signal.



• Definition of impulse signal:

$$\delta(t) = \begin{cases} 1, & t=0 \\ 0, & \text{else} \end{cases}$$

$\Longrightarrow$

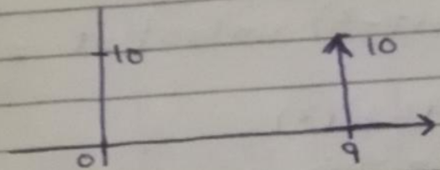
- General expression of impulse signal:

$$x(t) = K\,\delta(t-t_0)$$

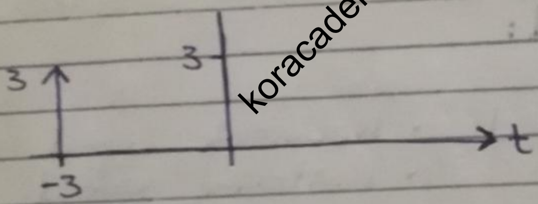
K → magnitude or amplitude

$\delta(t-t_0)$ → impulse signal shifted by t_0

- For example: $x(t) = 10\,\delta(t-9)$ → nine



$x(t) = 3\,\delta(t+3)$



Properties:

(i) $\delta(t) = \delta(-t)$ i.e $\delta(t)$ is even signal

(ii) $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

(iii) $\int_{a}^{b} K\,\delta(t-t_0)\,f(t)\,dt = K f(t_0) \longrightarrow (a)$

$\therefore$ where $a \leq t_0 \leq b$

or $\int_{a}^{b} K\,\delta(t)\,f(t)\,dt = K f(0) \longrightarrow (b)$

$\Longrightarrow$

this property (iii) is called the sifting property or sampling property

So the product of any function with impulse ftns, is equal to the value of that ftns at a point where impulse ftns exists.

↳ provided that the function is continous at point where impulse ftns exist

Exp: $\int_{0}^{5} 10\,\delta(t-3)\sin 100t\,dt = Kf(t_0)$

$Kf(3) = 10\sin(100)(3)$

$Kf(3) = 10\sin 300$

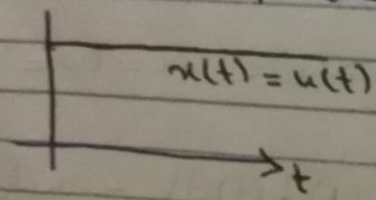
Unit step signal:

↳ Also called
- heaviside signal
- switching signal
- Identity signal

↳ represented by $u(t)$:

- it is defined as

$$x(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

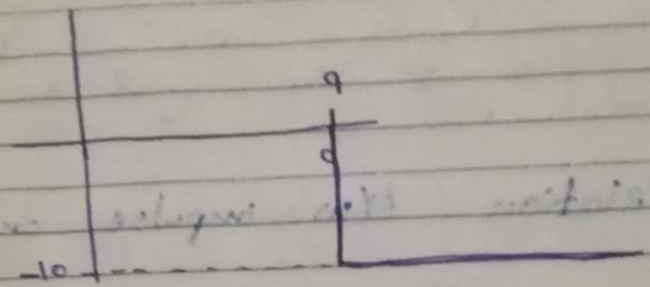


• Unit step signal give starting point to the signal so it is called the identity signal.

• General representation of $u(t)$:

$$x(t) = k\,u(t-t_0)$$

Exp: plot $-10u(t-9)$ nine

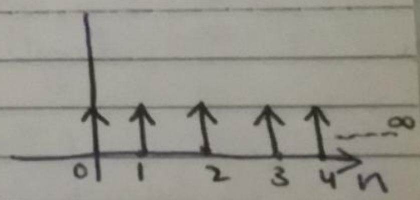


• **In discrete time:**

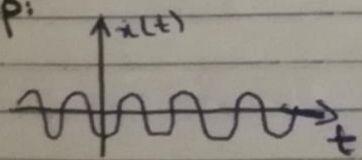
$u[n]$

here

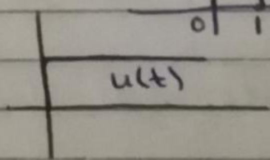
$$x[n] = \begin{cases} 1 , & n \geq 0 \\ 0 , & n < 0 \end{cases}$$



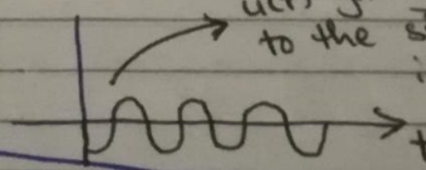
Exp:



X



y(t) =

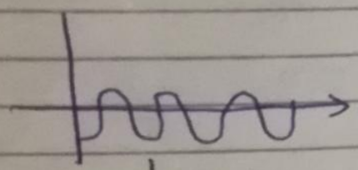


u(t) giving starting point to the signal so called identity or identification signal

or we can say that it make the system causal

Causal system:

↳ system which depend only on present and past values.



↳ when $u(t)$ is multplied with any signal, the signal become causal.

- **Relation b/w impulse and step signal**

$$u(t) = \int \delta(t)\, dt$$

$$\delta(t) = \frac{d}{dt} u(t)$$

Lecture 3

Fourier series:

- It is used for periodic signals.
- It is a way of representing a signal in terms of (infinite) **sine and cosine terms**

↳ But real time signals are non-periodic signals

↳ So **Fourier Transform** is used for the analysis of non-periodic signals.

↳ aperiodic

CTFT DTFT

Periodic Signal:

If we time shift a signal equal to time period, we get no change in a periodic signal.

ie $x(t) = x(t \pm T)$

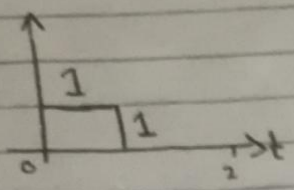
Fourier Series general representation:

$$\boxed{x(t) = dc + \text{sine and cosine terms}}$$

$\Longrightarrow$

time average of a ftn given us the DC value

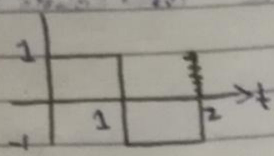
DC term:

- 

Area under curve = 1

DC value = $\frac{\text{Area}}{\text{time}}$

DC value = $\frac{1}{2}$

- 

$DC = \frac{0}{\text{time}} = 0$

Harmonics: Integer multiples of fundamental frequency.

Suppose:

$x(t) = A\sin\omega t + B\sin 2\omega t + C\sin 3\omega t$

$\omega \rightarrow$ fundemental frequency

$\omega = 2\pi f$ or $\omega = \frac{2\pi}{T}$

second/2nd harmonic or even harmonic

3rd harmonic or odd harmonic

∴ A, B & C are called co-efficients of harmonics, it defines the weights of the harmonics

∴ Suppose $x(t) = 2\sin\omega t + 7\sin 2\omega t + 11\sin 3\omega t$

this means that the 3rd harmonic is more dominant then 2nd and fundement harmonic

~~not~~ Fourier series can't be applied to every periodic signal.

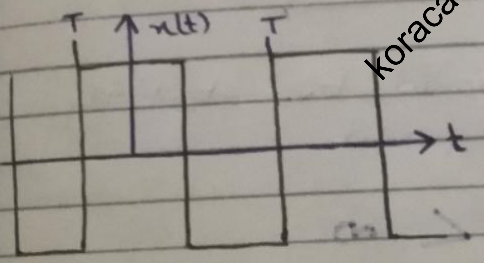
• Types of fourier series expansion:

1. Trigonometric Fourier series expansion
2. Exponential Fourier series expansion

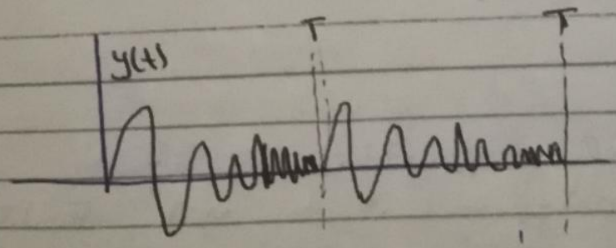
• Dirichlet's condition:

The periodic signal must fullfil / satisfy the Dirichlets conditions before we can apply the Fourier series.

1. Finite number of maxima and minima in one time period



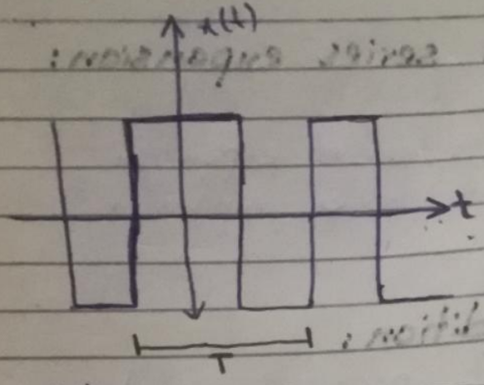
• only 2 maxima and minima in one period T-T. so F.series can be applied to this periodic signal



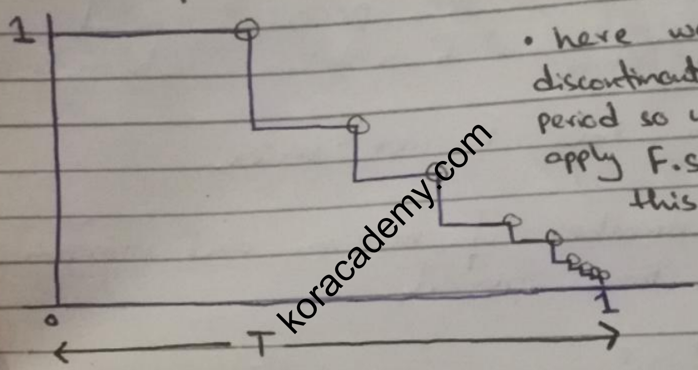
• infinite maxima and minima in one period so F.series can't be applyied to this periodic signal

2. Finite number of discontinuties in one time period.

⟹



• here we have only 2/finite number of discontinuities in one time period so F.series can be applied.



• here we have infinite discontinuities in one period so we can't apply F.series on this signal.

3 The function should be absolutely integrable over time period.

$$\int_T |x(t)| < \infty$$

→ As tan θ approaches to infinity at some points so it is not integrable absolutely.

Trignometric Fourier series:

$x(t)$ = ~~dc~~ sine and cosine terms

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

a_0 → dc term

$$\therefore a_0 = \frac{1}{T_0} \int_{T_0} x(t)\,dt \rightarrow \text{Area} \div T_0$$

a_0 → average or dc value

$$\therefore a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos n\omega_0 t\,dt$$

$$\therefore b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin n\omega_0 t\,dt$$

Exp:

$$x(t) = \underbrace{3}_{a_0/dc} + 2\sin\omega t + 3\cos\omega_0 t + 7\sin 2\omega_0 t + 5\cos 2\omega_0 t + \cdots$$

here

$a_0 = 3$, $a_1 = 3$, $b_1 = 2$

$a_2 = 5$, $b_2 = 7$

Important points about symmetries:

- If signal is symmetric about time axis, the area is zero so 'a_0' is also zero

i.e $a_0 = 0$ for symetric signal

(exto) — Symetric about x-axis : $(x,-y)$
Symetric about y-axis : $(-x,y)$
Symetric about origin : $(-x,-y)$

- If signal is even i.e symmetrical about y-axis there will be no '**bn**' terms b/c sine is an odd function
  - i.e if $x(t) = x(-t)$ then $b_n = 0$

- If the signal is odd signal i.e symetrical about origin then there will be no '**an**' terms b/c cosine is an even ftns.
  - if $x(t) = -x(t)$ then $a_n = 0$

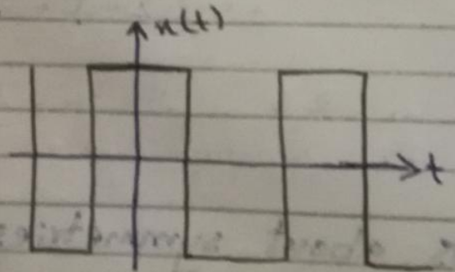
Conclusion:

(i) $a_0 = 0$ if signal symetric about time axis

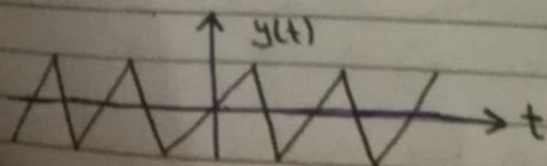
(ii) $b_n = 0$ if signal symetric about y-axis

(iii) $a_n = 0$ if signal symetric about ~~x-axis~~ origin

Exp:



- $a_0 = 0$
- $b_n = 0$
- a_n = find by formula



- $a_0 = 0$
- $a_n = 0$
- b_n = find

b/c c=0 in y=mx+c passing through origin

- Compact trigonometric series:

- from trigonometric series:

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

- if periodic signal is real:

$$\therefore a_n \cos n2\pi f_0 t + b_n \sin n2\pi f_0 t = C_n \cos(n2\pi f_0 t + \theta_n) \longrightarrow (i)$$

$$\therefore C_n = \sqrt{a_n^2 + b_n^2} \longrightarrow (ii)$$

$$\therefore \theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right) \longrightarrow (iii)$$

$$\therefore C_0 = a_0 \longrightarrow (iv)$$

- so for a compact trignometric series:

$$g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n2\pi f_0 t + \theta_n) \longrightarrow (v)$$

where $t_1 \leq t < t_1 + T_0$

Example $\Longrightarrow$

Exp 2-7: Find the compact trigonometric series representation of

$$e^{-t/2} \quad , \quad 0 \le t \le \pi$$

Sol: • $T_0 = \pi$, $w_0 = \frac{2\pi}{T_0} = \frac{2\pi}{\pi}$

$w_0 = 2 \text{ rad/sec}$

• $f_0 = \frac{1}{T_0} = \frac{1}{\pi}$

$$g(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n w_0 t + b_n \sin n w_0 t)$$

$$g(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos 2nt + b_n \sin 2nt) \quad \text{—(a)}$$

for a_0:

$$a_0 = \frac{1}{T_0} \int_{T_0} g(t)\,dt$$

$$a_0 = \frac{1}{\pi} \int_0^{\pi} g(t)\,dt$$

$$a_0 = \frac{1}{\pi} \int_0^{\pi} e^{-t/2}\,dt$$

$$a_0 = \frac{1}{\pi} \left[\frac{e^{-t/2}}{-1/2} \right]_0^{\pi}$$

$$a_0 = \frac{-2}{\pi} \left[e^{-t/2} \right]_0^{\pi}$$

$$a_0 = -\frac{2}{\pi} \left[e^{-\pi/2} - e^{-0} \right] = a_0 = 0.504$$

$$\therefore \int e^{at}\sin bt\,dt = \frac{e^{at}}{a^2+b^2}\left[a\sin bt - b\cos bt\right]$$

for a_n:

$$a_n = \frac{2}{T_0}\int_{T_0} g(t)\cos n\omega_0 t\,dt$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} e^{-t/2}\cos 2nt\,dt$$

using integration formula

$$\therefore \int e^{at}\cos bt\,dt = \frac{e^{at}}{a^2+b^2}\left[a\cos bt + b\sin bt\right]$$

$$a = -1/2 \quad , \quad b = 2n$$

$$a_n = \frac{2}{\pi}\,\frac{e^{-t/2}}{(1/2)^2+(2n)^2}\left[-\tfrac{1}{2}\cos 2nt + 2n\sin 2nt\right]_0^{\pi}$$

$$a_n = \frac{2}{\pi}\cdot\frac{e^{-t/2}}{\frac{1}{4}+4n^2}\left[-\tfrac{1}{2}\cos 2nt + 2n\sin 2nt\right]_0^{\pi}$$

$$a_n = \frac{2}{\pi}\cdot\frac{4}{1+16n^2}\left[-\tfrac{1}{2}e^{-t/2}\cos 2nt + 2e^{-t/2}n\sin 2nt\right]_0^{\pi}$$

$$a_n = \frac{2}{1+16n^2}\cdot\frac{4}{\pi}\left[-\tfrac{1}{2}e^{-t/2}\cos 2nt + 2e^{-t/2}n\sin 2nt\right]_0^{\pi}$$

$$a_n = \frac{2}{1+16n^2}\left[-\frac{2}{\pi}e^{-t/2}\cos 2nt + \cancel{\frac{8n}{\pi}e^{-t/2}\sin 2nt}^{\,0}\right]_0^{\pi}$$

$$a_n = \frac{2}{1+16n^2}\left[-\frac{2}{\pi}e^{-t/2}\cos 2nt\right]_0^{\pi}$$

$$a_n = \frac{2}{1+16n^2}\left[-\frac{2}{\pi}\left(e^{-\pi/2}\cos 2\pi n - e^{-0}\cancel{\cos 0}^{\,1}\right)\right]$$

$$a_n = \frac{2}{1+16n^2}\left[-\frac{2}{\pi}\left(e^{-\pi/2}\right)\right] \Longrightarrow$$

$$a_n = \frac{2}{1+16n^2}\left(-\frac{2}{\pi}(0.207)\right)$$

$$a_n = \frac{2}{1+16n^2}(0.504) \quad \text{—— (i)}$$

for b_n:

$$b_n = \frac{2}{\pi}\int_{\pi} e^{-t/2} \sin 2nt \, dt$$

$$b_n = 0.504\left(\frac{8n}{1+16n^2}\right)$$

Now (a) $\Rightarrow$

$$g(t) = 0.504 + \sum_{n=1}^{\infty}\left(\frac{2}{1+16n^2}(0.504)\cos 2nt + \frac{8n}{1+16n^2}(0.504)\sin 2nt\right)$$

Now converting to compact F.S:

$$a_0 = 0.504 \quad , \quad b_n = \frac{8n}{1+16n^2}(0.504)$$

$$a_n = \frac{2}{1+16n^2}(0.504)$$

here: C_0 :

$$C_0 = a_0 = 0.504 \quad \text{—— (ii)}$$

for C_n:

$$C_n = \sqrt{a_n^2 + b_n^2} \implies$$

$$C_n = 0.504\sqrt{\frac{4}{(1+16n^2)^2} + \frac{64n^2}{(1+16n^2)^2}}$$

$$C_n = 0.504\sqrt{\frac{4+64n^2}{(1+16n^2)^2}}$$

$$C_n = 0.504\sqrt{\frac{4\cancel{(1+16n^2)}}{(1+16n^2)^{\cancel{2}}}}$$

$$C_n = \frac{0.504(2)}{\sqrt{1+16n^2}}$$

for θ_n:

$$\theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$$

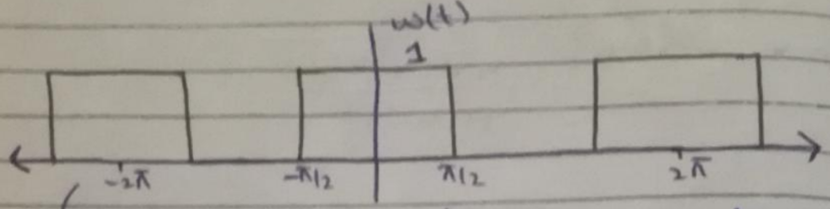
$$\theta_n = \tan^{-1}(-4n)$$

$$\theta_n = -\tan^{-1}(4n)$$

Now compact Fourier series is:

$$g(t) = 0.504 + 0.504\sum_{n=1}^{\infty}\frac{2}{\sqrt{1+16n^2}}\cos(2nt - \tan^{-1}4n)$$

Exp 2.8: Find the compact trignometric fourier series of periodic square wave shown:



• as symetric about y axis so **bn=0**

$T_0 = 2\pi \quad , \quad w_0 = \frac{2\pi}{T_0} \Rightarrow w_0 = 1$

so $g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n w_0 t + 0$ —(A)

so as bn=0 so only find a_0 & a_n

for a_0:

$$a_0 = \frac{1}{T_0}\int_{T_0} g(t)\,dt$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} 1\,dt$$

$$a_0 = \frac{1}{2\pi}[t]_{-\pi/2}^{\pi/2}$$

$$a_0 = \frac{1}{2\pi}[\pi/2 + \pi/2]$$

$$\mathbf{a_0 = 1/2}$$

for a_n:

$$a_n = \frac{2}{T_0}\int_{T_0} g(t)\cos n w_0 t\,dt$$

$$a_n = \frac{2}{2\pi}\int_{-\pi/2}^{\pi/2} 1 \cos nt \, dt$$

$$a_n = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \cos nt \, dt$$

$$a_n = \frac{1}{\pi}\left[\frac{\sin nt}{n}\right]_{-\pi/2}^{\pi/2}$$

$$a_n = \frac{1}{n\pi}\left[\sin nt\right]_{-\pi/2}^{\pi/2} = \frac{1}{n\pi}\left[\sin n\pi/2 - \sin n{-\pi/2}\right]$$

$$a_n = \frac{1}{n\pi}\left[\sin n\pi/2 + \sin n\pi/2\right] = \frac{1}{n\pi}\left[2\sin n\pi/2\right]$$

$$a_n = \frac{2}{n\pi}\sin\frac{n\pi}{2} \quad \text{— (i)}$$

↳ if $n =$ even , $a_n = 0$

So if $n =$ odd

- for $n = 1, 5, 9 \ldots$, $a_n = \frac{2}{n\pi}$
- for $n = 3, 7, 11 \ldots$, $a_n = -\frac{2}{n\pi}$

So put in equation (A)

$$g(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos t - \frac{1}{3}\cos 3t + \frac{1}{5}\cos 5t - \frac{1}{7}\cos 7t \ldots\right)$$

$$\therefore -\cos x = \cos(x - \pi)$$

$$g(t) = 1/2 + \frac{2}{\pi}\left(\cos t + 1/3\cos(3t-\pi) + 1/5\cos 5t + \frac{1}{7}\cos(7t-\pi)\right)$$

↳ (B)

$$g(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos t + \frac{1}{3}\cos(3t-\pi) + \frac{1}{5}\cos 5t + \frac{1}{7}\cos(7t-\pi) \ldots\ldots\right) \rightarrow (B)$$

here $C_0 = \frac{1}{2}$

for C_n:

- if $n =$ even ; $C_n = 0$
- if $n =$ odd ; $C_n = \frac{2}{\pi n}$

for θ_n:

- if $n = 1, 5, 9$; $\theta_n = 0$
- if $n = 3, 7, 11$; $\theta_n = -\pi$

So eq (B) is already in compact trigonometric form

Exponential Fourier series expansion:

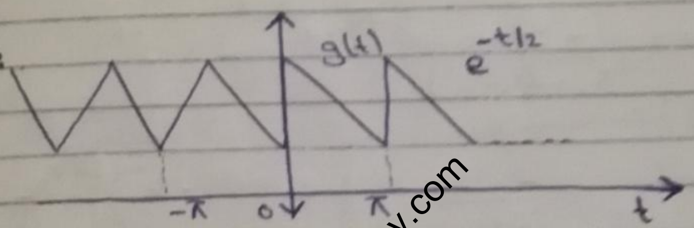
- Fourier series is represented as:

$$g(t) = \sum_{n=-\infty}^{\infty} D_n e^{jnw_0 t} \longrightarrow (A)$$

where

$$D_n = \frac{1}{T_0} \int_{T_0} g(t)\, e^{-jnw_0 t}\, dt \longrightarrow (B)$$

Exp:



Find exponential Fourier series

- here from graph $T_0 = \pi$

$$w_0 = \frac{2\pi}{T_0} = \frac{2\pi}{\cancel{\pi}} \Rightarrow w_0 = 2$$

$$g(t) = \sum_{n=-\infty}^{\infty} D_n e^{j2nt} \text{ ——— } (A)$$

$$\& \; D_n = \frac{1}{T_0} \int g(t)\, e^{-j2nt}\, dt$$

$$D_n = \frac{1}{\pi} \int_0^{\pi} e^{-t/2}\, e^{-j2nt}\, dt$$

$$D_n = \frac{1}{\pi} \int_0^{\pi} e^{-(1/2 + j2n)t}\, dt$$

$$D_n = \frac{1}{\pi}\left[\frac{e^{-(1/2+j2n)t}}{-(1/2+j2n)}\right]_0^{\pi}$$

$$D_n = \frac{1}{\pi}\left[\frac{e^{-(1/2+j2n)t}}{-\left(\frac{1+j4n}{2}\right)}\right]_0^{\pi}$$

$$D_n = \frac{-2}{\pi[1+j4n]}\left[e^{-(1/2+j2n)t}\right]_0^{\pi}$$

$$D_n = \frac{-2}{\pi[1+j4n]}\left[e^{-\pi/2}\cdot e^{-j2n\pi} - e^{-0}\right]$$

$$D_n = \frac{-2}{\pi[1+j4n]}\left[0.207\cdot e^{-j2n\pi} - 1\right]$$

According to Euler transformation:

$$\therefore e^{-j2n\pi} = \cos 2n\pi - j\cancel{\sin 2n\pi}^{\,0}$$

$$e^{-j2n\pi} = \cos 2n\pi$$

$$e^{-j2n\pi} = 1$$

$$D_n = \frac{-2}{\pi(1+j4n)}\left[0.207(1) - 1\right]$$

$$D_n = \frac{0.504}{1+j4n} \quad \text{—— (i)}$$

(A) $\Rightarrow g(t) = 0.504 \sum_{n=-\infty}^{\infty} \frac{1}{1+j4n} e^{j2nt}$

time domain doesn't give us enough info so we transform to frequency domain to get more details. This is called transformation

Lecture 4

Fourier Transform:

It is a mathematical tool for frequency analysis of aperiodic signals.

- Suppose we have a function $g(t)$:

$$g(t) \xrightarrow{\text{F. tranform}} G(\omega) \text{ or } G(f)$$

$G(\omega)$ → angular frequency | rad/sec

$G(f)$ → frequency

where

$$G(\omega) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt \longrightarrow (A)$$

$G(\omega)$ → F.T

→ mathematical representation of fourier transform

- **Inverse Fourier transform: (IFT)**

- $G(\omega) \xrightarrow{\text{IFT}} g(t)$

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega)\, e^{j\omega t}\, d\omega \longrightarrow (B)$$

- $G(f) \xrightarrow{\text{IFT}} g(t)$

$$g(t) = \int_{-\infty}^{\infty} G(f)\, e^{j\omega t}\, df \longrightarrow (C)$$

$\Longrightarrow$

• Dirichlets condition:

for F. transform we should have the following conditions satisfied:

1. Finite number of maxima and minima over any finite interval.

2. Finite number of discontinuities over any finite interval.

3. Signal should be absolutely integrable.

$$\int_{-\infty}^{\infty} |g(t)|\, dt < \infty$$

Note:

These conditions are sufficient but not but not necessary for the existance of Fourier transform

↳ It means that if all three conditions are satisfied than for sure ~~to~~ Fourier transform exist.

↳ but there are some signals which violates one or two conditions but still have F.T.

↳ For example sinc function

i.e $\frac{\sin x}{x} = \text{sinc}(x)$

it violate condition 3 but still FT exist

Note:
- So in general we say that any signal which can be generated practically, its FT will exist.

→ So the physical existance of a signal is sufficient for existance of FT but even this condition is also not necessary for existance of FT.

→ for example Impulse signal don't physically exist but its FT exist

- Also $u(t)$ is not absolutely integrable but still its FT exist.

Properties of Fourier Transform:

1. Linearity:
As FT have integration and integration is a linear operation so FT obey Linearity.

- if $g_1(t) \xrightarrow{FT} G_1(f)$, $g_2(t) \xrightarrow{FT} G_2(f)$

constants: a_1, a_2

$$\boxed{a_1 g_1(t) + a_2 g_2(t) \rightleftharpoons a_1 G_1(f) + a_2 G_2(f)}$$

$\Rightarrow$

2. Conjugation. (Conjugate symmetry property)

- if $g(t)$ is real function:

  $G(f)$ is FT

  and $G(-f)$ is the complex conjugate

i.e

$$G(-f) = G(f)$$

- $$|G(-f)| = |G(f)|$$

  ↳ this means amplitude is an even function.

- $$\theta_g(-f) = -\theta_g(f)$$

  ↳ this means phase is an odd function

Note: This property is only valid for real time $g(t)$

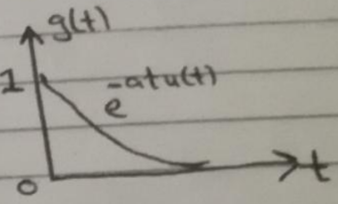
Exp: Find the FT of the signal $e^{-(at)}u(t)$.

Sol:

$$G(\omega) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt$$

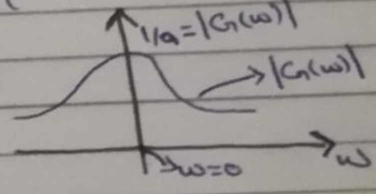
$$G(\omega) = \int_{-\infty}^{\infty} e^{-(at)}\, u(t)\, e^{-j\omega t}\, dt$$

∴ As when $u(t)$ is multiplied it ranges from $0 \longrightarrow \infty$



$$G(\omega) = \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}\, dt$$

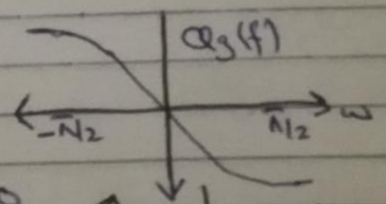
$$G(\omega) = \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$



$$G(\omega) = \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_0^{\infty}$$

$$G(\omega) = -\frac{1}{a+j\omega}\left[\cancel{e^{-\infty}}^{\,0} - \cancel{e^{-0}}^{\,1}\right]$$

$$G(\omega) = -\frac{1}{a+j\omega}(0-1)$$



$$G(\omega) = \frac{1}{a+j\omega}, \quad a>0$$

↳ odd ftns symmetrical about origin

$$|G(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}, \quad \theta_g(\omega) = \tan^{-1}(\omega/a)$$

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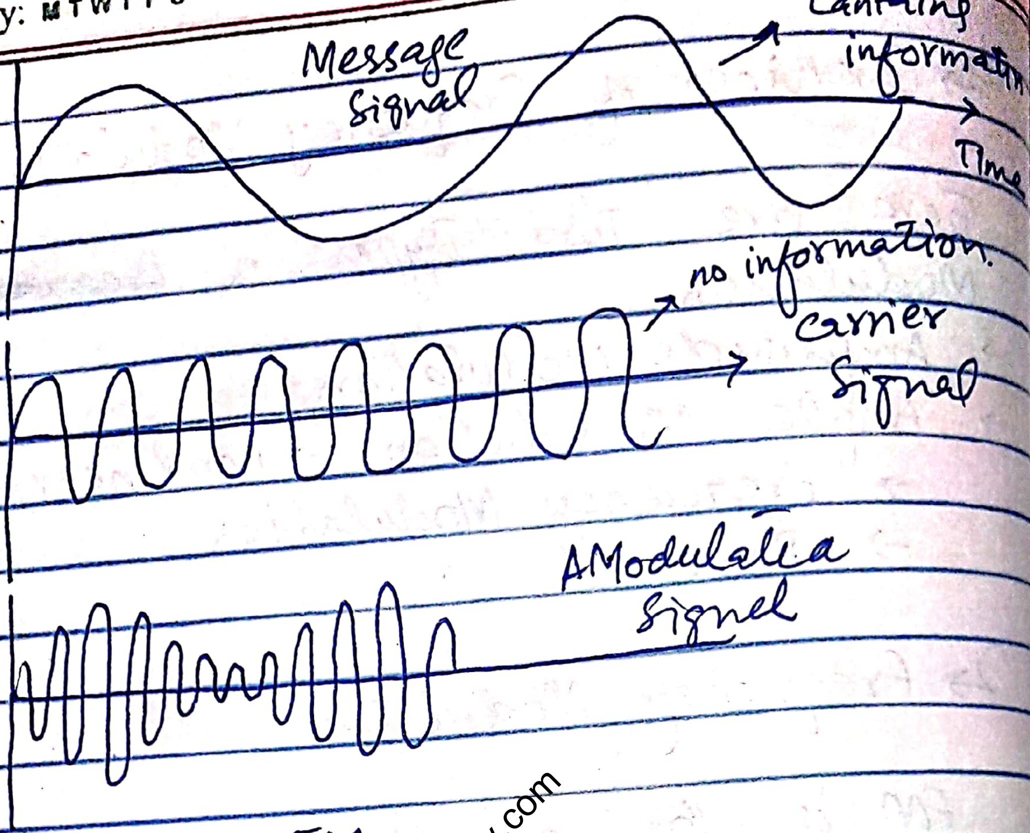
2. Overview of Frequency Modulation

There are Two types of ~~Frequency~~ Modulation
→ Amplitude Modulation
→ Angle Modulation → non linear
→ Frequency Modulation
→ Phase "

2. Frequency Modulation:

FM is the encoding of information in a carrier wave by varying the instantaneous frequency of the wave. So, in this case, the information is in the frequency variations of the carrier.
In AM, the Amplitude of the carrier signal varies w.r.t message signal.
In FM, the frequency of the carrier signal varies w.r.t message signal. So in this case amplitude and phase remains constant.

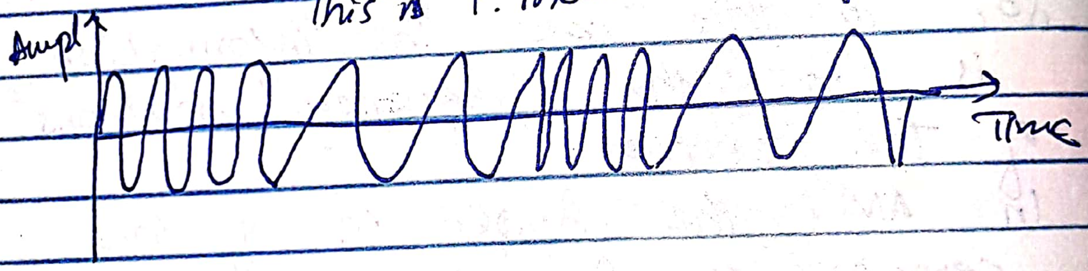
In AM



In FM

Same message & Carrier Signal

This is F. modulated Signal.



2) So, in frequency Modulation the amount of change in the frequency of the carrier signal is determined by the amplitude of the message signal

Let us assume that the carrier signal has frequency deviation of 3KHz. In such a case carrier signal will ~~be~~ move up and down by 3KHz.

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The resting frequency or center of a transmitter is defined as the output frequency with no modulating signal applied. It is actually allotted frequency to the transmitter.

When the message signal is applied the carrier signal, deviates up & down from its resting frequency.

→ The change of carrier frequency up or down is called frequency deviation. (Δf)

→ The total variation of frequency from highest to lowest is called carrier swing and is 2 times the frequency deviation.

→ In FM Broadcasting System, it has been internationaly agreed to limit maximum frequency deviation to 75 kHz on each side of centre frequency.

FM Broadcasting Range.

88 MHz ——— 108 MHz → 20 MHz

Channel BW is actually 200 kHz or 0.2 MHz

No. of channels = $\frac{20\ MHz}{0.2\ MHz}$ = 100 channel

⇒ Angle Modulation - False start concept of instantaneous frequency.

→ In FM, the instantaneous frequency of the carrier is varied in accordance with the amplitude of the message signal.

False Start

- In 1920, broadcasting was underdeveloped. There was active search for reduction of noise.
- Science thought that noise is proportional to bandwidth of the side bands, so less BW will mean less Noise.

- So there goal was to find the modulation scheme that requires less bandwidth.
- This leads to invention of FM
- The understanding was the carrier signal frequency would be varied with time.
- $w(t) = w_c + k m(t)$ where k is arbitrary constant.

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→ The understanding was that by controlling k_f we can control band width

- However experimental results showed that FM Bandwidth is always greater than (at best equal to) AM bandwidth.

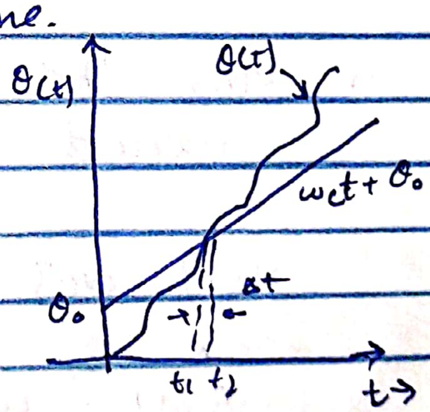
2) Instantaneous frequency is the frequency at any instant of time. It vary with time at every instant.

Consider a sinusoid with fixed frequency and phase.

$$\varphi(t) = A\cos(\omega_c t + \theta_0)$$

slope $= \omega_c$
y intercept $= \theta$

The angle has a linear relation with time.



What if frequency is not fixed

e.g → 2 is inst. freq.

$x(t) = \cos(2t-1)$

$y(t) = \cos(2t^2 - 5t + 7)$

↓

Here we need to find derivation

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$$\omega_i(t) = \frac{d\theta}{dt}$$

$\therefore$ $\theta(t) = \omega c t + \theta_0$

$\omega_i(t) = \omega c$

When frequeny is changing with time, the instantaneous frequency is equal to slope of tangent line at that point

$$\theta(t) = \int_{-\infty}^{t} \omega_i(\alpha)\, d\alpha.$$

→ Now changing the angle is the basis for two typs of Modulation

→ FM

→ Phase Modulation (PM)

In PM, the ~~angule~~ angle is varied linearly with m(t)

$\theta(t) = \omega c t + \theta_0 + k_p m(t)$

↳ Message Signal

$\hat{\theta}(t) = \omega c t + k_p m(t)$

$\varphi_{PM}(t) = A \cos[\omega c t + k_p m(t)]$

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→ kp is a parameter that specifies how much change in the angle occurs for every change in m(t).

Now, instantaneous angular frequency is

$$\omega_i(t) = \frac{d\theta}{dt} = \omega_c + k_p \dot{m}(t)$$

Hence in PM, the instantaneous angular frequency will be varying linearly with time derivative of the modulating signal.

If the instantaneous angular frequency varies linearly with modulating signal we have FM

$$\omega_i(t) = \omega_c + k_f m(t)$$

$$\theta(t) = \int_{-\infty}^{t} [\omega_c + k_f m(\alpha)] d\alpha$$

$$= \omega_c t + k_f \int_{-\infty}^{t} m(\alpha) d\alpha$$

$$\phi_{FM}(t) = A\cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha) d\alpha\right]$$

2> Relation b/w FM & PM.

→ FM & PM are similar and inseparable

→ In both FM & PM, the angle of carrier is varried w.r.t message signal.

→ In PM, the variation is proportional to m(t)

→ Where as in FM, the variation is proportional to m(t) integral.

$$m(t) \rightarrow \boxed{\int} \xrightarrow{\int m(\alpha)\,d\alpha} \boxed{\text{Phase Modulator}} \rightarrow \varphi_{FM}(t)$$

Frequency Modulator

$$m(t) \rightarrow \boxed{d/dt} \xrightarrow{\dot{m}(t)} \boxed{\text{Frequency Modulator}} \rightarrow \varphi_{PM}(t)$$

Phase Modulator.

2> Power of Angle Modulated wave

→ The amplitude of the angle M. wave remains constant. So, the power is $A^2/2$ regardless of the value of k_p or k_f

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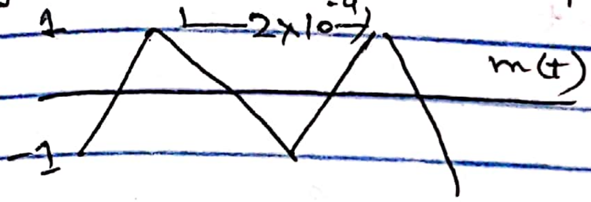
Example 5.1: Sketch FM & PM for modulating Signal Shown in figure. $k_f = 2\pi \times 10^5$ $k_p = 10\pi$ $f_c = 100MHz$



As we know that

$$w_i = w_c + k_f m(t)$$

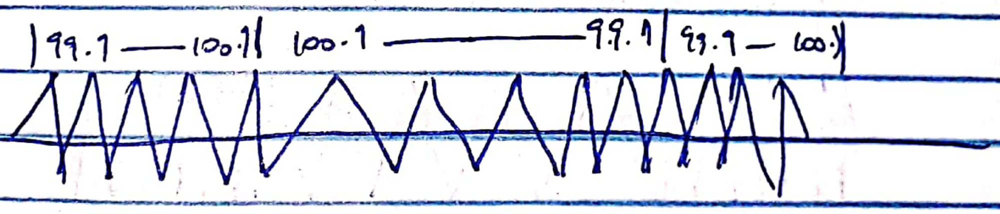
$$f_i = f_c + \frac{k_f}{2\pi} m(t)$$

$$f_i = f_c + 10^5 m(t)$$

$$f_i = 10^8 + 10^5 m(t)$$

$$f_{i\,max} = 10^8 + 10^5 (1) = 100.1 \text{ MHz}$$

$$f_{i\,min} = 10^8 + 10^5 (-1) = 99.9 \text{ MHz}$$

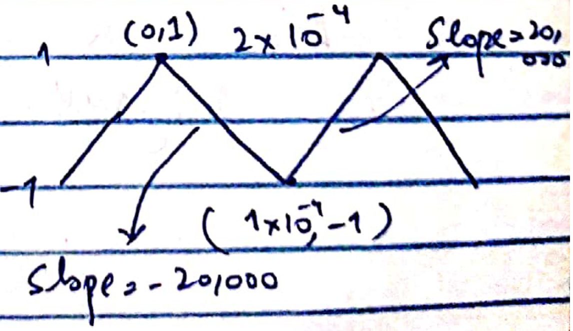


Now PM for m(t) is FM for $\dot{m}(t)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-2}{1\times10^{-4}}$$

$$m = -20{,}000$$



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Then

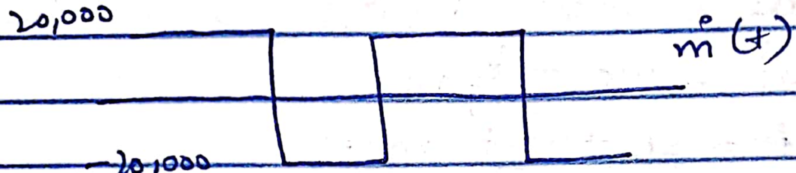
$y = mx + c$

$y = -20000t + c$ ∴ As $c = 1$

$y = -20000t$

And

$\dot{m}(t)$ will be

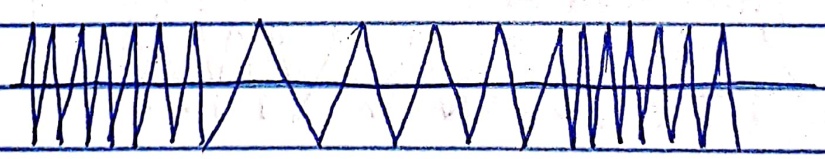


$f_i = f_c + \frac{k_p}{2\pi}\dot{m}(t)$

$f_i = 10^8 + 5\,\dot{m}(t)$

$f_{i_{max}} = 10^8 + 5(20{,}000) = 100.1\ MHz$

$f_{i\,min} = 10^8 + 5(-20{,}000) = 99.9\ MHz$



This is indirect Method b/c we have continuous signal.

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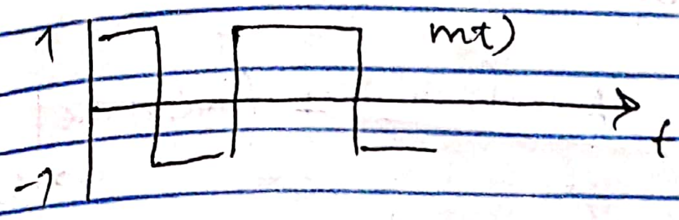
Example 5.2: Direct Method of PM

Sketch FM & PM

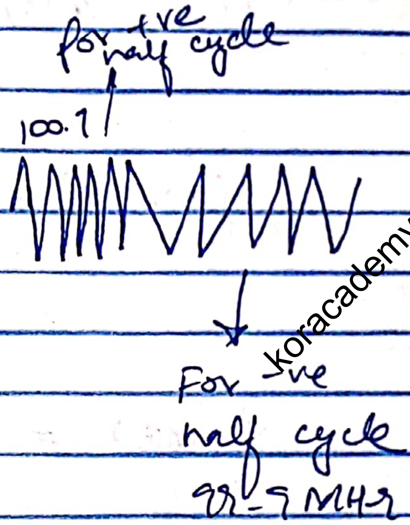
$k_f = 2\pi \times 10^5$ $k_p = \pi/2$



$$f_i = f_c + \frac{k_f}{2\pi} m(t)$$

$f_{i\,(max)} = 100.1$ MHz

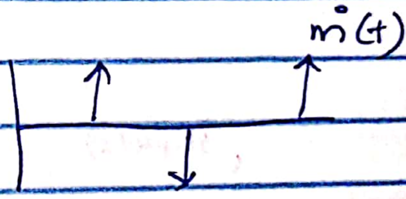
$f_{i\,(min)} = 99.9$ MHz



$\varphi_{PM} = A\cos[\omega_c t + k_p\, m(t)]$

$= A\cos[\omega_c t + \frac{\pi}{2} m(t)]$

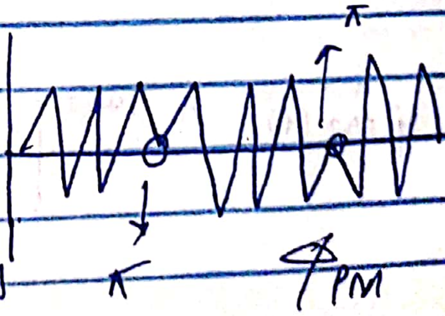
So $m(t) = 1$



$\varphi_{PM} = A \sin \omega_c t$

So $m(t) = -1$

$\varphi_{PM} = -A \sin \omega_c t$



$= k_p m_d$ = The amount of phase change at the time of discontinuity

$= \pi/2\,(2) = \pi$

Phase shift

key

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lecture # 09

⇒ Bandwidth of Angle Modulated wave (NBFM).

→ BW of FM signal.

$$\varphi_{FM}(t) = A\cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha\right] \rightarrow (1)$$

Let $$a(t) = \int_{-\alpha}^{t} m(\alpha)\, d\alpha \rightarrow (2)$$

$$\hat{\varphi}_{FM}(t) = A e^{j\left[\omega_c t + k_f \int_{-\alpha}^{t} m(\alpha)\, d\alpha\right]}$$

$$= A e^{j[\omega_c t + k_f a(t)]} \rightarrow (3)$$

$$\hat{\varphi}_{FM}(t) = A e^{j\omega_c t} \cdot e^{jk_f a(t)} \rightarrow (4)$$

$$\therefore e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \text{ ----}$$

$$e^{jk_f a(t)} = 1 + jk_f a(t) + (-1)\frac{k_f^2 a^2(t)}{2!} - j\frac{k_f^3 a^3(t)}{3!}$$

$$\hat{\varphi}_{FM}(t) = A e^{j\omega_c t}\left[1 + jk_f a(t) - \frac{k_f^2 a^2(t)}{2!} - j\frac{k_f^3 a^3(t)}{3!} \cdots\right] \rightarrow (5)$$

Now if BW of $m(t)$ is $M(\omega) \to B$
then BW of $a(t)$ will be $A(\omega) \to B$

$a^2(t) \longrightarrow 2B$
$\downarrow$
$a^n(t) \longrightarrow nB$

→ The theoretical BW of FM is infinite.

→ But we can find a narrow Band Angle Modulation signal

$$|k_f a(t)| << 1$$

$$\hat{\varphi}_{FM}(t) = Ae^{j\omega_c t}[1 + jk_f a(t)]$$

$$e^{j\omega_c t} = \cos\omega_c t + j\sin\omega_c t$$

$$\hat{\varphi}_{FM}(t) = A[\cos\omega_c t + j\sin\omega_c t](1 + k_f a(t))$$

$$\hat{\varphi}_{FM}(t) = A(\cos\omega_c t + j\sin\omega_c t + j k_f a(t)\cos\omega_c t$$
$$- k_f a(t)\sin\omega_c t$$

we have to find Real Terms

$$\varphi_{FM}(t) = Re(\hat{\varphi}_{FM}(t))$$

$$= A[\cos\omega_c t + (-k_f a(t)\sin\omega_c t)] \to (6)$$

for

$$\varphi_{PM}(t) = A[\cos\omega_c t - k_p m(t)\sin\omega_c t] \to (7)$$

Now

$$\varphi_{AM}(t) = \left(A + m(t)\right)\cos\omega_c t$$

$$= A\cos\omega_c t + m(t)\cos\omega_c t$$

$$BW = 2B$$

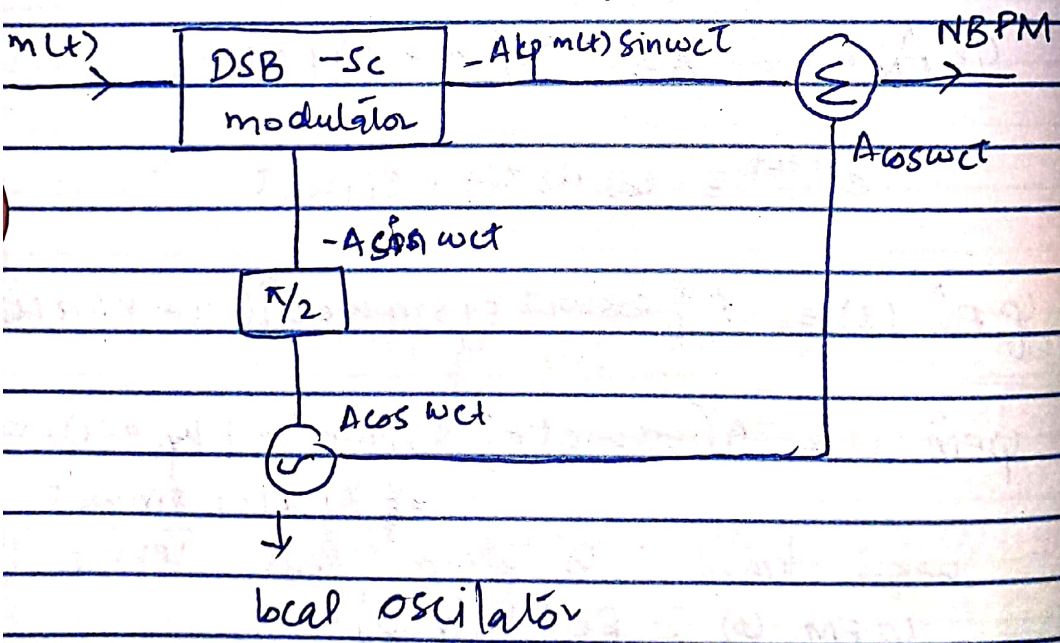
and also for NB PM wave

$$BW = 2B$$

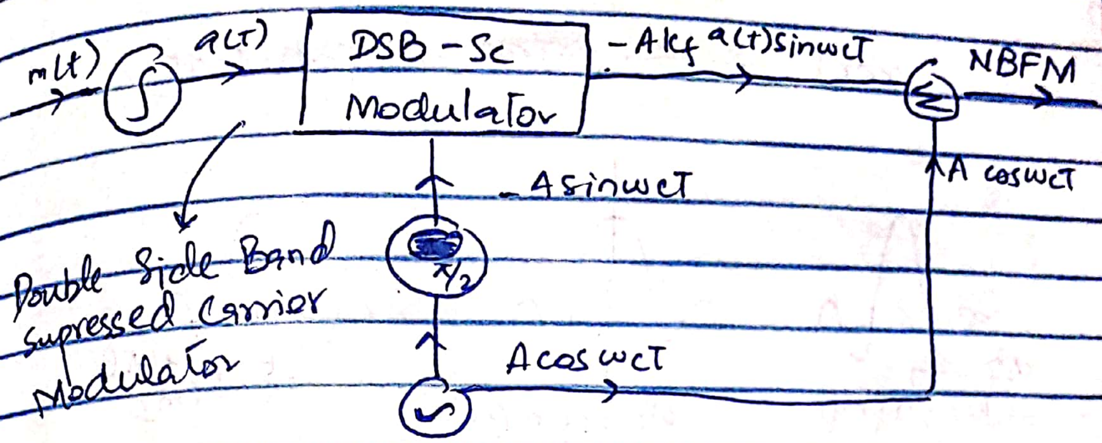
So, NB angle Modulation has BW of 2B.

→ Phase Modulation

$$\varphi_{PM}(t) = A\cos\omega_c t - Ak_p m(t)\sin\omega_c t$$



→ Frequency Modulation

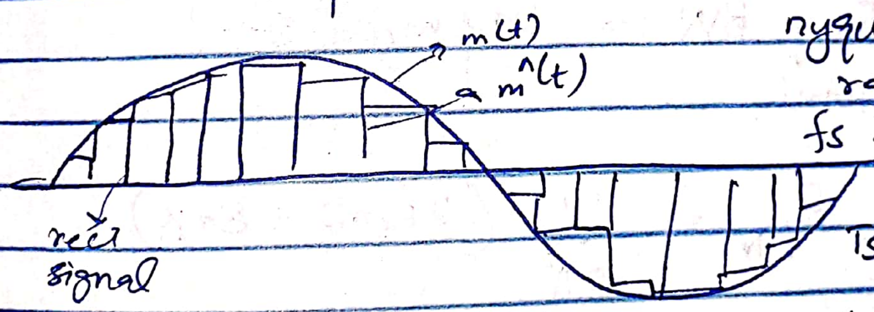


2) wide Band FM.

Here $|k_f a(t)| \ll 1$ is not true

$$w_c - k_f m_p \leq w \leq w_c + k_f m_p$$

$2 k_f m_p$ X



nyquist Sample rate

$f_s \geq 2 f_m$

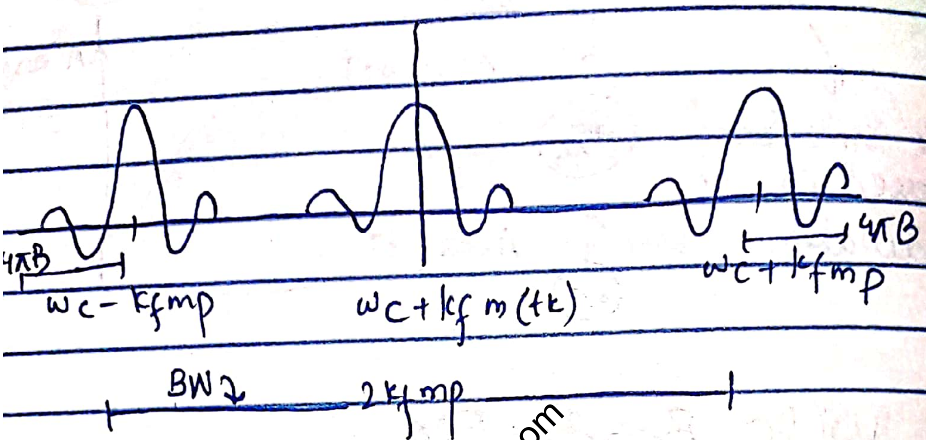
$f_s \geq 2B$

$T_s \leq 1/2B$

To ensure that $\hat{m}(t)$ has all the information of m(t), the cell width in $\hat{m}(t)$ must be no greater than the nyquist interval of $1/2B$ seconds. Thus m(t) is approximated by constant amplitude pulses (cells), of width

$$T = 1/2B \text{ seconds}$$

The fourier transform of rect —— sinc function



The spectrum of each sinusoid pulse spreads at on either side of the frequency by

$$2\pi / T = 2\pi \times 2B = 4\pi B$$

The Total BW $= 2k_f m_p + 8\pi B$

$$B_{FM} = \frac{1}{2\pi}\left(2k_f m_p + 8\pi B\right)$$

$$= \frac{k_f m_p}{\pi} + 4B$$

The frequency deviation is Δf

$$\Delta f = \frac{k_f m_p}{2\pi} \quad \text{so}$$

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$B_{FM} = 2\Delta f + 4B \longrightarrow$ (A)

$B_{FM} = 2(\Delta f + 2B) \longrightarrow$ (B)

In case NB $\Delta f \approx 0$

$B_{NBFM} \approx 4B$

But

$B_{NBFM} = 2B$

Actually

$B_{WBFM} = 2(\Delta f + B) \longrightarrow$ (C)

$\hookrightarrow$ Carson's Rule

$2(\Delta f + \frac{\Delta f}{\beta})$

$6k = 2(\frac{\beta\Delta f + \Delta f}{\beta})$

$(2+1)\Delta f$

$\frac{64}{3} = \Delta f$

3 KHz

deviation Ratio

$$\beta = \frac{\Delta f}{B}$$

$$B_{WBFM} = 2B\left(\frac{\Delta f}{B} + 1\right)$$

$$= 2B(\beta + 1) \longrightarrow (D)$$

So, these equations are used to find the BW of Phase Modulating Signal. as well as Frequency modulating Signal

In case PM

$\omega_i = \omega_c + k_p m'(t)$

$\Delta f = k_p m_p$

$m_p = |m'(t)|$.

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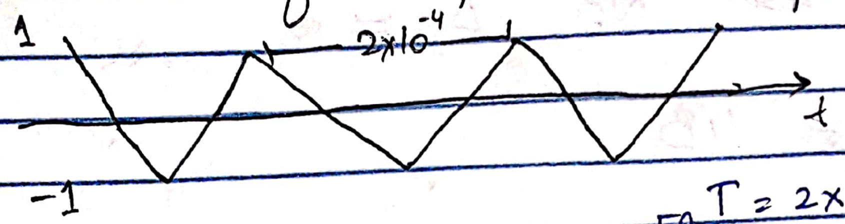
Example 5.3:-

Estimate B_{FM} & B_{PM} for Modulating Signal $m(t)$ for $k_f = 2\pi \times 10^5$ $k_p = 5\pi$



$T = 2 \times 10^{-4}$

fundamental frequency $f_0 = \frac{1}{2\times10^{-4}} = 5\,KHz$

Solution:

$m_p = 1$

$m(t) = \sum_n c_n \cos n\omega_0 t$

$$c_n = \begin{cases} 8/\pi^2 n^2 & , n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

we take the 3rd hardmonic as

$B = 3 \times f_0 = 15\,KHz$

$B_{FM} = 2(\Delta f + B)$

$\Delta f = k_f m_p / 2\pi$

$\Delta f = \frac{2\pi \times 10^5 \times 1}{2\pi} = 10^5 = 100\,kHz$

$B_{FM} = 2(100\,KHz + 15\,KHz)$

$= 230\,KHz$

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or

$$B_{FM} = 2B(\beta + 1)$$

$$\beta = \frac{\Delta f}{B} = \frac{100\ kHz}{15\ kHz}$$

$$= 2(15)\left(\frac{100}{15} + 1\right)$$

$$= 30\left(\frac{115}{15}\right)$$

$$B_{FM} = 230\ kHz \longrightarrow (1)$$

Now

$$B_{PM} = 2(\Delta f + B)$$

$$\Delta f = k_p m_p' / 2\pi \qquad m_p' = |\dot{m}(t)|$$

$$\Delta f = \frac{5\pi \times 20{,}000}{2\pi} \qquad y = -20{,}000 \quad y' = +20{,}000$$

$$\Delta f = 50\ kHz$$

$$B_{PM} = 2(50\ kHz + 15\ kHz)$$

$$B_{PM} = 130\ kHz \longrightarrow (2)$$

$$B_{PM} = 2B(\beta + 1)$$

$$\beta = \Delta f / B = 50\ kHz / 15\ kHz$$

$$= 2(15\ kHz)\left(\frac{50}{15} + 1\right)$$

$$B_{PM} = 130\ kHz$$

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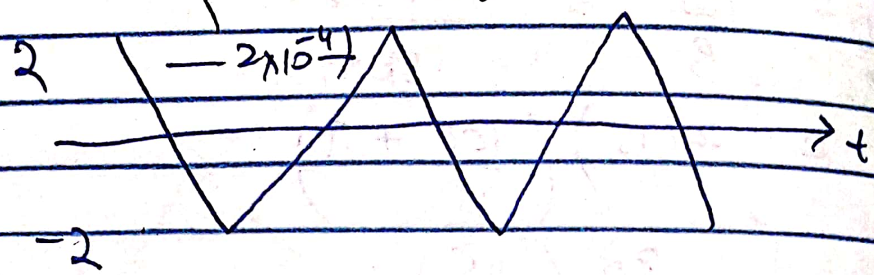
(b) Repeat the problem, if amplitude of $m(t)$ is doubled.



$K_f = 2\pi \times 10^{5}$

$k_p = 5\pi$

$B_{FM} = 2(\Delta f + B)$

$B = 15\,kHz$

$$\Delta f = \frac{K_f\, m_p}{2\pi}$$

$$\Delta f = \frac{2\pi \times 10^5 (2)}{2\pi}$$

$$\Delta f = 200\,kHz$$

$$B_{FM} = 2(200 + 15)$$
$$= 430\,KHz$$

$B_{PM} = 2(\Delta f + B)$

$$\Delta f = \frac{K_p\, m_p}{2\pi}$$

$$m_p = |\dot{m}(t)|_{max}$$

$y = -40{,}000 + 1$ $\quad$ $y = 40000 + c$

$\dot{y} = -40{,}000$ $\quad$ $\dot{y} = 40{,}000$

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$$\Delta f = \frac{5\pi \times 40{,}000}{2\pi} = 100\,kHz$$

$$B_{PM} = 2(100\,kHz + 15) = 230\,kHz$$

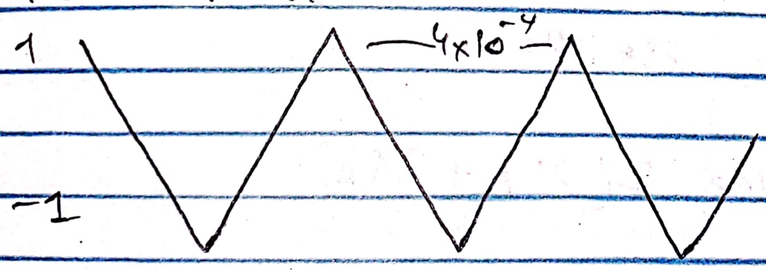
Doudling the amplitude of message signal can approximately double the bandwidth of the frequeny of phase modulated signal.

2) Effect of Time expansion on PM

Example 5.4

Repeat 5.3 if $m(t)$ is time expanded by a factor of 2

i.e $T = 4\times10^{-4}$



Sol:

$$B_{FM} = 2(\Delta f + B)$$

$f_0 = \frac{1}{T} = 2.5\,kHz$

we take 3rd harmonic

$B = 3f_0 = 7.5\,kH$

$\boxed{B = 7.5\,kHz}$

$$\Delta f = \frac{k_f m_p}{2\pi}$$

$$= \frac{2\pi\times10^5(1)}{2\pi} = 100\,kHz$$

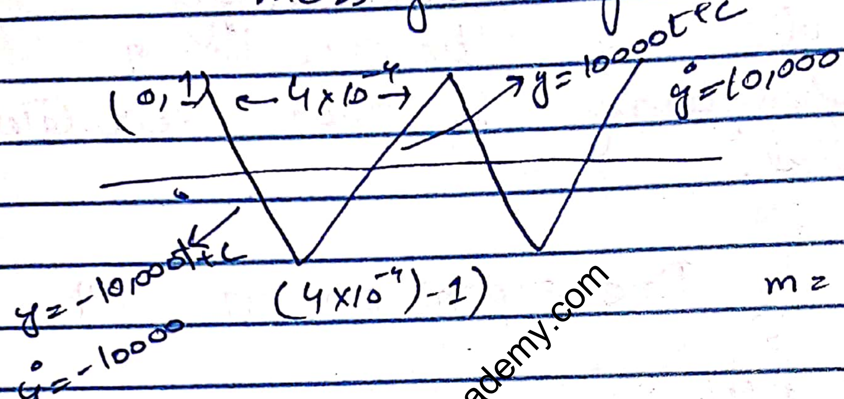
$$= 2(100k + 7.5k)$$

$$B_{FM} = 215\,kHz$$

$$B_{PM} = 2(\Delta f + B)$$

$$\Delta f = \frac{k_p m_p}{2\pi} \qquad m_p = |\dot{m}(t)|_{max}$$

we need to Take derivative of message signal.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_p = 10{,}000 \qquad m = \frac{-2}{2\times10^{-4}} = -10{,}000$$

$$\Delta f = \frac{5\pi \times 10{,}000}{2\pi}$$

$$\Delta f = 25\,kHz$$

$$B_{PM} = 2(25k + 7.5k)$$

$$B_{PM} = 65\,kHz.$$

The time expansion half the PM bandwidth.

PM → Phase Spectrum is dependent on the Spectrum of message signal.

FM is very little dependent.

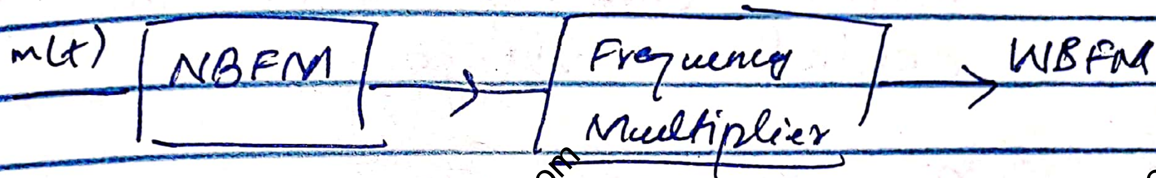
Lecture #10

Generation of FM waves.

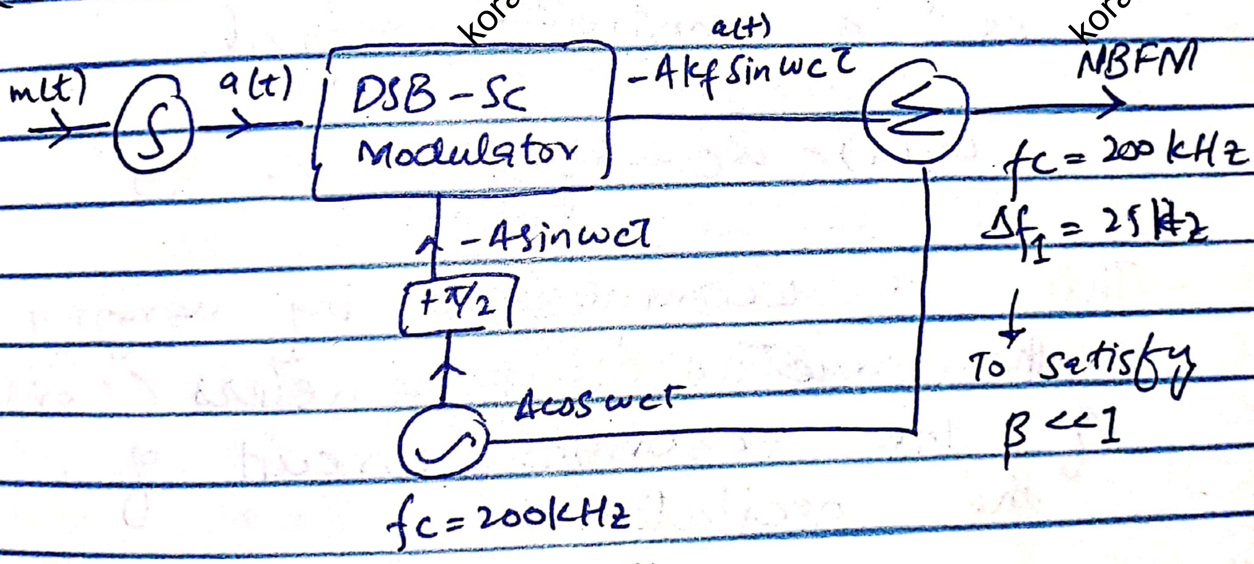
⇒ Indirect Method of FM Generation

(a)- First NBFM is generated

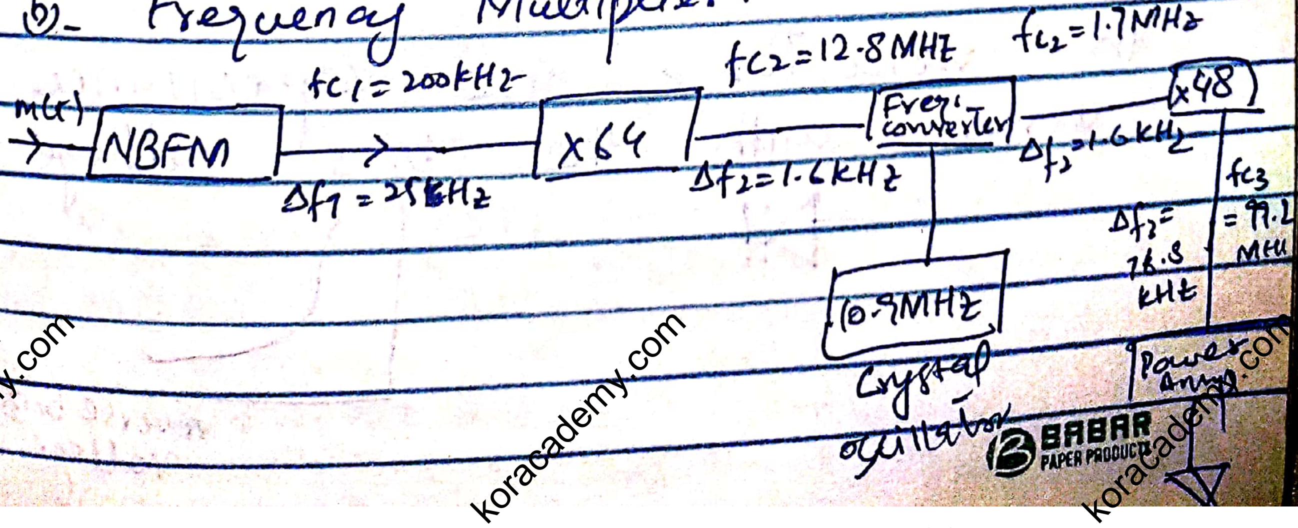
(b)- NBFM is then converted to WBFM using Frequency Multiplier.



(a)- NBFM Generation:



(b)- Frequency Multiplier:



2. Direct Generation of FM:

FM is generated by using VCO (voltage controlled oscillator)

which is sinusoidal oscillator with reactive elements (e.g capacitive element)

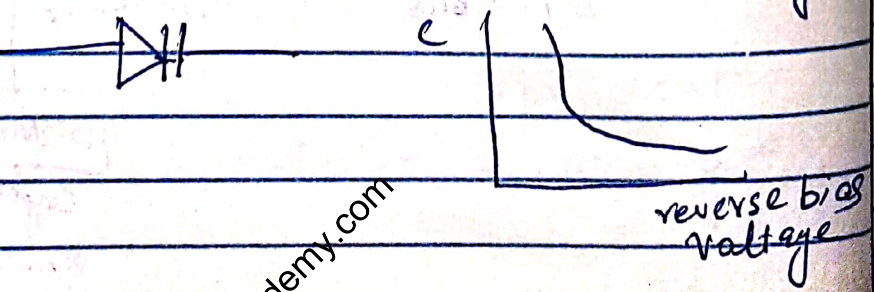
In VCO, the oscillator frequency varies linearly with the control voltage

We can generate FM wave by using modulating signal $m(t)$ as a control signal.

$$\omega_i(t) = \omega_c + k_f\, m(t) \quad \text{--- (i)}$$

This is accomplished by varying the reactive parameters (C or L) of the resonant circuit of the oscillator

In varactors, the capacitance varies with the bias voltage



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This capacitance can be approximated as a linear ft of bias voltage over a limited range

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

if the capacitance C is varied by modulating signal $m(t)$ then,

$$C = C_0 - Km(t)$$

$$\omega_0 = \frac{1}{\sqrt{L(C_0 - km(t))}} = \frac{1}{\sqrt{LC_0\left(1 - \frac{km(t)}{C_0}\right)}}$$

$$= \frac{1}{\sqrt{LC_0}\left(1 - \frac{km(t)}{C_0}\right)^{1/2}}$$

$$\omega_0 = \frac{1}{\sqrt{LC_0}}\left(1 - \frac{km(t)}{C_0}\right)^{-1/2} \longrightarrow (2)$$

Using binomial Expansion

$$(1+x)^n = 1 + nx + \cdots \qquad x < 1$$

So,

$$= \frac{1}{\sqrt{LC_0}}\left[1 + \frac{1}{2}\frac{km(t)}{C_0}\right] \longrightarrow (3)$$

$$\omega_0 = \omega_c\left(1 + \frac{Km(t)}{2C_0}\right) \longrightarrow (4)$$

$$\omega_0 = \omega_c + \frac{k\omega_c m(t)}{2C_0} \longrightarrow (5)$$

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Compare it to i)

$$K_f = \frac{k\omega_c}{2C_o}$$

$$k = \frac{2K_f C_o}{\omega_c}$$

$$\Delta C = k\, m_p$$

$$\Delta C = \frac{2K_f C_o m_p}{\omega_c}$$

$$\frac{\Delta C}{C_o} = \frac{2K_f m_p}{\omega_c}$$

$$\frac{\Delta C}{C_o} = \frac{2\Delta f}{f_c}$$

So, capacitance deviation provides the frequency deviation

This direct method has poor frequency which is controlled by the feedback.

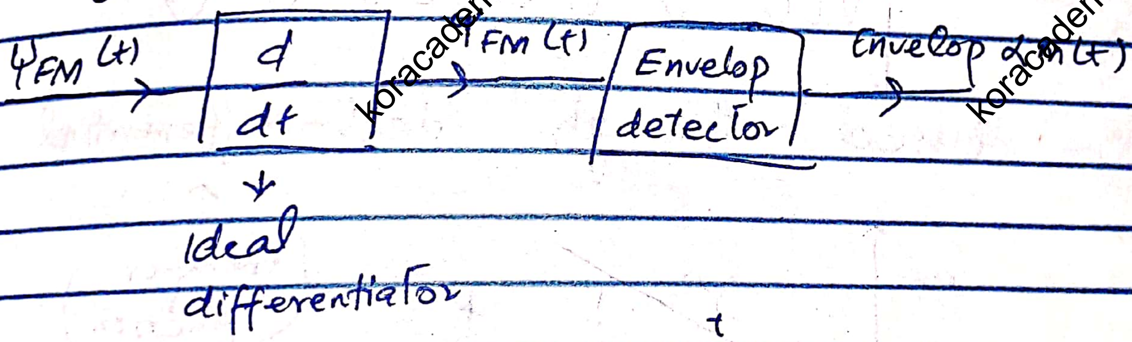
Demodulation of FM ~~wave~~

Two methods
- → Frequency descrimination
- → Phase lock loop (PLL)

Frequency discrimination Method:

i. Slope detection followed by envelope detection.

ii. The FM demodulator produce an o/p voltage dependent on frequency.



$$\varphi_{FM}(t) = A\cos\left(\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right)$$

let assume constant amplitude.

$$\frac{d}{dt}\varphi_{FM}(t) = -A\sin\left(\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right)\frac{d}{dt}\left(\omega_c t - -\right.$$

$$\dot{\varphi}_{FM}(t) = -A\sin\left(\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right)\left[\omega_c + k_f\, m(t)\right]$$

$$\dot{\varphi}_{FM}(t) = -A[\omega_c + k_f m(t)]\sin\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right] \to (1)$$

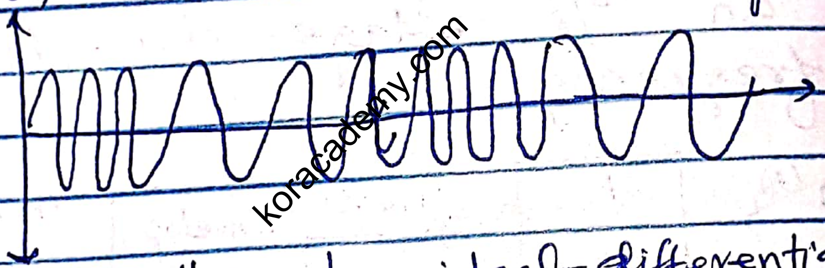
Envelop $= A(\omega_c + k_f m(t))$

$\Delta\omega = k_f m_p < \omega_c$

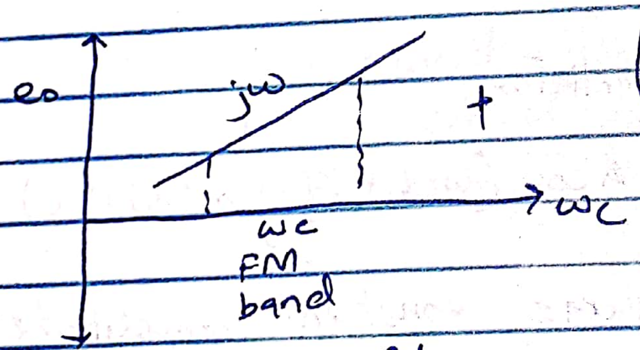
$\omega_c + k_f m(t) > 0$ for all t

In practice a high-pass filter can act as a ideal - differentiator

Now,



Passed through ideal-differential
(transfer function)

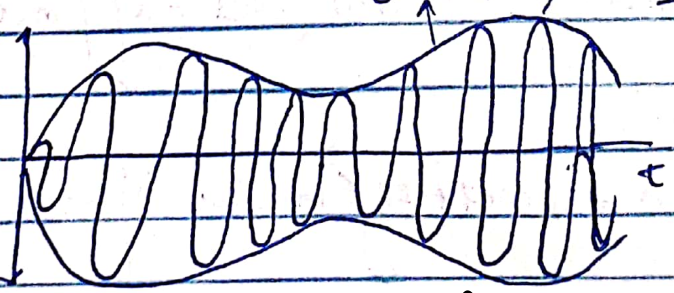


as a result.

$A[\omega_c + K_f m(t)]$

$= A\omega_c + A K_f m(t)$

DC



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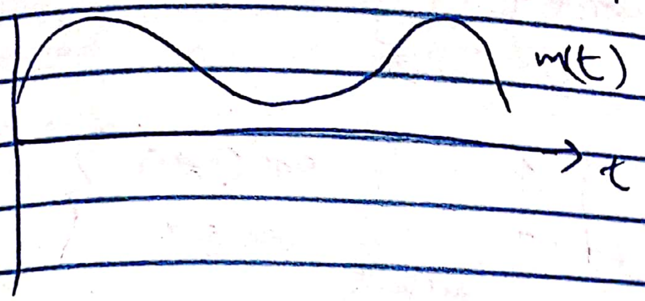
which is passed through DC blocker and low pass filter

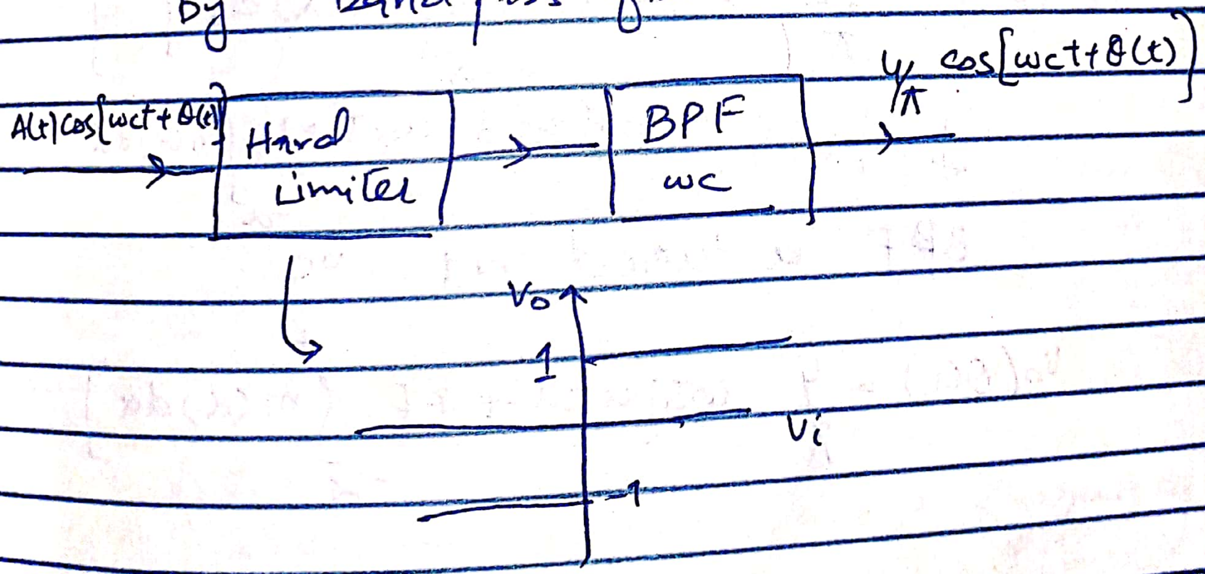


This is the messaged signal

2) Bandpass Limiter

(i) To eliminate amplitude variations of angle modulated wave.

(ii). It consist of hard limiter followed by band pass filter.



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The input of hard Limiter

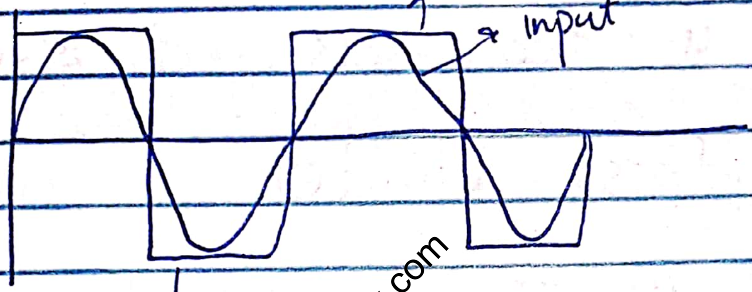
$$V_i(t) = A(t)\cos\theta(t)$$

$$\theta(t) = \omega_c t + k_f\int_{-\infty}^{t} m(\alpha)\,d\alpha$$

The signal

$$V_o(\theta) = \begin{cases} 1 & , \cos\theta \geq 0 \\ -1 & , \cos\theta < 0 \end{cases}$$

The output of HL



$$V_o(\theta) = \frac{4}{\pi}\left[\cos\theta - \frac{1}{3}\cos 3\theta + \frac{1}{5}\cos 5\theta\right]$$

$$V_o(\theta(t)) = \frac{4}{\pi}\left\{\cos\left[\omega_c t + k_f\int_{-\infty}^{t} m(\alpha)\,d\alpha\right] - \frac{1}{3}\cos 3\left[\omega_c t + k_f\int_{-\infty}^{t} m(\alpha)\,d\alpha\right]\right\}$$

BPF tuned at ω_c

$$V_o(\theta(t)) = \frac{4}{\pi}\cos\left[\omega_c t + k_f\int_{-\infty}^{t} m(\alpha)\,d\alpha\right]$$

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2) Phase lock loop Method:-

PLL is a feedback control system where the quantity fed back is phase.

PLL is used to track the phase and frequency of the incoming signal.

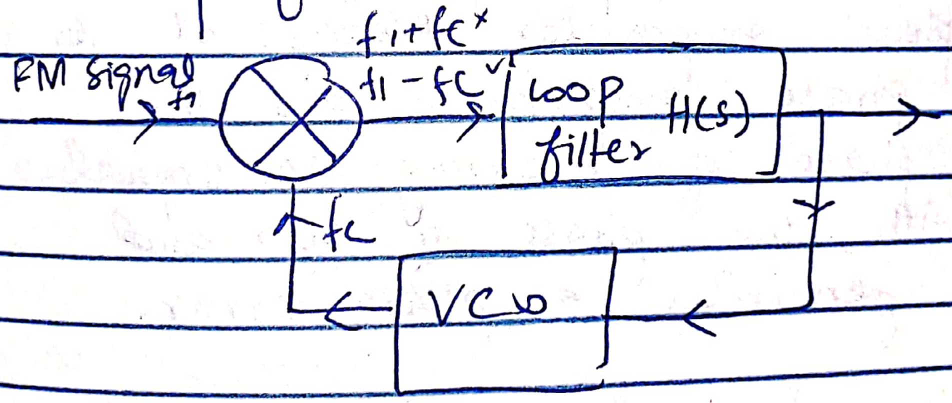
It is used to achieve frequency synchronism and phase synchronism.

There are 3 components

→ voltage controlled oscillator (VCO)

→ A multiplier serving as phase detector or phase comparator

→ loop filter which is basically LPF.



VCO:

→ is an oscillator whose frequency can be controlled by external source.

→ The oscillating frequency varies linearly with input voltage in VCO.

→ If input voltage to VCO is $e_o(t)$, its output voltage will be a sinusoid of frequency given as:

$$\omega(t) = \omega_c + c\, e_o(t)$$

ω_c → Free running frequency when $e_o(t) = 0$

→ The output of loop filter which is a voltage signal is fed to the input of the VCO.

Multiplier:

(Phase detector)

→ Multiplies the incoming FM by the output of VCO.

→ Also called phase detector or phase comparator because it do phase comparison b/w the phase of incoming FM signal with the phase of VCO and generates a phase error.

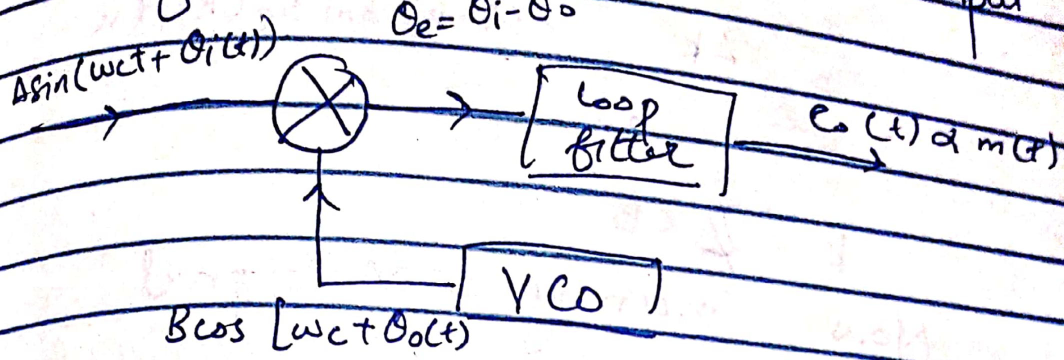
LPF: Loop filter:

→ Also call LPF

→ It filters high frequency components contained in multiplier output

→ It also produces a voltage proportional to the error signal.

→ The output of the loop filter which is voltage signal is fed to the input of VCO



After X -

$$= AB\sin(\omega_c t + \theta_i)\cos(\omega_c t + \theta_o)$$

$$= \frac{AB}{2}\left[\sin(\theta_i - \theta_o) + \sin\underbrace{(2\omega_c t + \theta_i + \theta_o)}_{\times \ \text{LPF}}\right]$$

$$= \frac{1}{2} AB \sin(\theta_i(t) - \theta_o(t))$$

convolution

$$e_o(t) = h(t) * \frac{1}{2} AB \sin[\theta_i(t) + \theta_o(t)]$$

$$c\, e_o(t) = \frac{1}{2} AB \int_0^t h(t-x)\sin[\theta_i(x) - \theta_o(x)]\,dx$$

$\longrightarrow c$ (ii)

$$\omega_{vco} = \omega_c + c\,e_o(t)$$

VCO output

$$B\cos(\omega_c + \theta_o(t))$$

$$\omega_{vco} = \omega_c + \dot{\theta}_o(t)$$

$$\dot{\theta}_o(t) = c\,e_o(t)$$

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$$\dot{\theta}_o(t) = \frac{1}{2} cAB \int_0^t h(t-x) \sin(\theta_i(x) - \theta_o(x)) dx$$

$$= Ak \int_0^t h(t-x) \sin \theta_e(x) dx$$

$k = \frac{1}{2} cB$

Now incoming FM Signal

$= A \sin [w_c t + \theta_i(t)]$

$$\theta_i(t) = k_f \int_{-\infty}^t m(\alpha) d\alpha$$

$$\theta_o(t) = k_f \int_{-\infty}^t m(\alpha) d\alpha - \theta_e$$

As

$\theta_e = \theta_i - \theta_o$

$\theta_o = \theta_i - \theta_e$

As Assume $\theta_e \ll$

$$\theta_o(t) = k_f \int_{-\infty}^t m(\alpha) d\alpha$$

As previously

$\dot{\theta}_o(t) = c e_o(t)$

$$e_o(t) = \frac{\dot{\theta}_o(t)}{c}$$

$$\theta_o(t) = k_f\, m(t)$$

Thus.

$$e_o(t) = \frac{k_f\, m(t)}{c}$$

So

$$e_o(t) \propto m(t)$$

Lecture # 11

2) Interference in Angle Modulated Wave

Assume we have unmodulated sinosidal carrier $A\cos w_c t$ interfere with $I\cos(w_c + w)t$

then

$$v(t) = A\cos w_c t + I\cos(w_c + w)t$$

$$= A\cos w_c t + I[\cos w_c t\cos wt - \sin w_c t \sin wt]$$

$$= A\cos w_c t + I\cos w_c t\cos wt - I\sin w_c t\sin wt$$

$$= [A + I\cos wt]\cos w_c t - I\sin w_c t\sin wt$$

$$= E_r(t)\cos[w_c t + \varphi_d(t)]$$

where $\varphi_d(t) = \tan^{-1}\left(\frac{I\sin wt}{A + I\cos wt}\right)$

where $(I \ll A)$

$$\varphi_d(t) = \frac{I}{A}\sin wt$$

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Instantaneous frequency

$$= \omega_c t + \dot{\varphi}_d(t)$$

$E_r(t)\cos(\omega_c t + \varphi_d(t)) \rightarrow$ Phase demodulator $\rightarrow y(d(t)) = \varphi_d(t)$

$E_r(t)\cos(\omega_c t + \varphi_d(t)) \rightarrow$ Frequency demodulator $\rightarrow y_d(t) = \dot{\varphi}_d(t)$

$$y_d(t) = \frac{I}{A}\sin\omega t \quad \text{for PM}$$

$$y_d(t) = \frac{I\omega}{A}\cos\omega t \quad \text{for FM}$$

$$\text{Interference} \propto \frac{1}{A}$$

So, angle modulated systems suppress weak interference better than AM.

→ Capture Effect:

For two transmitters with carrier frequency separation less than the audio range, the stronger carrier suppresses (captures) the weak carrier.

Hence in AM, the interference level should be kept below 35dB where as in FM it needs to be below 6dB

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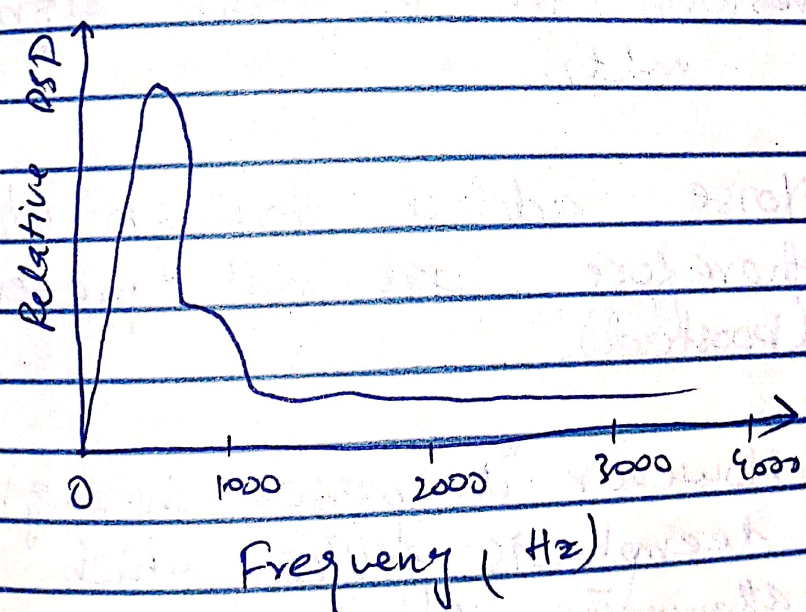
AM	FM
$A = 1W$ (Power)	(Carrier power) $A = 1W$
$I = \frac{1}{1000}$ W	$I = 0.25$ W
$10 \log (I/A) = -30 dB$	$10 \log \left(\frac{1}{4}\right) = -6 dB$

2) Interference due to Channel Noise :

→ Noise has a constant power spectral density i.e all frequency components have same amplitude (I)

→ Noise PSD is concentrated at higher frequency

→ Audio signal m(t) PSD is concentrated at lower frequencies (below 2.1 kHz)



2) PDE Process :

→ At the transmitter, the weaker high frequency components (beyond 2.1 kHz) of the audio signal are boosted before modulation by a Pre-emphasis filter of $H_p(j\omega)$.

→ A the receiver, the demodulator output is passed through a de-emphasis filter of transfer function $H_d(\omega) = 1/H_p(j\omega)$.
Thus, the de-emphasis filter undoes the pre-emphasis by attenuating the high frequency components (beyond 2.1 kHz) and thereby restore the original signal $m(t)$.

∴ Noise added through Channel therefore was not pre-emphasis (boosted).

→ However it passes through deemphasis filter which attenuates ~~its~~ high frequency component where most of the noise power is concentrated

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→ This the process of PDE leve the desired signal untouched but reduces the noise power considerably.

2→ Pre-emphasis & de-emphasis filter:

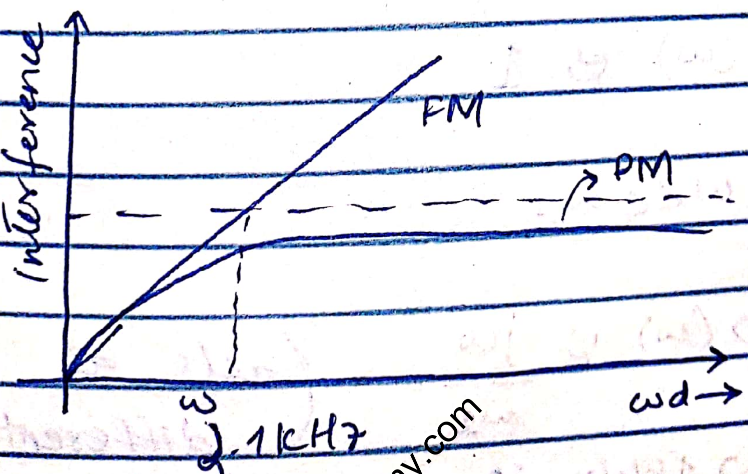
→ At lower frequencies, FM has smaller interference.

$$y_d(t) = \frac{I\omega \sin \omega t}{A}$$

→ At higher frequencies, PM has smaller interference

we want our system to behave like FM at lower frequencies and PM at higher frequencies.

→ This is accomplished by Pre-emphasis and De-emphasis filter.



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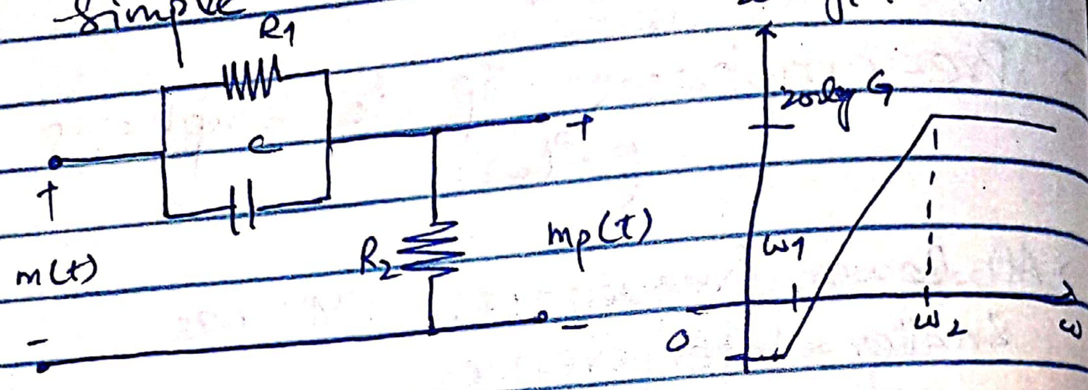
→ Pre-emphasis Filter :

- These filters can be realized by simple RC circuit.



$\omega_1 = 2.1 kHz$

$\omega_2 = 30 kHz$

$$Hp(\omega) = k \frac{j\omega + \omega_1}{j\omega + \omega_2} \quad , \quad k = gain = \frac{\omega_2}{\omega_1}$$

$$Hp(\omega) = \left(\frac{\omega_2}{\omega_1}\right) \frac{j\omega + \omega_1}{j\omega + \omega_2}$$

for lower frequency $\omega \ll \omega_1$

$$Hp(\omega) \simeq 1$$

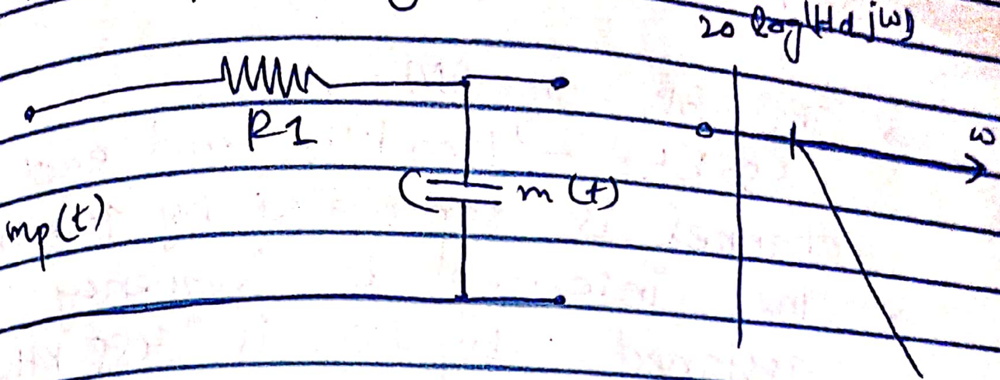
$$\omega_1 \ll \omega \ll \omega_2$$

$$Hp(\omega) \simeq \frac{j\omega}{\omega_1}$$ (acts at differentiator)

So 0 – 2.1 kHz is FM

2.1 — 3 kHz is PM

→ De-emphasis filter :-



$$Hd(\omega) = \frac{\omega_1}{j\omega + \omega_1}$$

$\omega << \omega_2$ $\quad Hp(\omega) \approx \frac{j\omega + \omega_1}{\omega_1}$

$Hp(\omega) \cdot Hd(\omega) \approx 1$

0 — 15 KHz

overall we are able to recieve the signal.

2) Superheterodyne AM Receiver:

→ The BW of AM is 550 kHz — 1600 kHz and each channel is separated by 10 kHz

→ The intermediate frequency assigned by FCC is 455 kHz

$$f_{if} = 455 \text{ KHz}$$

→ The process of mixing two signals of different frequencies to produce a new frequency is called as heterdyning.

→ The superheterodyne receiver converts high frequency signal to a fixed lower frequency which is called as intermediate frequency.

↳ It consists of 5 sections

i) - RF sections

ii). A frequency converter

iii). An intermediate frequency (IF) amplifier

iv). An envelope detector

v) - Audio amplifier.

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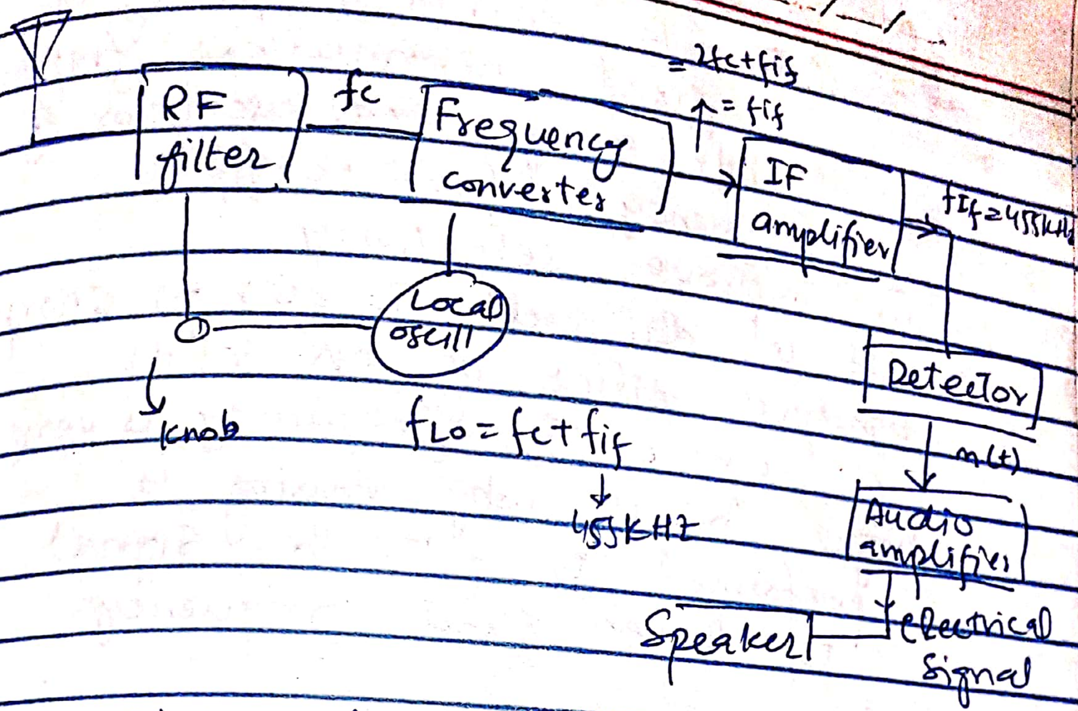
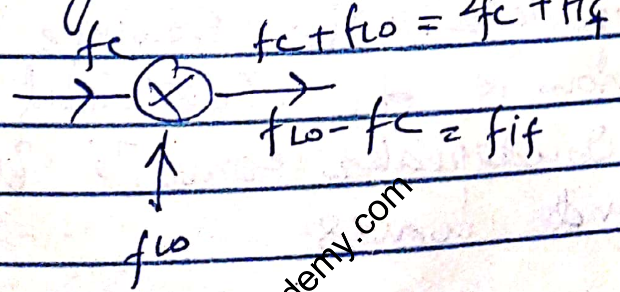


→ Receiving Antenna: Recieves EM signal and converts EM signal into electrical signal.

→ RF Section:

- Tunable Filter & amplifier.
- Picks up the desired frequency band (desired station), rejects other frequencies (other stations)
- Also amplify signal

→ frequency Converter:



→ Translate the carrier frequency f_c to a fixed intermediate frequency of 455 kHz using local oscillator of frequency $f_{LO} = f_c + f_{if}$

- To improve selectivity
- It is difficult to design sharp bandpass filter of BW 10kHz if centre frequency is very high. That is why mixing is performed to get the signal to lower fixed frequency.

2→ IF Section

→ Suppress adjacent channel interference

→ Amplify Signal

→ Envelop dector: Perform the demodulation to extract the message signal from carrier signal.

→ Audio Amplifier: Amplifies the message signal

→ Loudspeaker: The message signal till now is an electrical signal. The loudspeaker converts it to sound waves.

→ Image Frequency:

$$2F_{if} = 2 \times 455 kHz = 910 kHz$$

$$F_{image} = f_c + 2f_{if}$$

Assume

$$= 600 kHz + 2 \times 455 kHz$$
$$= 600 kHz + 910 kHz$$
$$= 1510 kHz$$

Let

$f_c = 600 kHz$	$f_c = 1000 kHz$	$f_c = 1510 kHz$
$f_{LO} = f_c + f_{if} = 1055 kHz$	$f_{LO} = 1455 kHz$	$f_{LO} = 1055 kHz$
$f_{if} = \|f_{LO} - f_c\| = 455 kHz$	$f_{if} = \|f_{LO} - f_c\| = 455 kHz$	$f_{if} = \|f_{LO} - f_c\| = 455 kHz$

So $f_c = 1510 kHz$ is called the image of $f_c = 600 kHz$

So, we perform Image frequency Suppression (910 kHz) and is done by RF section.

Why Superheterodyne?

$f_{LO} > f_c$

→ Smaller tuning range Smaller tuning ratio

550 kHz —— 1600 kHz

95 kHz — 1145 kHz 1005 kHz —— 2055 kHz

$\frac{1145}{95} = 12.05$ $\frac{2055}{1005} = 2.045$

2) Characteristics of Good Reciever:

→ Selectivity
→ Sensitivity
→ Fidelity

(1) Selectivity:
• The ability of a device to respond to particular frequency without interference from other frequencies.
• So how well the receiver picks up the right frequency without interference of other frequency.

• Greater selectivity implies greater rejection of unwanted signal frequencies so less interference.
• The ability to separate desired station from undesired one.

(2) Sensitivity:
The sensitivity of a reciever is the measure of the ability of reciever to pick weak signal and amplify it.

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So it is measure of the minimum signal strength that a receiver can detect.
Good Sensitivity means that a reciever can detect low power signal and process it successfully

2) Fidelity:
Ability of reciever to reproduce all the frequency components of the message signal
High fidelity implies less distortion.

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Stereophonic FM BroadCasting:

→ The FM broadcasting range is from 88 MHz - 108 MHz with a separation of 200 kHz b/w adjacent stations and a peak frequency of 75 kHz.

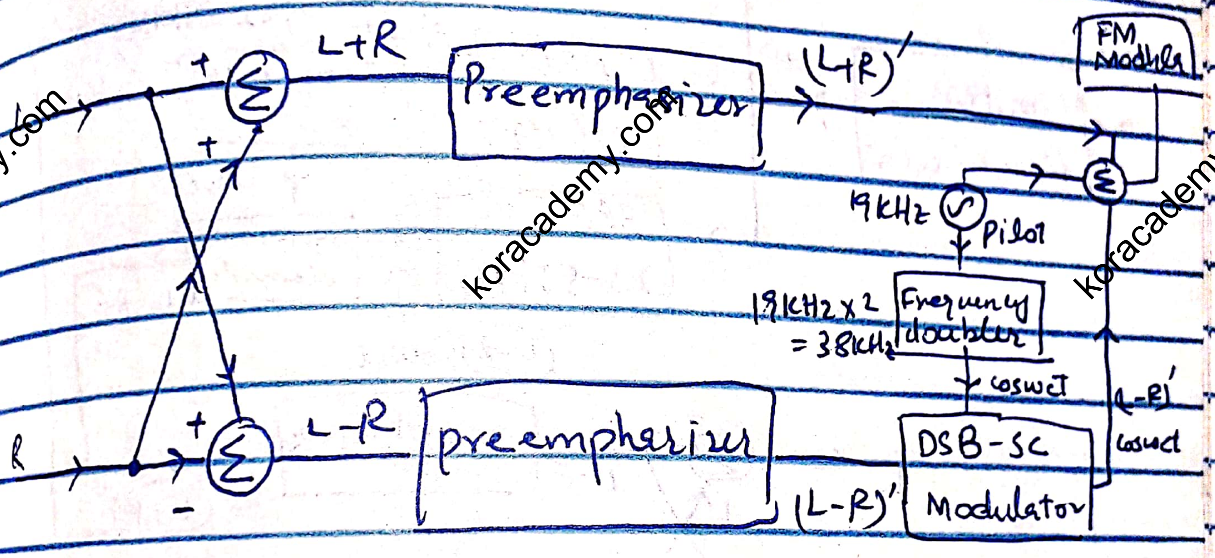
→ The intermediate frequency assigned by FCC is 10.7 MHz

→ A monophonic FM reciever is identical as super heterodyne AM reciever except that the envelope detector is replaced by frequency discriminator or PLL followed by a de-emphasizer.

→ Earlier FM broadcasting were monophonic. Sterophonic FM broadcasting was proposed later. FCC ruled out that stereophonic System had to be compatible with original monophonic system.

FM Stereo Transmitter:

→ So in FM steror transmitter we have Two signals
→ 'L' which is left signal intended for the left receptor i.e left ear
→ 'R' which is right signal intended for the right receptor i.e right ear.

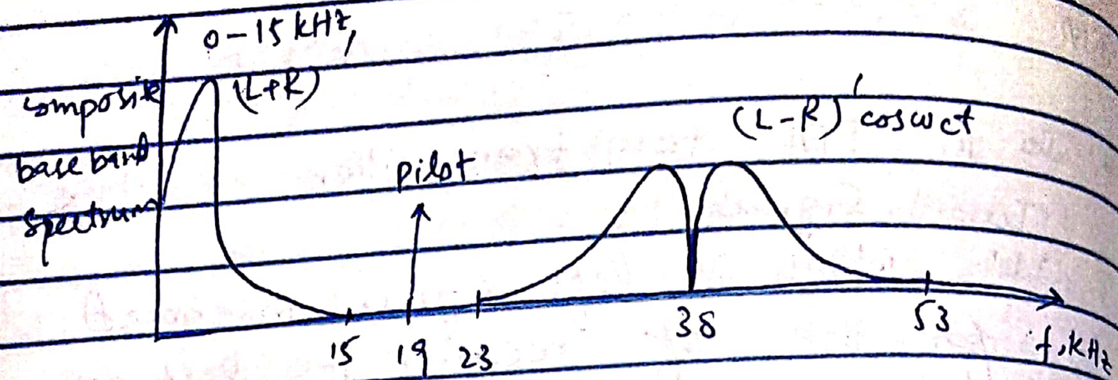


$m(t) = (L+R)' + \text{pilot} + (L-R)' \cos w_c t$

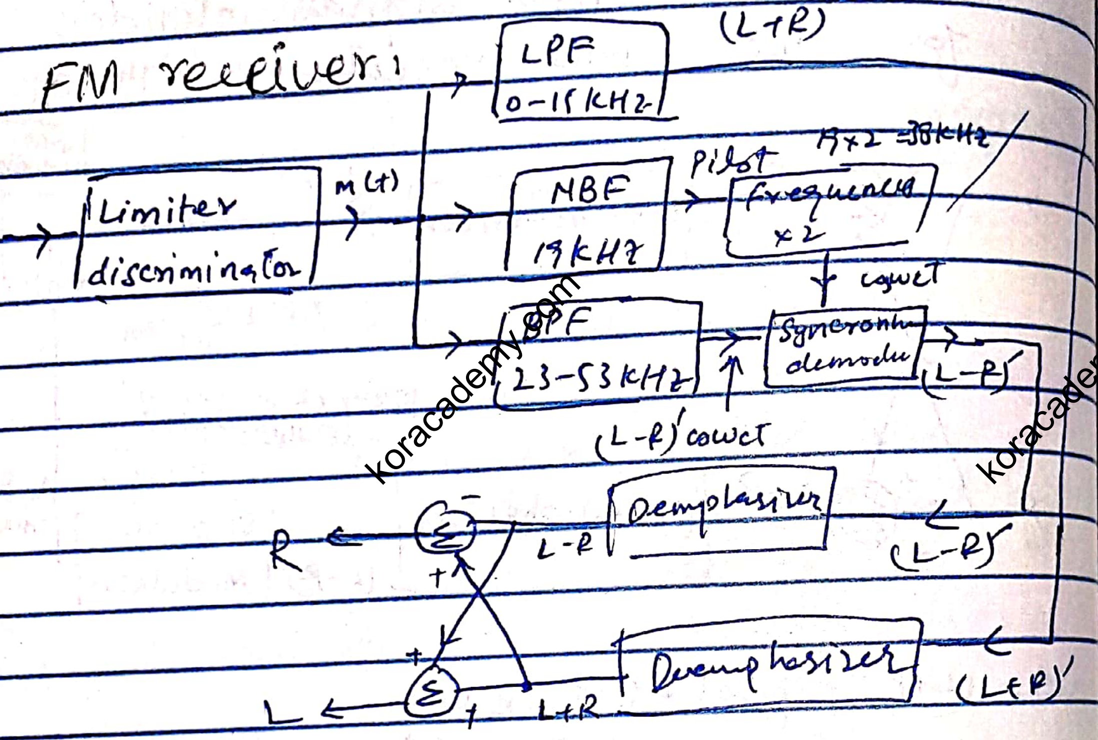
amplitude of pilot

$m(t) = (L+R)' + \alpha \cos \frac{w_c T}{2} + (L-R)' \cos w_c t$ ↳ Base band signal.

The reason for using a pilot of 19kHz rather than 38kHz is that it is easier to separate the pilot at 19kHz because there is no signal component within 4kHz of that frequency.



FM receiver:



→ A pilot is a single frequency signal transmitted over a communication system and helps in regeneration of carrier signal on the receiver side. By doing so we will be able to coherently demodulate the signal (L-R)' cosωct

→ If no 19 kHz pilot tone is present, then any signal in the 23–53 kHz range are ignored by a stereo reciever.

2) The peak amplitude of the composite message signal in stereo broadcaste is almost the same as that of monophonic signal (if we ignore pilot amplitude). That's why the frequency deviation which is proportional to peak signal amplitude remains almost the same.

$$\Delta f \propto \text{Peak amplitude } (A_p)$$

$$|m(t)|_{max} = A_p + A_p + \alpha$$

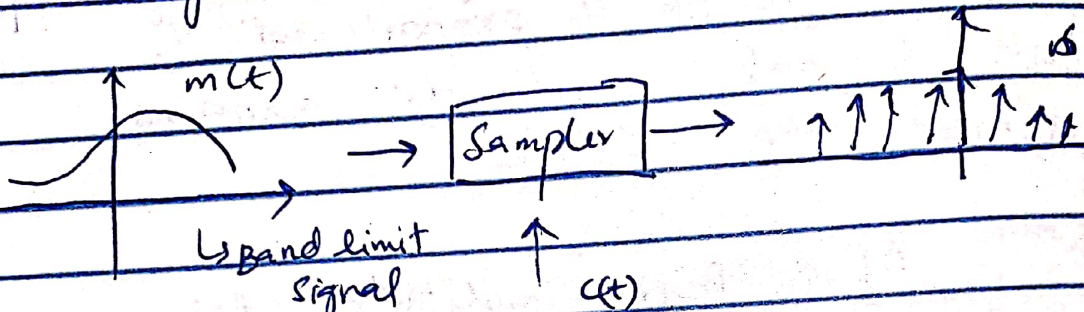
$$|m(t)|_{max} = 2A_p + \alpha$$

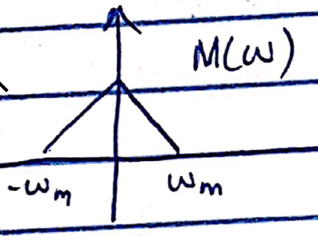
Monophonic

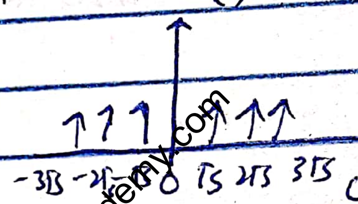
$$|m(t)|_{max} = 2A_p$$

→ Sampling & Sampling Theorem:

→ used to convert continuous time signal to discrete time signal.







$$c(t) = \sum_{-\infty}^{\infty} \delta(t - nT_s)$$

T_s = Sampling period

$$\omega_s = \frac{2\pi}{T_s} = \text{Sampling frequency}$$

$$s(t) = m(t) * c(t)$$

$$\therefore C(\omega) = \omega_c \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)$$

$$S(\omega) = \frac{1}{2\pi}\left[M(\omega) * C(\omega)\right]$$

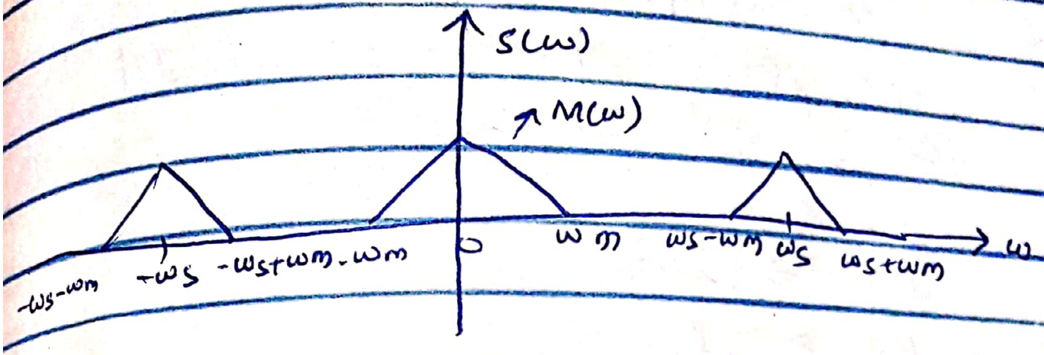
$$= \frac{1}{2\pi}\left[M(\omega) * \omega_c \sum_{-\infty}^{\infty} \delta(\omega - n\omega_s)\right]$$

$$S(\omega) = \frac{\omega_s}{2\pi}\left[\sum_{n=-\infty}^{\infty} M(\omega) * \delta(\omega - n\omega_s)\right]$$

$$\therefore \quad x(t) * \delta(t - t_1) = x(t - t_1)$$

$$S = \frac{1}{T_S}\left[\sum_{n=-\infty}^{\infty} M(\omega - n\omega_s)\right]$$

$$S(\omega) = \frac{1}{T_S}\left[\cdots + M(\omega+\omega_s) + M(\omega) + M(\omega-\omega_s) \cdots\right]$$



(i) $\omega_s - \omega_m > \omega_m$

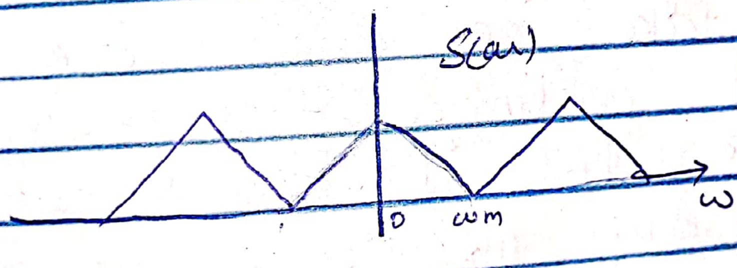
$\omega_s > 2\omega_m$

ω_s → Sampling frequency

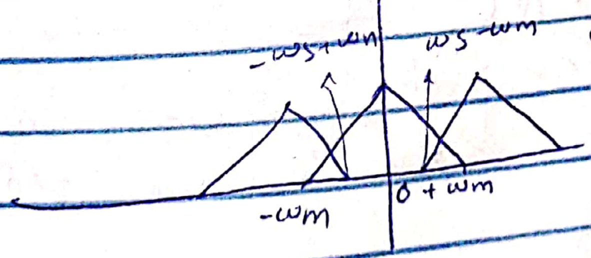
ω_m → Maximum frequency of msg

(ii) $\omega_s - \omega_m = \omega_m$

$\omega_s = 2\omega_m$



(iii) $\omega_s < 2\omega_m$



overlaping

Sampling theorem: (Nyquist)

To recover the original signal from the sampled signal the sampling frequency should be ≥ to the $2\omega m$.

$$\omega s \geq 2\omega m$$

$$\rightarrow fs \geq 2fm$$

↳ Pulse Code Modulation: (PCM)

convert Analog ⟶ Digital.

- An analog signals amplitude can take on any value in the continuous range. This means that it can take on any infinite number of values.
- On the other hand digital signal amplitude can only take a finite number of values.

There are 3 basic steps in PCM

(a) - Sampling
(b) - Quantization
(c) - Encoding

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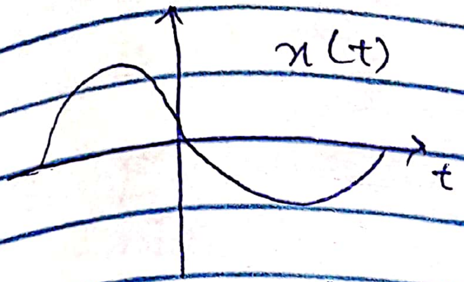
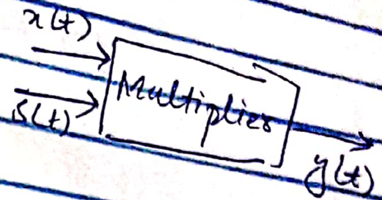
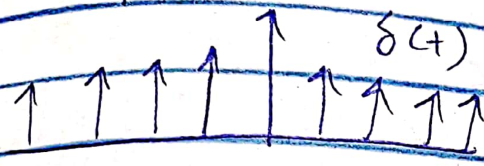
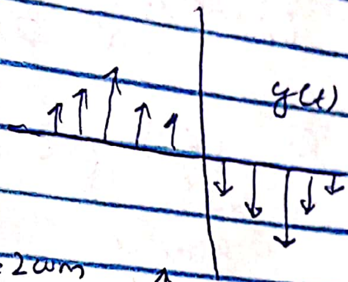
→ Sampling:

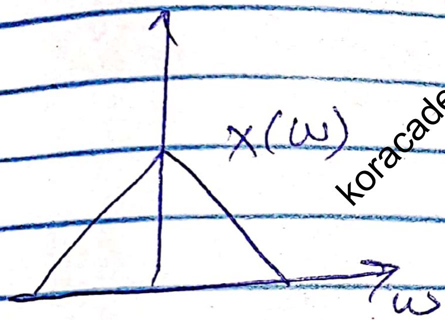
- convert continuous time signal to discrete time signal
- Nyquist rate is followed to recover the signal back : $f_s \geq 2f_m$



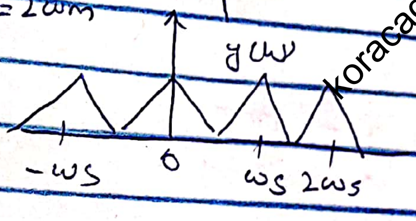








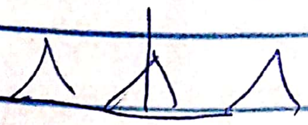
: $\omega_s = 2\omega_m$



→ Aliasing Effect:

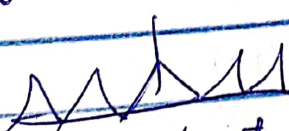
The ovelapping region in case of under sampling represents aliasing effect, which can be removed by

- $f_s > 2f_m$
- By using anti-aliasing filter



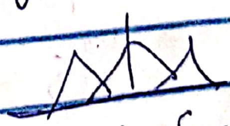
over-sampling

$f_s > 2f_m$



perfect

$f_s = 2f_m$



under sampling

$f_s < 2f_m$

2) Quantization:-

→ Rounding off the values of to one of the closest permissible number (or quentized levels)

Range → $-m_p$ to m_p

$$\Delta V = \frac{2m_p}{L}$$

$\Delta V = q = m$

$L = 16$ $q = 2$

step size

2) Encoding :-

Because we have $L = 16$ So, each of the quantized level will be assigned a 4 digit Binary code

0 → 0000
1 → 0001
2
3
4
5
6
7
8
9
10
11
12
13
14
15 → 1111

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PCM

→ The audio signal b/w is about 15 kHz but above 3.4 kHz signals can be suppressed with effecting the audio signal

→ In this case, sampling rate is taken as 8 kHz, instead of 6.8 kHz Nyquist rate to avoid unrealized filter required for signal reconstruction

→ Each sample is finally quantized into 256 levels which requires a group of 8 binary impulse to encode each sample thus telephone signal requires 8000×8 = 64000 binary pulses per second

→ The compact Disc is also an application of PCM

→ The sampling rate is 44.1 kHz and number of levels are 65,536.

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2) Advantages of Digital Communication

→ Can withstand channel noise, distortion much better than analog as long as they are within limits.

→ The amplifiers used in analog, amplifies signal and also noise. In digital, regenerative repeaters are used. This allows to carry transmission over longer distances.

→ Hardware implementation is flexible and easy because of microprocessor, digital switching and large scale integrated circuit.

→ Simple implimentation of error correction techniques

→ Multiplexing is easy

→ Inherently more efficient than analog in exchanging SNR for bandwidth

→ Digital storage is easy

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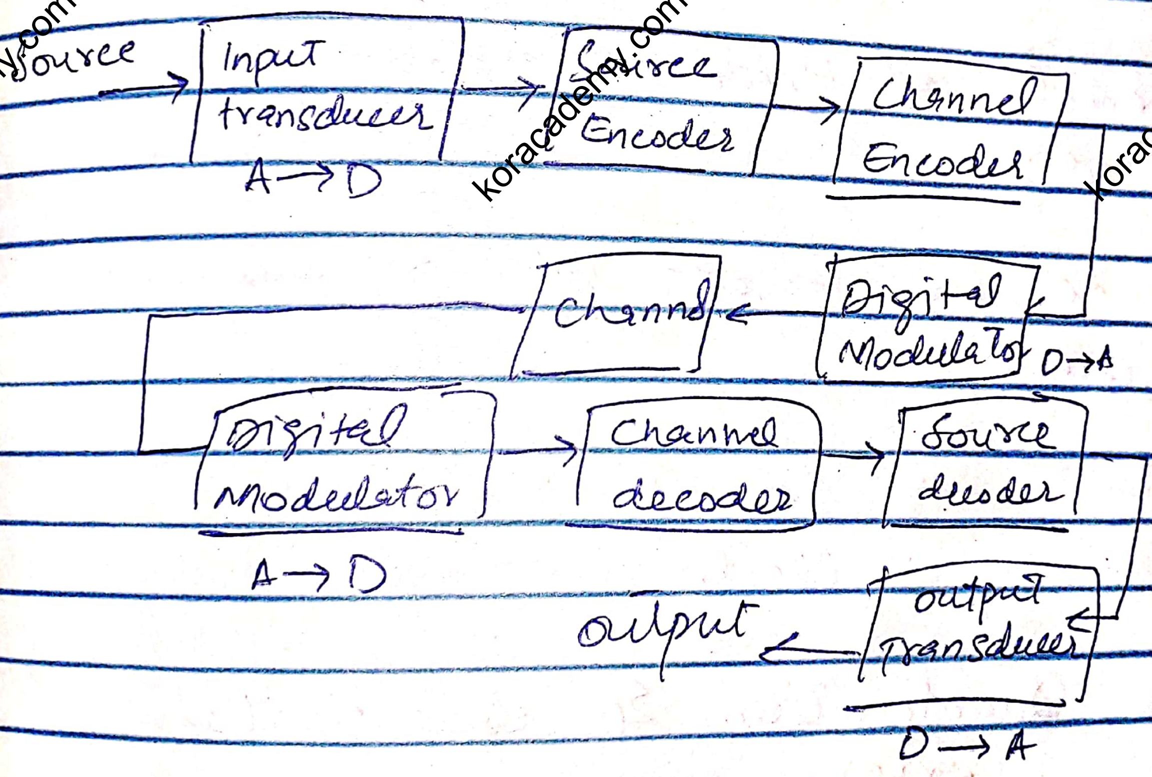
→ Reproduction of digital signals is extremely easy and reliable without deterioration.

→ Digital systems are expanding at a rapid rate so the cost of hardware continues to halve every 3 years while performance capacity double the same time.



Source → Sound

Input T → Converts To E-signal A↔D

Source Encoder → Compression of data i.e redundant bits are removed

This helps in effective utillization of BW

Day: M T W T F S

Date. _ _/ _ _/ _ _

Channel Encoder → error detection & correction for this purpose it adds redundant bits

Digital Modulator → D → Analog

Channel → Medium

Digital demod → A → D

Channel decoder → Error correction

Source decoder → converts bits → ~~wavy~~ waves

output transducer → Digital → Analog
Electrical → Sound.

2) Quantization & Quantization error.

→ In quantization we limit the amplitude of the message signal & m(t) to the range (mp, -mp)

→ The amplitude signal range is divided into L uniformly spaced

→ The Sample value is approximated by the midpoint of the interval in which it lies

→ Increasing number of quantization levels reduces step size which decreases quantization error.

$$\boxed{\frac{\Delta v}{2} = \frac{\tau}{2}} \rightarrow \text{maximum quantization error.}$$

Example 6.2:

BW = 3KHz

Sampled rate = 33-1/3 % higher than Nyquist rate

Sol: BW = 3KHz

Nyquist rate = 2 × 3kHz = 6 kHz

Sampling rate = $6KHz + \left(\frac{33.3}{100} \times 6kHz\right)$

= 8 kHz

$$\frac{0.5\, mp}{100} = \frac{\Delta v}{2} \quad (\text{error})$$

$$\therefore \Delta v = \frac{2mp}{L}$$

$$\frac{mp}{L} = \frac{0.5}{100} mp$$

$$\boxed{L = 200} \text{ levels}$$

L = 256 quantized levels

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n-bit encoder

$2^n = 256$

$2^8 = 256$

$n = 8$

$n = \log_2 256$

$\boxed{n = 8}$

we need bits per sample

$C = 8 \times 8000 = 64{,}000$ bit/sec

2 bits/sec per Hz

$BW_{(min)} = \frac{C}{2} = \frac{64000}{2} = 32\ kHz$

(b) $C_M = 24 \times 64000$

$= 1.536$ M bits/sec

$BW_M = \frac{1.536}{2} = 0.768$ M Hz

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2) Non-uniform quantization & Companding.

compressing + expanding

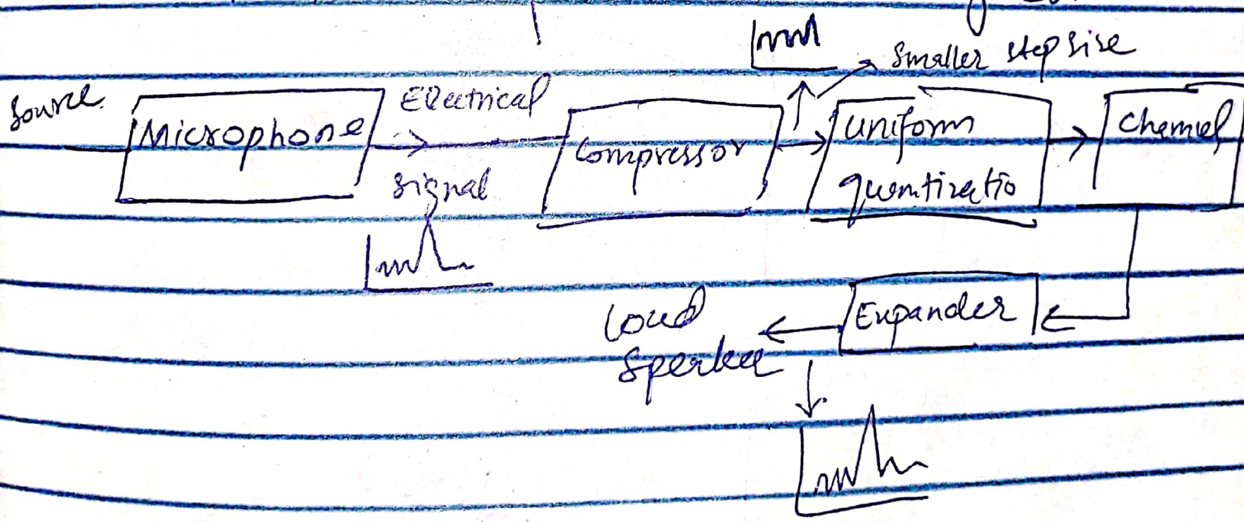
→ In uniform quantization, we have constant step size where quantization error is

$$\frac{\Delta V}{2} = \frac{q}{2} = \frac{m_p}{L} \qquad L = \text{no. of quantized level}$$

→ In non-uniform quantization, we have variable step size

→ Companding is used to achieve non uniform quantization

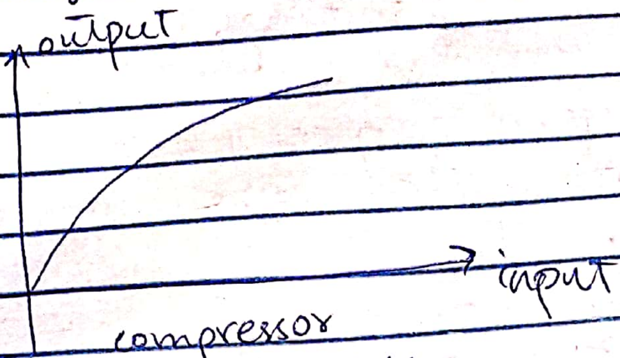
→ The process of companding is to compress the signal, perform uniform quantization and then expand the signal.



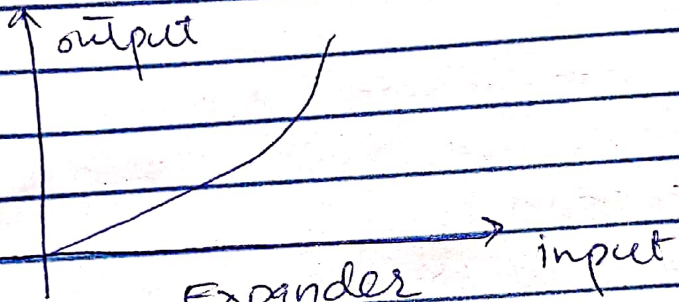
→ Quantization error is proportional to step size

→ Smaller step size, less quantization error/noise, which implies better SNR

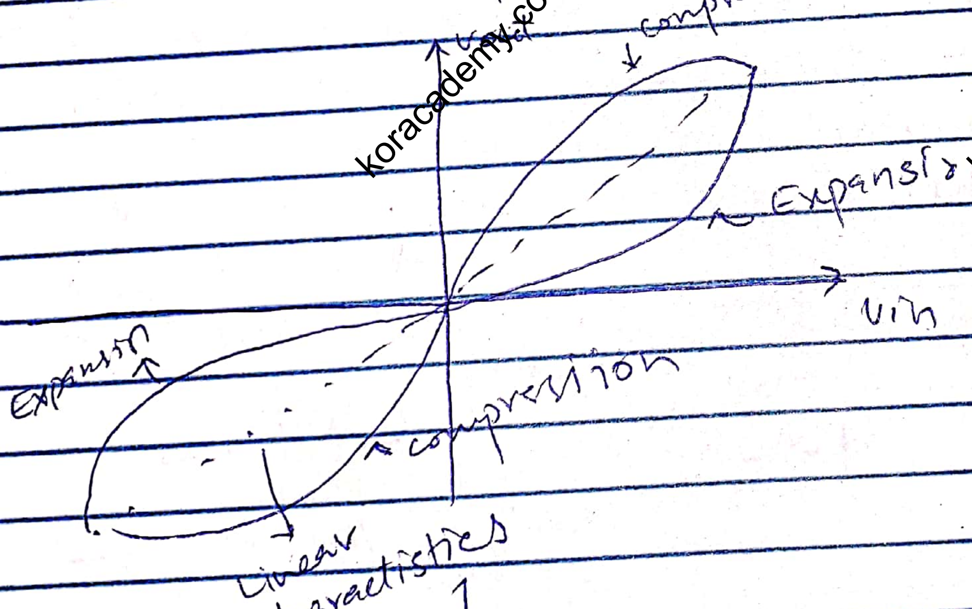
∴ SNR (Signal to Noise Ratio)

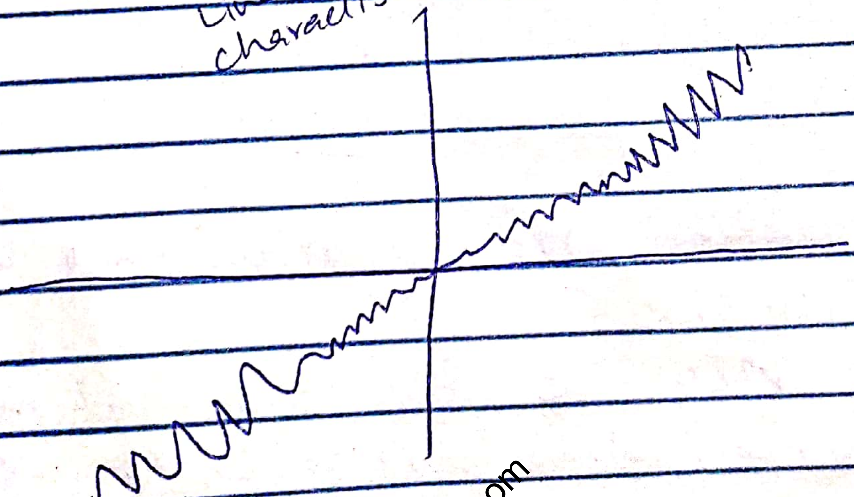


compressor characteristic



Expander





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→ The effect of quantization noise can be reduced by increasing the number of quantization interVals in low amplitude region as compared to high amplitude region

2. Compression Laws.

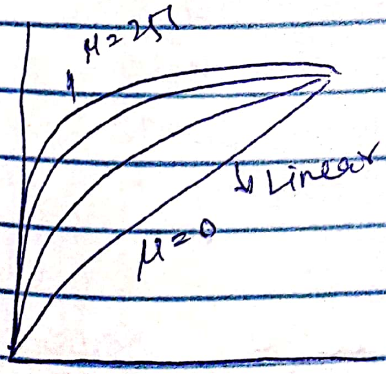
μ-Law: Used in North America & Japan

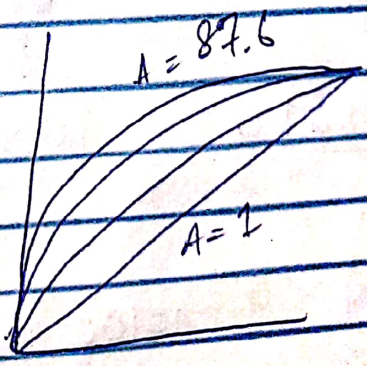
A-Law = used in Europe and rest of world

→ μ & A ~~laws~~ are compression parameters and ~~eliminates~~ determines the degree of compression.

→ The range of μ is 0-255 while the rage of A is 1-100

→ In case μ the optimum value is 288 while in " A the " " " 87.6





→ Increas L
→ Increase μ or A

SNR improves.

2> ASCII code:

→ American standard code for information interchange

→ ASCII code is a seven bit code so range is from 0 – 127

1 1 1 1 1 1 1 → 127

2^6 2^5 2^4 2^3 2^2 2^1 2^0

64 + 32 + 16 + 8 + 4 + 2 + 1 = 127

Do character coding of "RED" using 32-ary communication?
(5-bit)

R	E	D
82	69	68
1010010	1000101	1000100

64 0 16 0 0 2 0 = 82

10100	10100	01011	00010	0000
20	20	11	2	0

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2) Line Coding Scheme:

→ Converting digital data to digital Signal

→ Digital to digital communication

→ Converting sequence of bits into digital signal and then again converting digital signal back to digital data.

Necessary Characteristics of LC:

→ Small transmission B/W
→ low " power
→ Error correction & detection ability
→ No DC component.
→ Should by self-synchronized

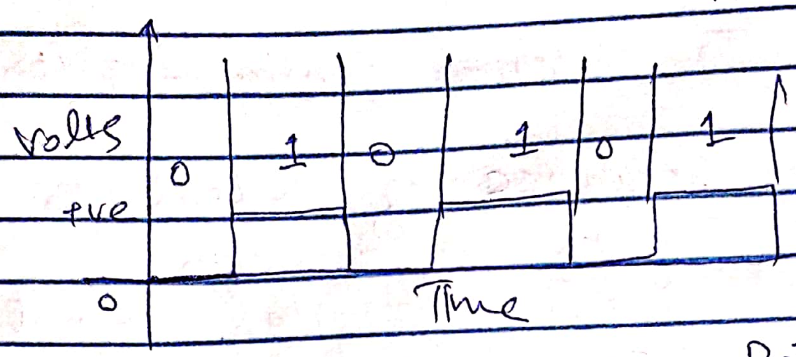
Types

1) - Unipolar (+ve, 0)
2) - Polar (+ve, -ve)
3) - bi-polar (+ve, 0, ∓ve)

(1) - uni-polar (ON-OFF keying)

1 represents +ve voltage:
0 " 0 "

Non-return to zero

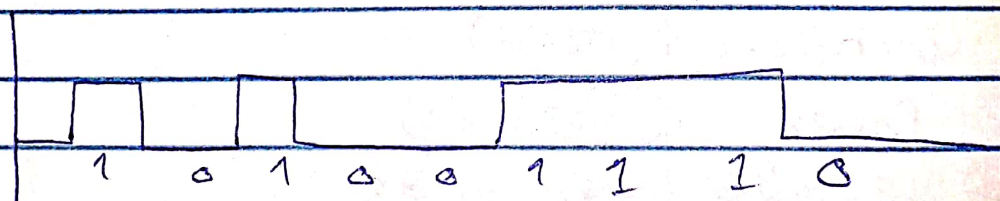


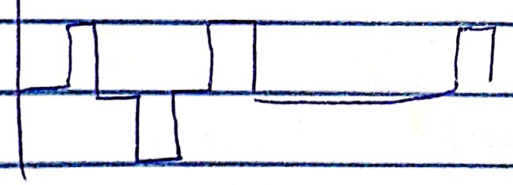
Return to zero
0 — 0 V
1 → +ve V for half time

(2) - Polar

0 — -ve
1 — +ve } NRZ

1 — +ve → half time return to zero
0 — -ve → " " " " "





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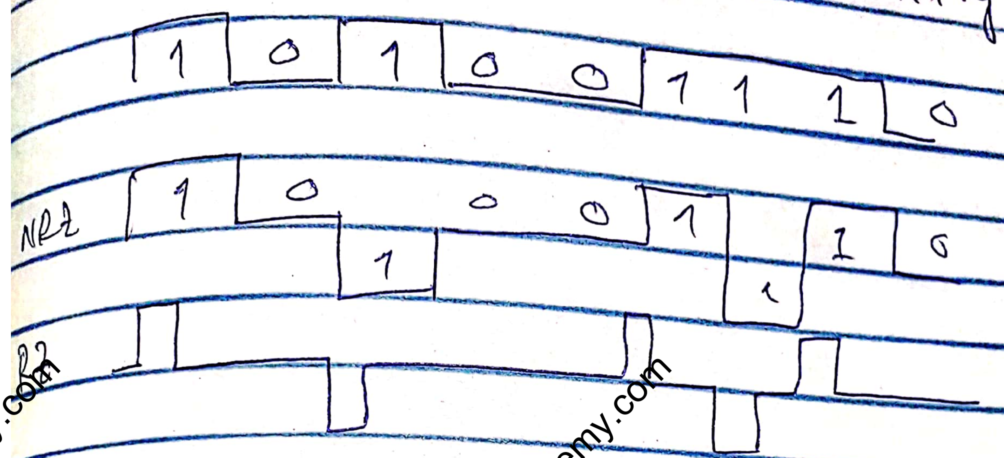
2) Bi-polar

(+ve, 0, -ve)

Alterate mark inversion (AMI)
called ternary in LAB
Sometimes called pseudo-ternary



2) Mächester

logic-1 has 2 halves -- -ve for first half
+ve " second
logic-0 is +ve for first half Se
-ve " second "